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Algebraic Approach to the Problem of Fault Accommodation in Nonlinear Systems

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Abstract: The problem of fault accommodation is considered for a class of nonlinear systems whose purpose is tracking the prescribed trajectory. In the framework of solving above problem, two main tasks are investigated involving algebraic tools (algebra of functions). The first task is nonlinear model reduction followed by adaptive observer design for fault estimation. The second one is asymptotic model matching to accommodate the faults.

1. INTRODUCTION

An increasing demand on reliability and safety for critical purpose control systems calls for the use of fault tolerant control (FTC) techniques. The goal of FTC is to determine such control law that preserves the main performances of the system in faulty case while the minor performances may degrade. An active approach to FTC is associated with on line fault detection and estimation followed by control law accommodation (Patton, 1997; Chen and Patton, 1999; Blanke *et. al.*, 2003). During last years different solutions were proposed to FTC problem involving the methods of optimal control (Staroswiecki, *et. al.*, 2006), H-infinity optimization (Weng *et. al.*, 2006), model matching (Staroswiecki, 2005), adaptive control (Jiang *et. al.*, 2003).

This paper deals with a class of systems whose purpose is tracking the prescribed trajectory. Different kinds of technical objects such as missiles, vehicles and manipulators belong to this class. The problem under consideration is formulated as follows: for a given reference model of the system and a given control it is necessary to find a new control such that the output of faulty system is asymptotically converging to the corresponding output produced by the reference model. Above problem is closely related to asymptotic model matching whose solution for nonlinear affine systems has been considered by Isidori (1989) within the framework of geometric approach. Nonlinear system transformation to so-called normal form is a basis for this solution.

In this paper, algebraic approach (algebra of functions) is proposed to solve FTC problem for nonlinear dynamic systems. It will allow extending the solution of asymptotic model matching task on the class of nonlinear systems that is more general than affine ones. Notice that algebraic approach was applied earlier for solving different tasks dealing with nonlinear system analysis, transformation and fault diagnosis (see e.g. Zhirabok and Shumsky, 1987; 1989; 1993; so on).

An idea of solution considered in the paper is: firstly, to extend the state space of nonlinear system by parameters subjected to fault action, secondly, to use an adaptive observer for estimating both system states and parameters and, thirdly, to accommodate the faults involving asymptotic model matching techniques. Solution of the last two tasks is based on nonlinear model transformations. As a result, studying these transformations is the central point of investigations.

The paper is organized as follows. In Section 2, description of FTC method is given. Section 3 is devoted to algebraic methods for nonlinear model transformations. Illustrative example is given in Section 4. Section 5 concludes the paper.

2. FTC METHOD DESCRIPTION

Consider the system specified by equations of the form

$$\dot{x}(t) = f(x(t), u(t), \vartheta(t)), \quad y(t) = h(x(t)) \tag{1}$$

where $x(t) \in X \subseteq R^n$ is the vector of state, $u(t) \in U \subseteq R^p$ is the input vector, $y(t) \in Y \subseteq R^l$ is the output vector, $\vartheta(t) \in \Theta \subseteq R^q$ is the parameter vector given for fault representation, f and h are nonlinear vector functions assumed to be smooth. Let for healthy system $\vartheta(t) = \vartheta^0$, where ϑ^0 is the nominal value of the parameter vector. The model (1) under $\vartheta(t) = \vartheta^0$ is called nominal or reference system model and is written as follows

$$\dot{x}(t) = f(x(t), u(t)), \quad y(t) = h(x(t)) \,. \tag{2}$$

Faults in the system result in $\vartheta(t) \neq \vartheta^0$. It is assumed that after abrupt fault distorted parameter vector takes constant value ϑ^f.

Assume that the output of reference model (2) under control $u(t)$ provides the desirable system trajectory. Consider a way of the new control $u^{'}(t)$ generation such that the output of the system (1) with $\vartheta(t)=\vartheta^f$ is asymptotically converging to the corresponding output produced by the reference model.

Let

$$\dot{\vartheta}(t)=0 \tag{3}$$

and introduce extended system model (1), (3). This model is considered as the base for adaptive observer design as well as for asymptotic model matching. Notice in advance, that adaptive observer design needs in observability of the extended model (see Section 3). If it is not a case, using appropriate coordinate transformation

$$z(t)=\varphi(x(t),\vartheta(t)) \tag{4}$$

one may find equivalent (in the sense of input-output behaviour) observable model of the form

$$\dot{z}(t)=f_*(z(t),u(t)), \quad y(t)=h_*(z(t))\,. \tag{5}$$

For the model (5), consider equivalent transformation to normal form

$$\dot{\xi}^i_{r_i}(t)=g^i(\xi(t),u(t)), \quad \dot{\xi}^i_j(t)=\xi^i_{j+1}(t),$$

$$y_i(t)=\xi^i_1(t), \quad 1\le i\le l, \quad 1\le j\le r_{i-1},$$

$$\dot{\xi}^{l+1}(t)=g^{l+1}(\xi(t),u(t)) \tag{6}$$

with

$$\xi(t)=\begin{pmatrix}\xi^1(t)\\ \vdots \\ \xi^l(t)\\ \xi^{l+1}(t)\end{pmatrix}=\begin{pmatrix}\psi^1(z(t))\\ \vdots \\ \psi^l(z(t))\\ \psi^{l+1}(z(t))\end{pmatrix}=\psi(z(t))\,, \tag{7}$$

$$\sum_{i=1}^{l} r_i \le n_z$$

where n_z the dimension of the model (5) state vector. Denote

$$g(\xi,u')=\begin{pmatrix}g^1(\xi,u')\\ \vdots \\ g^l(\xi,u')\end{pmatrix}$$

and assume that for every ξ, $\xi=\psi(\varphi(x,\vartheta))$, $x\in X$, $\vartheta\in\Theta$, $u'\in U$

$$\operatorname{rank}\frac{\partial g(\xi,u')}{\partial u'}=p\,.$$

In this case nonlinear equation

$$g(\xi,u')=w \tag{8}$$

is solvable for u'. Let solution of (8) is written in the form

$$u'=\eta(\xi,w)\,. \tag{9}$$

Taking

$$w_i(t)=g^i(\psi(\varphi(x_{(R)}(t),\vartheta^0)),u(t))+$$
$$\sum_{j=1}^{r_i} c^i_j(\psi'^i_j(\varphi(x_{(R)},\vartheta^0))-\xi^i_j(t)), \quad 1\le i\le l \tag{10}$$

one obtains from (4), (6) – (8)

$$y^{(r_i)}_{(R)}(t)-y^{(r_i)}(t)=\sum_{j=1}^{r_i} c^i_j(y^{(j-1)}_{(R)}(t)-y^{(j-1)}(t))\,.$$

Subscript (R) in (10) and below accompanies the vectors of the reference (nominal) model (2). Thus, an appropriate choosing the roots δ^i_j, $1\le j\le r_i$, $1\le i\le l$, of characteristic equations

$$\prod_{j=1}^{r_i}(\delta-\delta^i_j)=\delta^{r_i}+c^i_{r_i}\delta^{r_i-1}+\ldots+c^i_2\delta+c^i_1, 1\le i\le l,$$

makes available asymptotical convergence of the system output to the corresponding output produced by the reference model (2).

So, the new control may be found by substituting (10) into (9). Under this, as soon as the vector ξ is immediately unavailable, its estimation obtained by adaptive observer has to be involved.

3. NONLINEAR MODEL TRANSFORMATIONS

This section deals with the tasks related to the realization of FTC method discussed above. It starts with brief introduction into algebraic tools in use. Then, the methods for extended model reduction and nonlinear transformation to normal form are consistently considered.

3.1 Mathematical preliminaries

Denote $\Im_S$ the set of smooth vector functions with domain S. For $\alpha,\beta\in\Im_S$, partial preordering relation $\le$ is defined as follows: $\alpha\le\beta$ if and only if there exists some differentiable function γ determined on the set of values of α such that $\beta=\gamma(\alpha)$. To verify if $\alpha\le\beta$, one can check the equality of ranks for functional (Jacobian) matrices $\partial\alpha/\partial s$ and $\partial(\alpha^T,\beta^T)^T/\partial s$: $\alpha\le\beta \Leftrightarrow \operatorname{rank}(\partial(\alpha^T,\beta^T)^T/\partial s)=\operatorname{rank}(\partial\alpha/\partial s)\ \forall s\in S$. If $\alpha\le\beta$ and $\beta\le\alpha$ then α and β are equivalent: $\alpha\cong\beta$. The relation $\cong$ splits the set $\Im_S$ on equivalent function classes. The set of equivalent function classes corresponds to the partial ordering set of partitions of S, and the first is a grid with zero, given by arbitrary one-to-one function (in particular, the identity function

$i(s) = s \ \forall s \in S$), and unity, given by arbitrary constant function ($c(s) = \mathrm{const} \ \forall s \in S$). The binary grid operation $\alpha \times \beta$ is defined as the operation of finding the maximum bottom for the functions α and β. This operation is calculated in a simple way: $\alpha \times \beta = (\alpha^{\mathrm{T}}, \beta^{\mathrm{T}})^{\mathrm{T}}$. For nominal system (2), binary relation $\Delta \subset \Im_X \times \Im_X$ and operator $\mathbf{M}$ are introduced as follows:

$$[(\alpha(x), \beta(x)) \in \Delta] \Leftrightarrow \left[u \times \alpha(x) \le \frac{\partial \beta(x)}{\partial x} f(x, u) \right]$$

$$(\mathbf{M}(\beta), \beta) \in \Delta, \quad (\alpha, \beta) \in \Delta \Rightarrow \alpha \le \mathbf{M}(\beta).$$

The way for operator $\mathbf{M}$ calculating is given by following theorem

Theorem 1 (Zhirabok and Shumsky, 1989). Let the vector function $(\partial\beta/\partial x) f(x,u)$ is written in the form $\zeta(\alpha_1(x), \alpha_2(x), \ldots, \alpha_s(x), u)$, where $\alpha_1(x), \alpha_2(x), \ldots, \alpha_s(x) \in R^1$ are the functions, satisfying following condition: there exist the values of input vector $u = u_1$, $u = u_2, \ldots, u = u_r$, such that the functional inequality

$$(\partial\beta/\partial x) f(x,u_1) \times (\partial\beta/\partial x) f(x,u_2) \times \ldots \times (\partial\beta/\partial x) f(x,u_r)$$

$$\le \alpha_i(x), \ i = 1, \ldots, s,$$

holds. Then $\mathbf{M}(\beta) \cong \alpha_1 \times \alpha_2 \times \ldots \times \alpha_s$.

The next theorem gives the rule for finding the vector function with so-called substitution property.

Theorem 2 (Zhirabok and Shumsky, 1987). Let there exists integer k such that

$$\varphi = \prod_{j=0}^{k} \mathbf{M}^j(h) \le \prod_{j=0}^{k+1} \mathbf{M}^j(h)$$

where $\mathbf{M}^0(h) = h$. Then the function φ satisfies conditions

$$(\varphi, \varphi) \in \Delta, \quad \varphi \le h \qquad (11)$$

and for every vector function $\varphi' \in \Im_X$ satisfying (11) functional inequality $\varphi' \le \varphi$ holds.

By evident manner, above algebraic tools are extended for model (1), (3).

3.2. Extended model reduction

Adaptive observer can be designed in the form of Luenberger observer on the base of the extended model (1), (3). Known nonlinear techniques (see e.g. Birk and Zeitz, 1988; Misawa and Hedrick, 1989; Jo and Seo, 2002; so on) can be applied to solve this problem involving the methods of global (perfect or partial) and local linearization. All these methods need in observability of the model in use. As soon as there is no guarantee that the model (1), (3) is observable (under observable nominal model (2)), the problem of model reduction (finding the observable image of unobservable model) arises.

There are different kinds of observability for nonlinear systems. Consider some of them with application to the extended model. Denote $H(x_{(E)}(t_0), u(\tau), T)$ output response for (1), (3) starting from $x_{(E)}(t_0)$ under control $u(\tau)$, $t_0 \le \tau \le t$ at time interval $T = [t_0, t]$, where $x_{(E)} = (x^{\mathrm{T}}, \vartheta^{\mathrm{T}})^{\mathrm{T}}$ is the state vector of the extended model (1), (3).

Definition 1 (observability). System (1), (3) is called observable (or, that is the same, weakly observable) if for every $x_{(E)}(t_0) \ne x'_{(E)}(t_0)$ there exist finite time interval T and control $u(\tau)$ such that

$$H(x_{(E)}(t_0), u(\tau), T) \ne H(x'_{(E)}(t_0), u(\tau), T). \qquad (12)$$

Definition 2 (strong observability). System (1), (3) is called strongly observable if there exists finite time interval T such that (12) holds for every control $u(\tau)$.

Definition 3 (local observability). System (1), (3) is called locally (locally strongly) observable if for every $x_{(E)}(t_0)$ and $x'_{(E)}(t_0)$ from small neighbourhood of $x_{(E)}(t_0)$ condition of definition 1 (2) holds.

Generally speaking, known methods of nonlinear observer design need, at least, in local strong observability condition that guarantees full rank of the nonlinear observability matrix (Birk and Zeitz, 1988). It is caused by the necessity to inverse the observability matrix under observer design. But at practice, local strong observability condition holds seldom. Usually, only the demand on local observability is satisfied. In this case, full rank condition for nonlinear observability matrix function is fulfilled almost for all values of its arguments, excluding only some points of singularity. From the practical point of view, it is not a fatal for nonlinear observer design if the methods for ill-defined matrix inversion can be applied.

Theorem 3. Let φ be the vector function, satisfying condition (11) written for the model (1), (3). For every $x_{(E)}(t_0) \ne x'_{(E)}(t_0)$ and arbitrary time interval T the equality

$$H(x_{(E)}(t_0), u(\tau), T) = H(x'_{(E)}(t_0), u(\tau), T)$$

holds if and only if

$$\varphi(x_{(E)}(t_0)) = \varphi(x'_{(E)}(t_0)).$$

Proofs of this and the next theorems are omitted due to the limited volume of the paper.

Corollary 1. The system (1), (3) is observable if and only if φ is one-to one function.

Corollary 2. The system (1), (3) is locally observable if and only if for every $x_{(E)} \in X \times \Theta$

$$\operatorname{rank} \frac{\partial \varphi}{\partial x_{(E)}} = n + q \cdot \tag{13}$$

Let condition (13) is violated. In this case the model (1), (3) has to be transformed to observable one; an appropriate coordinate transformation is given by following theorem.

Theorem 4. Let φ be the function "on" satisfying condition (11). In this case φ specifies coordinate transformation of (1), (3) to observable model (5) and the functions f_*, h_* of the model (5) are found from equations

$$f_*(\varphi(x_{(E)}), u) = \left(\partial\varphi/\partial x_{(E)}\right)\begin{pmatrix} f(x_{(E)}, u) \\ 0_{q\times 1}\end{pmatrix},$$

$$h_*(\varphi(x_{(E)})) = h(x) \cdot \tag{14}$$

Remark 1. φ is the function "on" if it does not contain functionally dependent components, i.e. the number of components of this function is strictly equal to the rank of functional matrix $\partial\varphi/\partial x_{(E)}$.

Given above is summarized in following algorithm.

Algorithm 1 (model reduction).

1. For the model (1), (3), find the function φ involving the rule of theorem 2.

2. If (13) holds go to the next step, otherwise go to the step 4.

3. Take identity function φ and the model (5) in the form (1), (3). End.

4. Obtain the function "on" by excluding functional dependent components from φ.

5. Find the model (5) from equations (14). End.

3.3. Transformation to normal form

Let

$$\psi^i_j = \mathbf{M}^{j-1}(h_i), 1 \le j \le r_i - 1, \tag{15}$$

$$\dot{\xi}^i_j = \frac{\partial \psi^i_j}{\partial z} f_* \cdot \tag{16}$$

Determine the indices (so-called relative degrees) r_i, $1 \le i \le l$, as follows

$$\partial \dot{\xi}^i_j / \partial u = 0 \; \forall j < r_i - 1 \quad \text{and} \quad \partial \dot{\xi}^i_{r_i} / \partial u \ne 0 \cdot \tag{17}$$

An algorithm for the model (5) transformation to normal form (6) is given below.

Algorithm 2 (transformation to normal form).

1. Calculate the functions ψ^i_j, ξ^i_j and relative degrees r_i, $1 \le i \le l$, according to (15) - (17).

2. If

$$\sum_{i=1}^{l} r_i = n_z,$$

then complete description of the normal form is found from

$$\dot{\xi}^i_{r_i} = \frac{\partial \psi^i_{r_i}}{\partial z} f_*, \quad \dot{\xi}^i_j = \xi^i_{j+1},$$

$$y_i = \xi^i_1, \quad 1 \le i \le l, \quad 1 \le j \le r_{i-1}. \tag{18}$$

End. Otherwise go to the next step.

3. Find the vector function ψ^{l+1} with number of components strictly equal to

$$n_z - \sum_{i=1}^{l} r_i$$

such that

$$\psi^{l+1} \times \prod_{i=1}^{l} \prod_{j=1}^{r_i} \mathrm{M}^j(h_i) \cong i \cdot$$

4. Find the normal form from (18) and

$$\dot{\xi}^{l+1} = \frac{\partial \psi^{l+1}}{\partial z} f_* \cdot \tag{19}$$

End.

Remark 2. $\dot{\xi}^i_j(t), 1 \le j \le r_i, 1 \le i \le l$, and $\dot{\xi}^{l+1}(t)$ are found from (18) and (19) respectively as the functions of $u(t)$ and $z(t)$. The last vector is estimated by adaptive observer.

4. EXAMPLE

Consider system of the form (1) with the functions

$$\dot{x} = \begin{pmatrix} -x_1^2 + \ln(1 + \vartheta_1 u_1 + \vartheta_2 u_2) \\ -x_1 x_2 + \ln(1 + \vartheta_1 u_1 - \vartheta_2 u_2)\end{pmatrix};$$

$$y = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}. \tag{20}$$

Nominal values of parameters are as follows $\vartheta_1^0 = \vartheta_2^0 = 1$. Using additional equations

$$\dot{\vartheta}_1(t) = 0, \; \dot{\vartheta}_2(t) = 0 \tag{21}$$

one obtains extended model (20), (21). For this model, application of algorithm 1 results in:

1. $\varphi = \prod_{j=0}^{1} \mathbf{M}^j(h) = h \times \mathbf{M}(h) = x_1 \times x_2 \times x_1^2 \times x_1 x_2 \times \vartheta_1 \times$

$\vartheta_2 \cong i(x, \vartheta) \le \prod_{j=0}^{2} \mathbf{M}^j(h)$, where $i(x, \vartheta) = x \times \vartheta$ is identity function. The function $\mathbf{M}(h)$ has been found by following manner. Involving the rule of theorem 1, the function $\frac{\partial h}{\partial (x, \vartheta)} \begin{pmatrix} f \\ 0_{2\times 1} \end{pmatrix}$ is fixed at $u_1 = u_2 = 0$, $u_1 = 1,\ u_2 = 0$, $u_1 = 0, u_2 = 1$ that gives the functions $x_1^2 \times x_1 x_2$, $(-x_1^2 + \ln(1+\vartheta_1)) \times (-x_1 x_2 + \ln(1+\vartheta_1))$, $(-x_1^2 + \ln(1+\vartheta_2)) \times (-x_1 x_2 + \ln(1-\vartheta_2))$. It may be easily seen that the functions $x_1^2,\ x_1 x_2,\ \vartheta_1,\ \vartheta_2$ are expressed from above functions. Then, as soon as $\frac{\partial h}{\partial (x, \vartheta)} \begin{pmatrix} f \\ 0_{2\times 1} \end{pmatrix}$ can be expressed as the function of $x_1^2 \times\ x_1 x_2 \times \vartheta_1 \times \vartheta_2$ and u (the function ζ in theorem 1), one obtains $\mathbf{M}(h) =\ x_1^2 \times x_1 x_2 \times \vartheta_1 \times \vartheta_2$.

2. Because φ is one-to-one function, condition (17) holds.

3. Observable extended model is taken in the form (20), (21) that corresponds to identity function φ. End.

To design adaptive observer, local linearization techniques (Birk and Zeitz, 1988) has been applied to (20), (21). Adaptive observer has been found in the form

$$\begin{pmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \dot{\hat{\vartheta}}_1 \\ \dot{\hat{\vartheta}}_2 \end{pmatrix} = \begin{pmatrix} -\hat{x}_1^2 + \ln(1 + \hat{\vartheta}_1 u_1^{'} + \hat{\vartheta}_2 u_2^{'}) \\ -\hat{x}_1 \hat{x}_2 + \ln(1 + \hat{\vartheta}_1 u_1^{'} - \hat{\vartheta}_2 u_2^{'}) \\ 0 \\ 0 \end{pmatrix} +$$

$$\begin{pmatrix} -2\hat{x}_1 - d_2 \\ -\hat{x}_2 \\ -0.5(1 + \hat{\vartheta}_1 u_1^{'} + \hat{\vartheta}_2 u_2^{'}) d_1 / u_1^{'} \\ -0.5(1 + \vartheta_1 u_1^{'} + \vartheta_2 u_2^{'}) d_1 / u_2^{'} \end{pmatrix} (\hat{x}_1 - y_1) +$$

$$\begin{pmatrix} 0 \\ -\hat{x}_1 - d_2 \\ -0.5(1 + \hat{\vartheta}_1 u_1^{'} - \hat{\vartheta}_2 u_2^{'}) d_1 / u_1^{'} \\ 0.5(1 + \vartheta_1 u_1^{'} - \vartheta_2 u_2^{'}) d_1 / u_2^{'} \end{pmatrix} (\hat{x}_2 - y_2)$$

where $d_1,\ d_2$ are the coefficients of characteristic equation and the symbol ^ accompanies the estimations of appropriate variables under new control $u^{'} \in U$.

Normal form for (20), (21) is found by application of algorithm 2:

1. From (19)-(21) one obtains $r_1 = 1$, $\psi_1^1 = x_1$; $r_2 = 1$, $\psi_1^2 = x_2$.

2. As soon as $r_1 + r_2 = 1 + 1 = 2 < n_z = 4$, step 3 has to be fulfilled.

3. Function ψ^3 with $n_z - r_1 - r_2 = 4 - 1 - 1 = 2$ components is taken as follows $\psi^3 = \vartheta_1 \times \vartheta_2$.

4. Normal form is found from (18), (19)

$$\dot{\xi}_1^1 = x_1^2 + \ln(1 + \vartheta_1 u_1 + \vartheta_2 u_2), \quad y_1 = \xi_1^1;$$

$$\dot{\xi}_1^2 = x_1 x_2 + \ln(1 + \vartheta_1 u_1 - \vartheta_2 u_2),\ y_2 = \xi_1^2;$$

$$\dot{\xi}^3 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \quad (22)$$

After this, new control is found from (22) and (9) as follows

$$\begin{pmatrix} u_1^{'} \\ u_2^{'} \end{pmatrix} = \begin{pmatrix} 1/2\hat{\vartheta}_1 & 1/2\hat{\vartheta}_1 \\ 1/2\hat{\vartheta}_2 & -1/2\hat{\vartheta}_2 \end{pmatrix} \begin{pmatrix} \exp((w_1 - \hat{x}_1^2)) - 1 \\ \exp((w_2 - \hat{x}_1 \hat{x}_2)) - 1 \end{pmatrix}$$

where $w_1,\ w_2$ are obtained from (22) and (10)

$$w_1 = x_{1(R)}^2 + \ln(1 + \vartheta_1^0 u_1 + \vartheta_2^0 u_2) + c_1^1 (x_{1(R)} - \hat{x}_1);$$
$$w_2 = x_{1(R)} x_{2(R)} + \ln(1 + \vartheta_1^0 u_1 - \vartheta_2^0 u_2) + c_1^2 (x_{2(R)} - \hat{x}_2).$$

Some above results are proved by simulation. Under simulation, the initial coordinates of system (20) are taken equal to 0. Initial estimations of coordinates and parameters are taken equal to 0 and 1 respectively. The constants are $d_1 = 9$, $d_2 = 6$, $c_1^1 = c_1^2 = 3$ that correspond to the roots of characteristic equations equal to -3. The controls are taken as follows $u_1(t) = 0.8$, $u_2(t) = 0.64$. To represent the faults, parameters $\vartheta_1,\ \vartheta_2$ are changed to $\vartheta_1 = 0.5$ and $\vartheta_2 = 0.7$ at $t = 30$ and $t = 60$ respectively. Simulation results are given in figures. Fig.1 illustrates the reference model behaviour under initial control, while Fig.2 and Fig.3 show the behaviour of faulty system under initial and new control respectively. The residuals $y_1 - y_{1(R)}$ and $y_2 - y_{2(R)}$ are given in Fig.4. It is clearly seen from Fig.4 that the outputs of faulty system under new control are asymptotically converging to appropriate outputs of the reference model.

5. CONCLUSION

In this paper, the method has been proposed for FTC in nonlinear systems whose purpose is tracking the prescribed trajectory. Realization of the method is connected with the tasks of nonlinear model reduction and asymptotic model matching. Algebra of functions based solutions have been proposed for these tasks. In contrast to former result (Isidori, 1989), the method of asymptotic model matching considered in the paper is applicable to more general class of nonlinear systems than affine ones.

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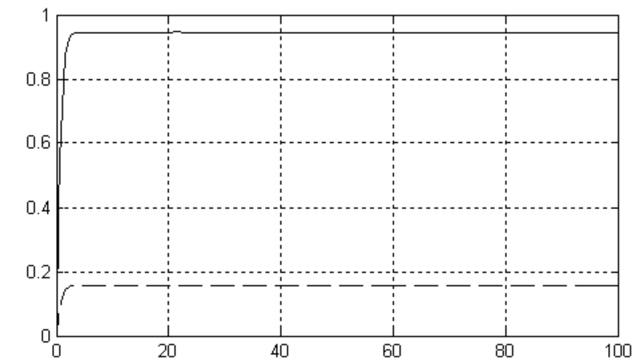

Fig. 1. Outputs of the reference model under initial control: $y_{1(R)}$ (-) and $y_{2(R)}$ (- -)

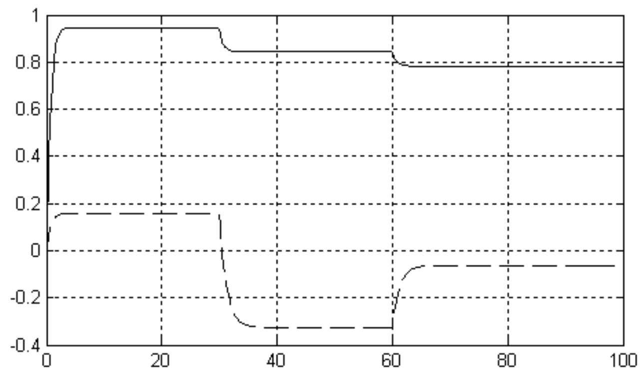

Fig. 2. Outputs of faulty system under initial control: y_1 (-) and y_2 (- -)

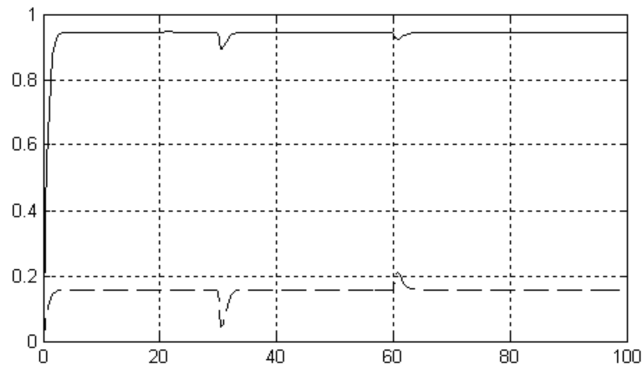

Fig. 3. Outputs of faulty system under new control: y_1 (-) and y_2 (- -)

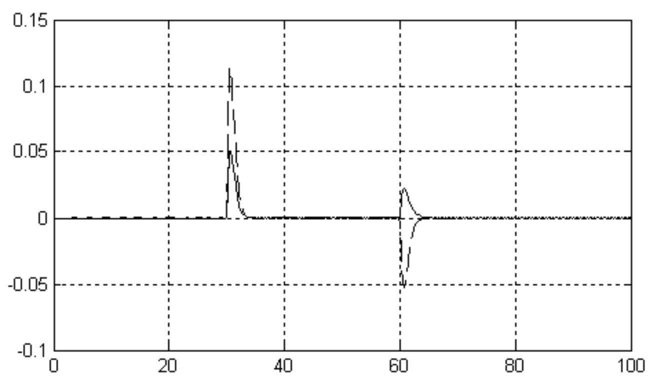

Fig. 4. The residuals under new control: $y_1 - y_{1(R)}$ (-) and $y_2 - y_{2(R)}$ (--)