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Minimum-time Feedforward Plus PID Control for MIMO Systems ⋆

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Abstract: In this paper we propose a technique for the determination of a feedforward control law to be applied to a closed-loop PID-based control system for a multi-input multi-output process in order to achieve a minimum-time transition of the outputs subject to constraints on both the control variables and the system outputs. The optimal command inputs are determined by suitably approximating the state variables and the input signals by means of Chebyshev series and by subsequently solving a constrained optimisation problem. Simulation results demonstrate the effectiveness of the methodology.

1. INTRODUCTION

The great majority of control loops in industrial settings are based on Proportional–Integral–Derivative (PID) controllers, because of the advantageous cost/benefit ratio they are able to provide. In order to help the operator to select the controller gains to address given control specifications, many tuning formulas, based on a simple model of the process, have been devised in the past, both for single-input single-output (SISO) (O'Dwyer (2006)) and multi-input multi-output (MIMO) systems (see for example (Luyben (1986); Dong and Brosilow (1997); Chen and Seborg (2002); Lee et al. (2004))).

However, it is also recognised that the performance achieved by PID controllers is determined also by the suitable implementation of those functionalities that have to (or can) be added to the basic PID control law in order to deal with practical issues (Visioli (2006)). One of these additional functionalities is the feedforward control action, which can be suitably applied in order to improve the set-point following performance, especially when the tuning of the PID parameters is devoted to the load disturbance rejection performance. Actually, while different techniques have been devised for the synthesis of a feedforward action in the context of SISO systems (see, for example, (Åström and Hägglund (2006); Wallen and Åström (2002); Visioli (2004); Piazzi and Visioli (2006))), this aspect has been somewhat overlooked for MIMO processes.

From another point of view, in order to achieve a high performance in practical applications, constraints on both the the system inputs and outputs should be considered explicitly in the design phase (Glattfelder and Schaufelberger (2003)). Indeed, there are always saturation limits for the actuators and many times the process variables cannot exceed given limit values in order to satisfy the control specifications.

In this paper we propose the application of a minimum-time feedforward control strategy to a closed-loop MIMO system with PID controllers where both actuator limits as well as constraints on the maximum overshoot and undershoot of the outputs are taken into account. In other words, we determine the command inputs to be applied to the closed-loop system in order to provide a minimum-time rest-to-rest transition from an equilibrium state to another (corresponding to desired transitions of the process outputs from a set-point value to another) subject to minimum and maximum constraints for the manipulated variables as well as for the process variables.

A Chebyshev approach is employed for this purpose (Vlassenbroeck (1988); Jaddu and Shimemura (1999)). In particular, the method is based on parameterising the state variables and the control variables by Chebyshev series. In this way the system dynamics is transformed into a system of algebraic equations and therefore the minimum-time control problem is reduced into a constrained optimisation problem.

The paper is organised as follows. In Section 2 the minimum-time constrained feedforward control problem is formulated. The use of the Chebyshev approach for synthesising the solution is explained in detail in Section 3. Simulation results are presented in Section 4 and conclusions are given in the Section 5.

Notation. $[\mathbf{v}]_i$ denotes the ith component of vector $\mathbf{v}$.

2. PROBLEM FORMULATION

For the sake of clarity we will assume that the MIMO process has two inputs and two outputs. The methodology can be extended easily to the case of m inputs and p outputs. The process is modelled by means of first-order plus dead-time (FOPDT) transfer functions where the dead time is approximated by a first-order Padè approximation, namely, the matrix transfer function is

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$$\mathbf{P}(s) = \begin{bmatrix} \frac{K_{11}}{T_{11}s+1}e^{-L_{11}s} & \frac{K_{12}}{T_{12}s+1}e^{-L_{12}s} \\ \frac{K_{21}}{T_{21}s+1}e^{-L_{21}s} & \frac{K_{22}}{T_{22}s+1}e^{-L_{22}s} \end{bmatrix} \cong \begin{bmatrix} \frac{K_{11}}{T_{11}s+1}\frac{-L_{11}/2s+1}{L_{11}/2s+1} & \frac{K_{12}}{T_{12}s+1}\frac{-L_{12}/2s+1}{L_{12}/2s+1} \\ \frac{K_{21}}{T_{21}s+1}\frac{-L_{21}/2s+1}{L_{21}/2s+1} & \frac{K_{22}}{T_{22}s+1}\frac{-L_{22}/2s+1}{L_{22}/2s+1} \end{bmatrix}. \tag{1}$$

This is a typical choice in industrial practice, since this model can describe well the dynamics of many (coupled) industrial processes.

The process is controlled by means of two (decentralised) output-filtered PID controllers (see Figure 1), namely, the controller transfer function is:

$$\mathbf{C}(s) = \begin{bmatrix} C_1(s) & 0 \\ 0 & C_2(s) \end{bmatrix}. \tag{2}$$

with

$$C_j(s) = K_{pj}\left(1 + \frac{1}{T_{ij}s} + T_{dj}s\right)\frac{1}{T_{fj}s+1} \tag{3}$$

where ($j = 1, 2$) K_{pj} is the proportional gain, T_{ij} is the integral time constant, T_{dj} is the derivative time constant and T_{fj} is the time constant of a first-order filter that makes the transfer function proper. The value of T_{fj} can be selected, once the other parameters are determined, such that the filter dynamics does not influence the dynamics of the PID controller and the effects of the measurement noise are reduced as much as possible.

It is worth noting that the output-filtered PID controller has been selected because it presents some advantages with respect to other control structures (Visioli (2006)), but the technique proposed in this paper can be applied to any other kind of PID controller.

The problem considered is to determine the signals $r_1(t)$ and $r_2(t)$, to be applied to the closed-loop system when a transition from an equilibrium point corresponding to the process variable values $\mathbf{y^0} = [y_1^0\ y_2^0]^T := [y_1(t_0)\ y_2(t_0)]^T$ to another equilibrium point corresponding to the process variable values $\mathbf{y^f} = [y_1^f\ y_2^f]^T := [y_1(t_f)\ y_2(t_f)]^T$ is required. In particular, a minimum-time (rest-to-rest) transition is required subject to given limits on the control variables and on the process variables. In the following, without loss of generality, we consider $t_0 = 0$ and $\mathbf{y^0} = \mathbf{0}$.

In order to pose the problem formally, it is convenient to consider the closed-loop system transfer function

$$\mathbf{T}(s) := (\mathbf{I} + \mathbf{P}(s)\mathbf{C}(s))^{-1}\mathbf{P}(s)\mathbf{C}(s) \tag{4}$$

and write a minimal state-space realization of it, that is:

$$\begin{aligned} \frac{d\mathbf{x}(t)}{dt} &= \mathbf{F}\mathbf{x}(t) + \mathbf{G}\mathbf{r}(t) \\ \mathbf{y}(t) &= \mathbf{H}\mathbf{x}(t) \end{aligned} \tag{5}$$

where $\mathbf{F}$, $\mathbf{G}$ and $\mathbf{H}$ are 12×12, 12×2 and 2×12 matrices, respectively. The considered time-optimal feedforward constrained control problem can be therefore expressed as follows:

$$\min_{\mathbf{r}(t)} t_f \tag{6}$$

subject to:

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{F}\mathbf{x}(t) + \mathbf{G}\mathbf{r}(t) \qquad 0 \le t \le t_f \tag{7}$$

$$\mathbf{x}(0) = \mathbf{x_0} \qquad \mathbf{x}(t_f) = \mathbf{x_f} \tag{8}$$

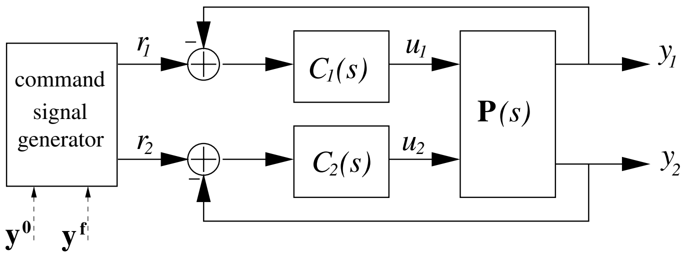

Fig. 1. The considered control scheme.

$$u_i^- \le u_i(t) \le u_i^+, \qquad i = 1, 2, \qquad t \in [0, t_f] \tag{9}$$

$$y_i^- \le y_i(t) \le y_i^+, \qquad i = 1, 2, \qquad t \in [0, t_f] \tag{10}$$

where $u_i(t)$, $i = 1, 2$ is expressed as a linear combination of the state vector variables $x_i(t)$ and of the command inputs $r_1(t)$ and $r_2(t)$, whilst u_i^-, u_i^+ and y_i^-, y_i^+ ($i = 1, 2$) are evidently the constraints for the control variables and the process variables respectively.

It can be demonstrated that this time-optimal control problem has a solution if

$$\begin{aligned} &\{0, y_i^f\} \subset (y_i^-, y_i^+), \qquad i = 1, 2 \\ &\{0, [\mathbf{P}^{-1}(0)\mathbf{y^f}]_i\} \subset (u_i^-, u_i^+), \qquad i = 1, 2 \end{aligned} \tag{11}$$

(see (Piazzi and Visioli (2001)) and (Piazzi and Consolini (2006b))). It is worth noting that the conditions (11) simply mean that the initial and final steady-state values of the control variables and of the process variables must be included in the range allowed for the control variables and process variables respectively (note that $[\mathbf{P}^{-1}(0)\mathbf{y^f}]_i$ is the final steady-state value of the ith component of the process input vector).

3. CHEBYSHEV OPTIMISATION

3.1 Chebyshev polynomials

The Chebyshev polynomials of the first kind are a set of orthogonal polynomials defined as the solutions to the Chebyshev differential equation and denoted $T_i(\tau)$. They are normalised such that $T_i(1) = 1$, $i = 0, 1, \ldots$ and in their trigonometric form they are expressed as:

$$T_i(\tau) = \cos(i \cdot \arccos(\tau)) \qquad \tau \in [-1, 1]. \tag{12}$$

They can be also defined by the recurrence relation:

$$\begin{aligned} T_0(\tau) &= 1 \\ T_1(\tau) &= \tau \\ T_{i+1}(\tau) &= 2\tau T_i(\tau) - T_{i-1}(\tau) \qquad i > 1. \end{aligned} \tag{13}$$

In order to use the Chebyshev polynomials of the first kind for the approximation of the system dynamics, the following time transformation is therefore necessary:

$$t = \frac{t_f}{2}(1 + \tau). \tag{14}$$

This transformation allows a change from the time domain $t \in [0, t_f]$ to the Chebyshev domain $\tau \in [-1, 1]$. The new system dynamics expressed in the Chebyshev domain is therefore (Jaddu and Shimemura (1999)):

$$\frac{d\mathbf{x}(\tau)}{d\tau} = \frac{t_f}{2}(\mathbf{F}\mathbf{x}(\tau) + \mathbf{G}\mathbf{r}(\tau)) \qquad -1 \le \tau \le 1 \tag{15}$$

with initial and final conditions:

$$\mathbf{x}(-1) = \mathbf{x^0} = \mathbf{0}, \qquad \mathbf{x}(1) = \mathbf{x^f}. \tag{16}$$

3.2 Approximation through Chebyshev series

Once the system dynamics has been rewritten in the Chebyshev domain, the next step is the expansion of both the state vector $\mathbf{x}$ and the control variables vector $\mathbf{r}$ through Chebyshev series of order h:

$$\mathbf{x_h}(\tau) = \frac{1}{2}\boldsymbol{\alpha_0} T_0(\tau) + \sum_{i=1}^{h} \boldsymbol{\alpha_i} T_i(\tau) \tag{17}$$

$$\mathbf{r_h}(\tau) = \frac{1}{2}\boldsymbol{\beta_0} T_0(\tau) + \sum_{i=1}^{h} \boldsymbol{\beta_i} T_i(\tau) \tag{18}$$

where $\tau \in [-1,1]$ and $\bar{\boldsymbol{\alpha}} := [\boldsymbol{\alpha_0}, \boldsymbol{\alpha_1}, \ldots, \boldsymbol{\alpha_h}]$ (with $\boldsymbol{\alpha_i} = [\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{i12}]^T$, $i = 0, \ldots, h$) and $\bar{\boldsymbol{\beta}} := [\boldsymbol{\beta_0}, \boldsymbol{\beta_1}, \ldots, \boldsymbol{\beta_h}]$ (with $\boldsymbol{\beta_i} = [\beta_{i1}, \beta_{i2}]^T$, $i = 0, \ldots, h$) are the unknown coefficients. The same order h has been assumed for both the state and the input for the sake of simplicity. The choice of h is related to the required accuracy. Actually, increasing its value yields to a better approximation (indeed, in principle, for h that tends to infinity, the approximation tends to the exact solution), but, from another point of view, increases also the complexity of the optimisation problem. The value of $h = 80$ has therefore been selected to provide a high accuracy with a reasonable computational effort.

3.3 Equality and inequality constraints

The approximation of the state and the input variables $\mathbf{x_h}$ and $\mathbf{r_h}$ is then substituted into the ordinary differential equation (15), yielding to:

$$\frac{d\mathbf{x_h}(\tau)}{d\tau} = \frac{t_f}{2}\left(\mathbf{F}\mathbf{x_h}(\tau) + \mathbf{G}\mathbf{r_h}(\tau)\right) \qquad -1 \le \tau \le 1. \tag{19}$$

as well as into the initial and final condition equations (16). The left hand side of equation (19) can be derived by considering that the derivative of the series (17) with respect to τ is given by

$$\frac{1}{2}\boldsymbol{\alpha'_0} + \sum_{i=1}^{h-1} \boldsymbol{\alpha'_i} T_i(\tau) \tag{20}$$

where the coefficients $\bar{\boldsymbol{\alpha'}} := [\boldsymbol{\alpha'_0}, \boldsymbol{\alpha'_1}, \ldots, \boldsymbol{\alpha'_{h-1}}]$ can be expressed in terms of the coefficients $\boldsymbol{\alpha}$ by means of the following formula (Fox and Parker (1972)):

$$\boldsymbol{\alpha'_{j-1}} - \boldsymbol{\alpha'_{j+1}} - 2r\boldsymbol{\alpha_j} = 0, \quad j = 1, \ldots, h-1. \tag{21}$$

Thus we obtain

$$\frac{t_f}{2}\left(\mathbf{F}\mathbf{x_h}(\tau) + \mathbf{G}\mathbf{r_h}(\tau)\right) = \frac{1}{2}\boldsymbol{\alpha'_0} + \sum_{i=1}^{h-1} \boldsymbol{\alpha'_i} T_i(\tau) \tag{22}$$

By equating the coefficients of same-order Chebyshev polynomials we obtain a system of $12 \times h$ nonlinear equality constraints (note that the final time t_f is unknown).
The substitution of $\mathbf{x_h}$ into the initial and final condition expression (16) yields to 2×12 additional equality constraints, namely, (Vlassenbroeck (1988))

$$\frac{1}{2}\boldsymbol{\alpha_0} + \sum_{i=1}^{h} (-1)^i \boldsymbol{\alpha_i} - \mathbf{x}(-1) = 0 \tag{23}$$

and

$$\frac{1}{2}\boldsymbol{\alpha_0} + \sum_{i=1}^{h} \boldsymbol{\alpha_i} - \mathbf{x}(1) = 0. \tag{24}$$

After having determined the expression of $\mathbf{u_h}(\tau)$ as a linear combination of the elements of $\mathbf{x_h}(\tau)$ and $\mathbf{r_h}(\tau)$, the inequality constraints (9)-(10) can be handled by rewriting them as

$$\begin{array}{ll} y_i^- - [\mathbf{H}\mathbf{x_h}(\tau)]_i \le 0 & i = 1,2 \\ [\mathbf{H}\mathbf{x_h}(\tau)]_i - y_i^+ \le 0 & i = 1,2 \\ u_i^- - [\mathbf{u_h}(\tau)]_i \le 0 & i = 1,2 \\ [\mathbf{u_h}(\tau)]_i - u_i^+ \le 0 & i = 1,2 \end{array} \tag{25}$$

and by defining four vectors of slack variables $\mathbf{w_i}(\tau) = [w_{i1}\ w_{i2}]$, $i = 1, \ldots, 4$. Thus, expression (25) can be rewritten as

$$\begin{array}{ll} y_i^- - [\mathbf{C}\mathbf{x_h}(\tau)]_i = -[\mathbf{w_1}(\tau)]_i^2 & i = 1,2 \\ [\mathbf{C}\mathbf{x_h}(\tau)]_i - y_i^+ = -[\mathbf{w_2}(\tau)]_i^2 & i = 1,2 \\ u_i^- - [\mathbf{u_h}(\tau)]_i = -[\mathbf{w_3}(\tau)]_i^2 & i = 1,2 \\ [\mathbf{u_h}(\tau)]_i - u_i^+ = -[\mathbf{w_4}(\tau)]_i^2 & i = 1,2 \end{array}. \tag{26}$$

At this point each slack variable can be expanded in a Chebyshev series with unknown coefficients $\bar{\boldsymbol{\gamma}}$ and by equating again the coefficients of same-order Chebyshev polynomials, a set of (nonlinear) equality constraint relations in $\bar{\boldsymbol{\alpha}}$, $\bar{\boldsymbol{\beta}}$ and $\bar{\boldsymbol{\gamma}}$ emerges. In this way an optimisation problem with only (nonlinear) equality constraints is obtained.
Alternatively, the Chebyshev series in (25) can be evaluated at a number of points τ_i, $-1 = \tau_0 < \tau_1 < \cdots < \tau_k = 1$, so that a set of inequality constraint relations in $\bar{\boldsymbol{\alpha}}$ and $\bar{\boldsymbol{\beta}}$ results. Although this approach is less rigorous, we preferred to use it because, overall, it requires a less computational effort.

3.4 Optimisation

By following the steps described before, the optimal control problem (6)-(10) is therefore transformed into a parameter optimisation problem which consists in finding t_f, $\bar{\boldsymbol{\alpha}}$ and $\bar{\boldsymbol{\beta}}$ in order to minimise the transition time t_f subject to the posed equality and inequality constraints. The minimum-time feedforward control law is then obtained by applying expression (18).
To solve this optimisation problem, a sequential quadratic programming (SQP) method, such as the one implemented in the function "`fmincon`" of Matlab can be used (Matlab (2006)). In this context the starting values of the parameters t_f, $\bar{\boldsymbol{\alpha}}$ and $\bar{\boldsymbol{\beta}}$, denoted respectively as t_f^0, $\bar{\boldsymbol{\alpha}}^0$ and $\bar{\boldsymbol{\beta}}^0$ can be selected through the Chebyshev interpolation of the state variables evolution of the system when a step signal (whose amplitude is that required at the final equilibrium point) is applied to each system input. The value of t_f^0 can be selected as the largest (2%) settling time of the outputs. Then, consider the $h+1$ Chebyshev nodes that are obtained as

$$\tau_i = \cos\left(\frac{\pi i}{h}\right) \qquad i = 0, \ldots, h. \tag{27}$$

These nodes are symmetrically distributed about $\tau = 0$. As far as i increases, they cluster towards the endpoints of the interval. If we denote by $\bar{x}_i(\tau)$ the interpolant of function $x_i(\tau)$, $i = 1, \ldots, n$ at the Chebyshev nodes, we have (Quarteroni and Valli (1997)):

$$\bar{x}_i(\tau) = \sum_{j=0}^{h} \alpha_{ji}^0 T_j(\tau) \tag{28}$$

where

$$\alpha_{ji}^0 = \frac{2}{hd_j} \sum_{k=0}^{h} \frac{1}{d_k} \cos\left(\frac{kj\pi}{h}\right) x(\tau_k) \tag{29}$$

with

$$d_i = \begin{cases} 2 & \text{for} \quad i = 0, h \\ 1 & \text{for} \quad i = 1, \ldots, h-1 \end{cases} \tag{30}$$

Having chosen step signals for the two inputs, for the Chebyshev series (18) the best initial fitting is obtained with β_{0i}^0 $(i = 1, 2)$ equal to the value of the step amplitude of the ith input multiplied by two and $\beta_{\mathbf{1}}{}^0 = \cdots = \beta_{\mathbf{h}}{}^0 = \mathbf{0}$.

Finally, it is worth stressing that the optimisation algorithm can be made faster by providing the explicit expression of the gradient of both the equality and inequality constraints with respect to t_f, $\bar{\boldsymbol{\alpha}}$ and $\bar{\boldsymbol{\beta}}$.

4. SIMULATION RESULTS

An illustrative example is presented to demonstrate the effectiveness of the proposed feedforward method. The following system (Chien et al. (1999); Chen and Seborg (2002)), which represents the model of an industrial-scale polymerization reactor, has been considered:

$$\mathbf{P}(s) = \begin{bmatrix} \dfrac{22.89}{4.572s+1} e^{-0.2s} & \dfrac{-11.64}{1.807s+1} e^{-0.4s} \\ \dfrac{4.689}{2.174s+1} e^{-0.2s} & \dfrac{5.80}{1.801s+1} e^{-0.4s} \end{bmatrix} \tag{31}$$

A decentralised PI control system, without a precompensator or any decoupling scheme, has been tuned by applying the BLT method (Luyben (1986)), yielding the following tuning parameters: K_{p1}=0.21, T_{i1}=2.26, K_{p2}=0.18, T_{i2}=4.25.

Then, in order to show the effectiveness of the proposed control strategy in the presence of constraints, boundaries on the control variables u_1 and u_2 have been considered, namely, $u_1^- = u_2^- = -0.13$ and $u_1^+ = u_2^+ = 0.13$. Three cases have been evaluated: in the first, a transition from 0 to 1 is requested for both the process variables, i.e., we selected $\mathbf{y^f} = [1\ 1]^T$. The response of the process (and the resulting control variables) when a step signal is applied to the two command inputs is shown in Figure 2. By applying the Chebyshev optimisation after setting $y_1^- = y_2^- = -0.01$ and $y_1^+ = y_2^+ = 1.01$, the two command inputs shown in Figure 3 have been obtained. The value of optimal transition time is $t_f^* = 22.8$. By using these two command inputs instead of the step signals, the process variables and the control variables plotted in Figure 4 resulted. It can be seen that there are active constraints either on the process inputs or on the process outputs (Piazzi and Consolini (2006a)) and, despite the optimal rest-to-rest transition time appears somewhat large, the actual transient response is very satisfactory with a small rise time and virtually no overshoot (due to the sensible constraints posed on the process variables).

In the second case, we have selected $\mathbf{y^f} = [1\ 0]^T$. The response obtained by applying a step signal to r_1 in shown in Figure 5. The Chebyshev optimisation with $y_1^- = y_2^- = -0.01$, $y_1^+ = 1.01$ and $y_2^+ = 0.01$ yields to the command inputs plotted in Figure 6 and to the corresponding control system response shown in Figure 7. The minimum rest-to-rest transition time is $t_f^* = 17.2$. Also in this case it can be seen that the constraints are not violated and that a very satisfactory performance is obtained in terms of rise time, settling time and overshoot. Further, the process variable y_2 is virtually not affected by the transition performed by y_1, that is, a substantial decoupling is achieved.

Finally, the case where $\mathbf{y^f} = [0\ 1]^T$ has been selected. The response obtained by applying a step signal to r_2 in shown in Figure 8, while the command inputs obtained by applying the Chebyshev optimisation (with $y_1^- = y_2^- = -0.01$, $y_1^+ = 0.01$ and $y_2^+ = 1.01$) and the corresponding process and control variables are shown in Figures 9 and 10 respectively. The achieved optimal transition time is $t_f^* = 26.9$. It appears that the considerations done in the previous case are also valid in this case.

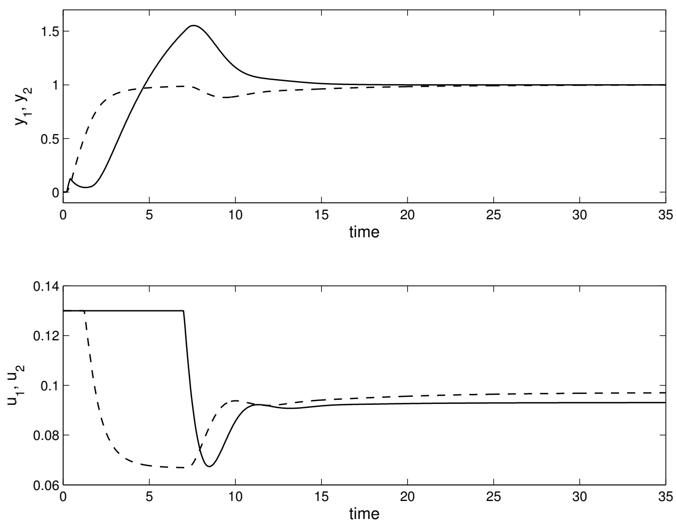

Fig. 2. Process variables and control variables for $\mathbf{y^f} = [1\ 1]^T$ when step signals are applied (y_1, u_1: solid line; y_2, u_2: dashed line).

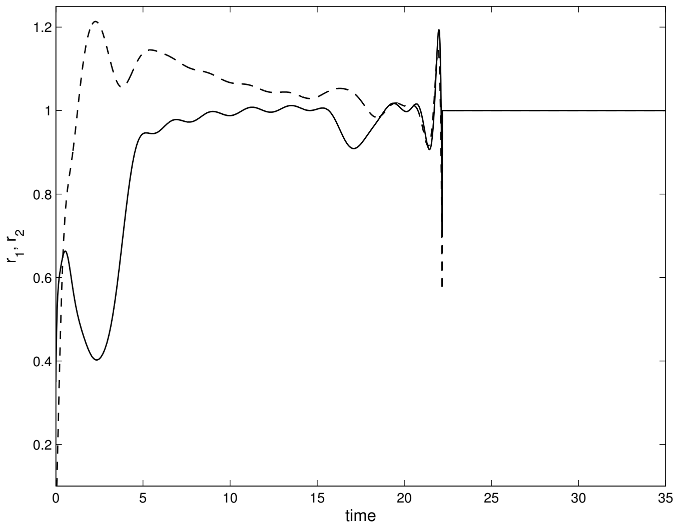

Fig. 3. Optimal command inputs for $\mathbf{y^f} = [1\ 1]^T$ (r_1: solid line; r_2: dashed line).

5. CONCLUSIONS

In this paper we have proposed a technique for the determination of a feedforward control law capable of achieving a minimum-time transition of a PID controlled MIMO

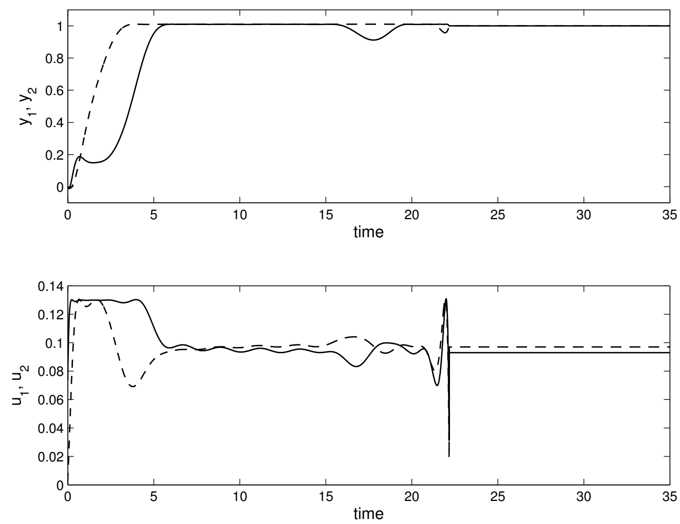

Fig. 4. Process variables and control variables for $\mathbf{y^f} = [1\ 1]^T$ when the Chebyshev optimisation is applied (y_1, u_1: solid line; y_2, u_2: dashed line).

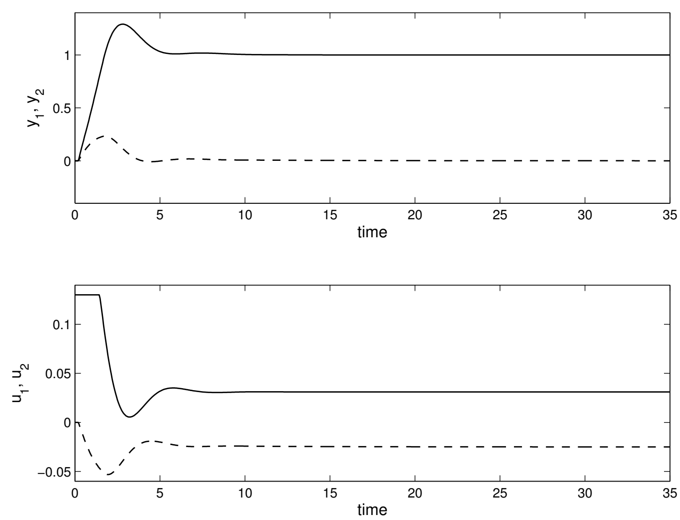

Fig. 5. Process variables and control variables for $\mathbf{y^f} = [1\ 0]^T$ when step signals are applied (y_1, u_1: solid line; y_2, u_2: dashed line).

process subject to constraints on both the control variables and the process variables. From the illustrative results it appears that, by posing sensible output constraints, a very satisfactory performance is obtained in terms of rise time, settling time and overshoot. Further, a decoupling of the system can be virtually achieved. The technique appears to be suitable to implement in Distributed Control Systems for those processes (for example, batch processes) where set-point transitions are known in advance.

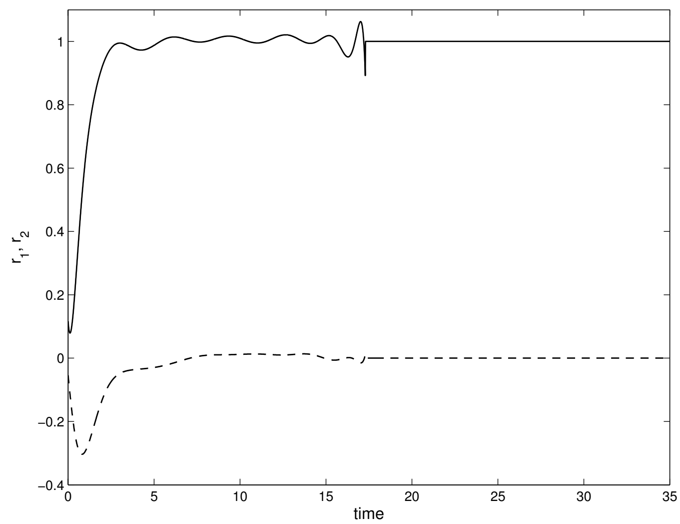

Fig. 6. Optimal command inputs for $\mathbf{y^f} = [1\ 0]^T$ (r_1: solid line; r_2: dashed line).

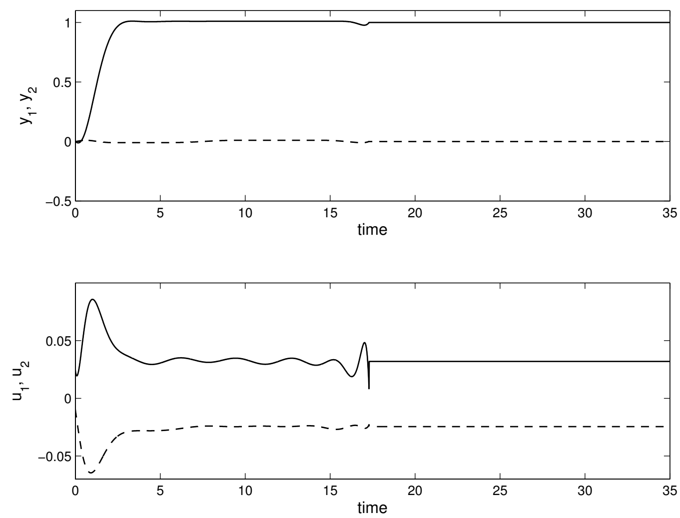

Fig. 7. Process variables and control variables for $\mathbf{y^f} = [1\ 0]^T$ when the Chebyshev optimisation is applied (y_1, u_1: solid line; y_2, u_2: dashed line).

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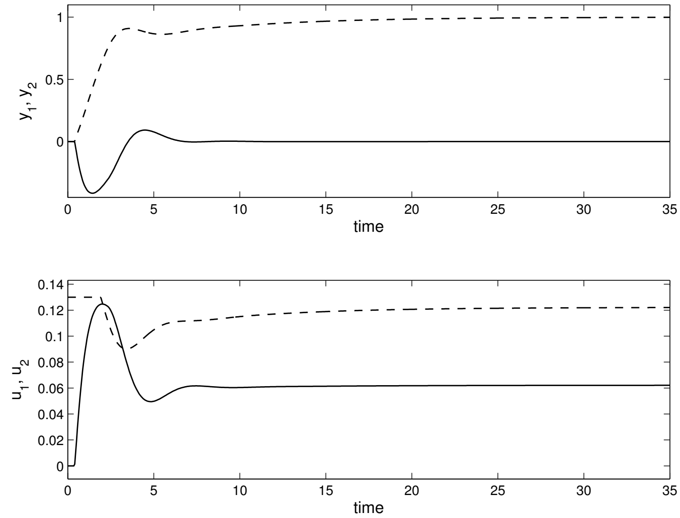

Fig. 8. Process variables and control variables for $\mathbf{y^f} = [0\ 1]^T$ when step signals are applied (y_1, u_1: solid line; y_2, u_2: dashed line).

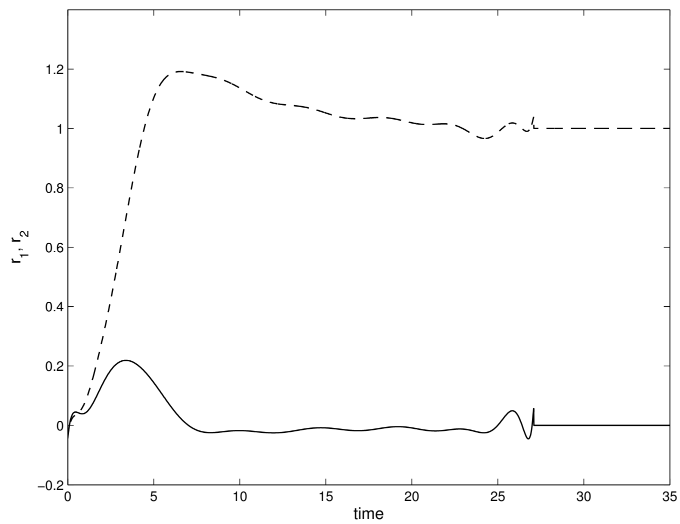

Fig. 9. Optimal command inputs for $\mathbf{y^f} = [0\ 1]^T$ (r_1: solid line; r_2: dashed line).

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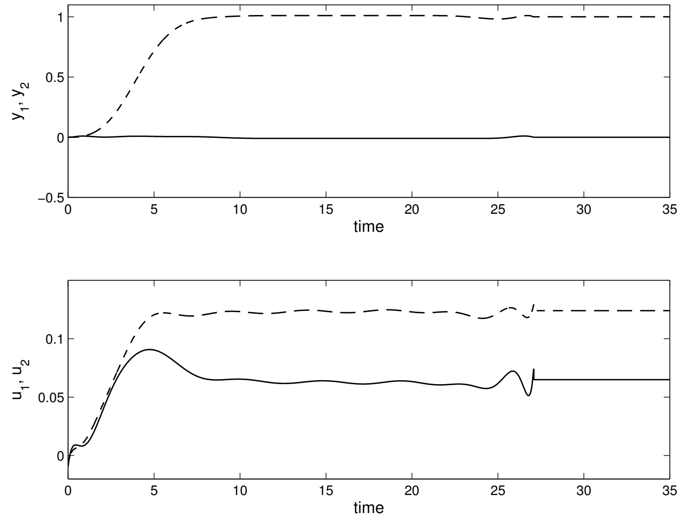

Fig. 10. Process variables and control variables for $\mathbf{y^f} = [0\ 1]^T$ when the Chebyshev optimisation is applied (y_1, u_1: solid line; y_2, u_2: dashed line).

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