Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Hybrid Model Predictive Control Applied to Switching Control of Burner Load for a Compact Marine Boiler Design

Brian Solberg * **Palle Andersen** ** **Jan M. Maciejowski** *** **Jakob Stoustrup** **

* *Department of Engineering, Aalborg Industries A/S, Gasværksvej 24, DK-9100 Aalborg, Denmark, (e-mail: bso@aalborg-industries.com)*
** *Department of Electronic Systems, Aalborg University, Fredrik Bajers Vej 7C, DK-9220 Aalborg Øst, Denmark, (e-mail: {pa,jakob}@es.aau.dk)*
*** *Department of Engineering, University of Cambridge, Trumpington Street, Cambridge CB2 1PZ, UK, (e-mail: jmm@eng.cam.ac.uk)*

Abstract: This paper discusses the application of hybrid model predictive control to control switching between different burner modes in a novel compact marine boiler design. A further purpose of the present work is to point out problems with finite horizon model predictive control applied to systems for which the optimal solution is a limit cycle. Regarding the marine boiler control the aim is to find an optimal control strategy which minimizes a trade-off between deviations in boiler pressure and water level from their respective setpoints while limiting burner switches. The approach taken is based on the Mixed Logic Dynamical framework. The whole boiler systems is modelled in this framework and a model predictive controller is designed. However to facilitate on-line implementation only a small part of the search tree in the mixed integer optimization is evaluated to find out whether a switch should occur or not. The strategy is verified on a simulation model of the compact marine boiler for control of low/high burner load switches. It is shown that even though performance is adequate for some disturbance levels it becomes deteriorated when the optimal solution is a limit cycle.

1. INTRODUCTION

The control of marine boilers mainly focuses on minimizing the variation of steam pressure and water level in the boiler, keeping both variables around some given setpoint. Up till now this task has been achieved using classical SISO controllers, one using the fuel flow to control the steam pressure and one using the feed water flow to control the water level.

A more efficient control can allow smaller water and steam volumes in the boiler implying lower production and running costs and a more attractive product. In (Solberg et al., 2005) a successful application of LQG control to the Mission$^{\text{TM}}$ OB boiler from Aalborg Industries A/S (AI) product range was shown.

The specific boiler concerned in the present work is a novel compact marine boiler from AI. The boiler is a side fired one-pass smoke tube boiler. The boiler consists of a furnace and convection tubes surrounded by water. At the top of the boiler steam is led out and feed water is injected. The compact boiler is equipped with a two-stage burner unit with two pressure atomizer nozzles of different size. With slight abuse of notation we refer to these nozzles as Burner 1 (the small nozzle) and Burner 2 (the large nozzle). This means that there are two burners and designing an appropriate switching strategy between these can allow for a high turndown ratio, defined as the ratio between the largest and lowest possible fuel flow, or equivalently burner load. However too much switching will increase actuator wear and decrease performance due to non-optimal combustion during burner start-up.

Unfortunately the maximum power generated by Burner 1, $\overline{Q}_l$, alone is lower than the minimum power generated by the combined operation of the burners, $\underline{Q}_h$. There are two power gaps. This means that, for a steam flow that corresponds to a steady state power consumption in one of these gaps, the burners have to follow some on/off switching scheme to keep the pressure around its reference value. The gaps will be defined as: *Gap-region 1* $Q_{ss} \in [0; \underline{Q}_1] := G_1 \subset \mathbb{R}$ and *Gap-region 2* $Q_{ss} \in [\overline{Q}_l; \underline{Q}_h] := G_2 \subset \mathbb{R}$. In the sequel we shall refer to these gap-regions a bit loosely using statements such as 'the disturbance-', 'the required fuel flow-', 'the energy request belong to a gap-region', which all translate into the equivalent formulation that the steady state power consumption can not be met exactly by any available fuel flow.

The challenge in this work is to design an appropriate burner switching strategy that minimizes pressure variations and hence fluctuations in steam quality without compromising water level performance to still allow the smaller boiler geometry. Such a task would normally have been approached using heuristic rules combined with hysteresis control, however we seek a more systematic design procedure. The control problem is complicated by the shrink-and-swell phenomenon which introduces non-minimum phase characteristics in the system, (Åström and Bell, 2000). This phenomenon is seen when e.g. the steam flow is abruptly increased. This causes the pressure to drop instantly, which in turn causes an expansion of steam bubbles below the water surface and further lowers the boiling point causing even more bubbles to be generated leading to an almost instant increase in the water level. However mass is removed from the boiler so eventually the water level will decrease. Similar behaviors can be observed when the feed water or fuel flow is changed.

The boiler system belongs to the special class of systems integrating logic and dynamics. Many methods along with traditional hysteresis and pulse width modulation (PWM) have been proposed for controlling these systems – see e.g. (Sarabia et al., 2005; Bemporad and Morari, 1999; Hedlund and Rantzer, 1999; Solberg et al., 2007a). If we do not accept large persistent deviations in pressure from the setpoint or if a goal is to bring the integrated pressure error to zero then for some steam loads the burners must switch on and off according to some pattern to compensate for the gap-regions. In particular this will introduce a limit cycle in the state trajectory.

Let us define the optimal solution as the solution achieved by using an integral cost functional taking the average over an infinite horizon.

The optimal solution will then be dependent on the current disturbance and states and can be a limit cycle. The period and amplitude of the pressure oscillation corresponding to the limit cycle will change with operating conditions. Therefore this solution can not be found by traditional hysteresis control which operates with fixed bounds on the pressure to switch the burners. Traditional PWM suffers from similar shortcomings as the switching period for such schemes are fixed and only the duty-cycle can vary. Further, normally PWM is seen in connection with a cascade control configuration where the inner loop, PWM, runs much faster than the outer process. Neither hysteresis control nor PWM explicitly consider that a cost is assigned to switching the burners which is essential in this problem setup. The method we describe in this paper does not suffer from these limited degrees of freedom. This method is based on Model Predictive Control (MPC) in combination with the Mixed Logic Dynamical (MLD) framework, which is an approach which allows standard tools to be applied to obtain an optimizing control law (Bemporad and Morari, 1999).

We show through simulations that this new method indeed does change behavior for different choices of the steam load.

The paper is organized as follows. First the marine boiler system is introduced and its control properties are discussed. Secondly the hybrid MPC controller is described. In the subsequent section the controller is validated in a simulation study where prediction mismatches are illustrated as consequence of a finite horizon cost. Further a comparison with traditional hysteresis control is made. Finally conclusion and future works are presented.

n_0	*Idle*: both burners are off and Burner 1 is ready to enter start-up sequence.
$n_{0,1}$	*Burner 1 start-up*: this state contains a sequence of events split into three time intervals. It takes 3 second from the electrode is ignited to the solenoid valve opens. Then the flame scanner must detect a flame within the next 5 seconds and finally the flame has 5 seconds to stabilize before release for modulation.
n_1	*Low load*: Burner 1 is on and Burner 2 is off.
$n_{1,2}$	*Burner 2 start-up*: this state is analogous to $n_{0,1}$.
n_2	*High load*: both burners are on.
$n_{1,0}$	*Shut down*: in this state Burner 1 is shut off followed by 30 seconds of purging.

Table 1. Function of states in the finite state machine describing the burner unit.

2. SYSTEM DESCRIPTION

The boiler consists of two logically separated parts, one containing the heating system and one containing the water-steam system. The heating system consists of the furnace and the convection tubes. The water-steam system consists of all water and steam in the boiler. These two systems are interconnected by the metal separating them i.e. the furnace jacket and the convection tube jackets.

The boiler is equipped with two actuator systems for feed water and burner control, respectively. The feed water flow dynamics are linearized in an inner cascade controller which allows the reference to the feed water flow to be used as a manipulated variable. The corresponding inner loop can easily be designed to be faster than the outer loop. The burner system is more complicated. It can operate in three modes; Mode 0: both burners off; Mode 1: Burner 1 on and Burner 2 off; Mode 2: both burners on.

The function of the burner unit can be described by a finite state machine. The state machine consists of six states: three representing the modes described above and another three describing transitions between these, see Fig. 1.

The function of each state is summarized in Table 1.

States n_1, n_2 are characterised by the continuous input variable, fuel, being controllable. In contrast transition states $n_{0,1}, n_{1,2}, n_{1,0}$ are governed by predetermined control sequences. To initiate a switch between modes, certain guards have to be satisfied, as shown in Fig. 1. In most cases this is just a matter of setting the Boolean variable corresponding to the specific burner being on or

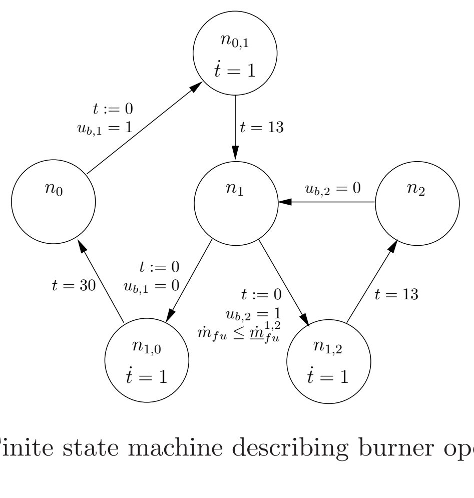

Fig. 1. Finite state machine describing burner operation.

off. However, to initiate a switch from Mode 1 to Mode 2, $n_1 \rightarrow n_{1,2}$, the combustion air flow and hence the fuel flow to Burner 1 has to be below a certain level, in order to be able to fire Burner 2.

2.1 Modeling

A detailed model of the boiler system can be found in (Solberg et al., 2005) and a thorough model analysis was presented in (Solberg et al., 2007b). In this section we shall not repeat these results but only summarize the details important to the current work.

The simplified model will consist of the state machine shown in Fig. 1. The model should describe the total fuel supply to the burners $\dot{m}_{fu} = \dot{m}_{fu,1} + \dot{m}_{fu,2}$ as this is assumed equivalent to the total power delivered from the burner unit. This model is different for each state in the finite state machine. In the transition states $n_{0,1}, n_{1,2}, n_{1,0}$ the fuel flow is constrained to move according to certain patterns. In n_0 there is no flow. In n_1 the fuel flow is equal to the flow to Burner 1. Finally in n_2 an underlying controller distributes the flow reference to the two burners in order to maximize efficiency. The total fuel flow can be assumed to be equal to the reference due to the much faster dynamics of the combustion process than that of the boiler water/steam part. We note here that the fuel flow rate constraints are different in n_1 and n_2. When the burners are on, an underlying controller adjusts the combustion air flow keeping a clean combustion with an oxygen percentage of the exhaust gas above three percent.

The model of the boiler presented in (Solberg et al., 2005) is presented here in a simplified version as studies have shown that both the flue gas part (furnace and convection tubes) and the metal separating the water/steam part from the flue gas have considerably faster dynamics than the desired closed loop bandwidth. Due to this fact the power delivered to the water/steam part is modelled as:

$$Q = \eta \dot{m}_{fu} \tag{1}$$

where η is a constant describing a combination of energy released in the combustion plus furnace and convection tubes heat transfer efficiency. η is in fact a function of the boiler load. However it turns out that in the specific boiler treated here η is approximately constant leading to (1).

The model of the water/steam part has the purpose of describing the steam pressure in the boiler p_s and the water level L_w. The modeling is complicated by the shrink-and-swell phenomenon, (Åström and Bell, 2000), which is caused by the distribution of steam bubbles under the water surface.

The total volume of water and steam in the boiler is given as: $V_t = V_w + V_s + V_b$, where V_w is the water volume, V_s is the volume of the steam space above the water surface and V_b is the volume of the steam bubbles below the water surface.

To capture the dynamics of the water/steam part the total mass and energy balances are considered. The total mass balance for the water/steam part leads to the following expression:

$$\left[(V_t - V_w)\frac{d\rho_s}{dp_s} + V_w\frac{d\rho_w}{dp_s}\right]\frac{dp_s}{dt} + (\rho_w - \rho_s)\frac{dV_w}{dt} = \dot{m}_{fw} - \dot{m}_s, \tag{2}$$

and the total energy balance for the water/steam part leads to:

$$\begin{pmatrix} \rho_w V_w \dfrac{dh_w}{dp_s} + h_w V_w \dfrac{d\rho_w}{dp_s} + \rho_s (V_t - V_w)\dfrac{dh_s}{dp_s} + \\ h_s(V_t - V_w)\dfrac{d\rho_s}{dp_s} - V_t + \rho_m V_m c_{p,m}\dfrac{dT_s}{dp_s} \end{pmatrix} \frac{dp_s}{dt} + \\ + (h_w\rho_w - h_s\rho_s)\frac{dV_w}{dt} = Q + h_{fw}\dot{m}_{fw} - h_s\dot{m}_s \tag{3}$$

where $\dot{m}_{fw}$ is the feed water flow, $\dot{m}_s$ is the steam flow,ρ is density, h is enthalpy and T is temperature, c_p is specific heat capacity and subscript m stands for metal. It should be noticed that energy accumulated in the boiler, furnace and convection tubes metal jackets are included in the balance for the water/steam part.

The two equations above only express the pressure and the water volume in the boiler. As the water level of interest in the control problem is given as: $L_w = (V_w + V_b - V_o)/A_{ws}$, another equation is needed for describing the volume of steam bubbles V_b in the water. (The water level is measured from the furnace top, V_o is the volume surrounding the furnace, and A_{ws} is the water surface area). To do this the mass balances for the steam bubbles and the water are combined with the empirical equation:

$$\dot{m}_{b\rightarrow s} = \gamma\frac{V_b}{V_w} + \beta\dot{m}_{w\rightarrow b}, \tag{4}$$

which expresses the amount of steam escaping the water surface, $\dot{m}_{b\rightarrow s}$ as function of the water volume, steam bubble volume and vaporization flow from water to bubbles $\dot{m}_{w\rightarrow b}$. This leads to the final differential equation describing the water/steam part:

$$\left((1-\beta)V_w\frac{d\rho_w}{dp_s} + V_b\frac{d\rho_s}{dp_s}\right)\frac{dp_s}{dt} + (1-\beta)\rho_w\frac{dV_w}{dt} + \\ + \rho_s\frac{dV_b}{dt} = (1-\beta)\dot{m}_{fw} - \gamma\frac{V_b}{V_w} \tag{5}$$

This equation introduces V_b in the model and thereby the shrink-and-swell phenomenon.

The shrink-and-swell phenomenon is only introduced through the variable V_b. From a physical point of view this seems natural as it is the steam bubbles that experience the non-minimum phase behavior and transfer this to the output water level, whereas the water volume/mass in the boiler does not exhibit the inverse response behavior.

In practice the water/steam circuit is closed and the steam flow is governed by several valves combined with pipe resistance. Therefore a variable $k(t)$ expressing pipe conductance and valve strokes is introduced. $\dot{m}_s$ is then given as:

$$\dot{m}_s(t) = k(t)\sqrt{p_s(t) - p_{dws}} \tag{6}$$

where the downstream pressure, p_{dws}, is the pressure in the feed water tank which is open and hence has ambient pressure, $p_{dws} = p_a$. $p_s(t) - p_{dws}$ is the differential pressure over the steam supply line.

The final model has the form:

$$F(\check{x})\dot{\check{x}} = h(\check{x}, \check{u}, \check{d}) \tag{7}$$

where $\check{x} = [p_s, V_w, V_b]^T$, $\check{u} = [\dot{m}_{fu}, \dot{m}_{fw}]$ and $\check{d} = k$. The temperature of the feed water is assumed constant and therefore not included in $\check{d}$.

A linear approximation of (7) can be generated for controller design. In (Solberg et al., 2007b) it was shown that the dynamics of the one-pass smoke tube boilers from AI, around the cross-over frequency has little dependency of the steam load. For this reason it suffices to focus on a controller design derived from one linear model hence leaving out any gain scheduling. Thus the sampled linear approximation of the marine boiler takes the form:

$$\check{x}(k+1) = \check{A}\check{x}(k) + \check{B}\check{u}(k) + \check{B}_d\check{d}(k) \tag{8a}$$

$$\check{y}(k) = \check{C}\check{x}(k) + \check{D}\check{u}(k) + \check{D}_d\check{d}(k) \tag{8b}$$

$$\check{x} \in \mathcal{X},\ \check{u} \in \mathcal{U}_{i(k)} \quad i(k) \in \{0,1,2\} \tag{8c}$$

where i is the current burner mode, $\check{y} = [p_s, L_w]^T$, $\mathcal{X} \subset \mathbb{R}^n$ and $\mathcal{U}_i \subset \mathbb{R}^m$ are compact sets describing constraints on state and inputs respectively.

2.2 Control Properties

For the marine boilers concerned the well known shrink-and-swell phenomenon from feed water flow to water level, (Åström and Bell, 2000), has not been observed in measurements. This means that this loop, in principle, is limited in bandwidth only by actuators and sensors (and model uncertainty).

Another property of the system is the high bandwidth in the response from the steam flow disturbance to the outputs. This complicates the controller design as it sets a requirement for a high closed loop bandwidth in order to suppress the effect of the disturbance. This means that the controller update frequency should be high limiting the time available between updates for on-line controller computations. In particular the controller sampling time is set to $T_s = 1$ second.

It is preferred to avoid the use of a flow sensor for steam flow measurement as such equipment is expensive. In (Solberg et al., 2005) it was shown that relying on an estimate of this flow provides satisfactory performance.

Regarding the control structure, it would be preferred to leave the burner switching to an underlying burner control system which delivers the requested fuel flow. However due to the long sequences associated with burner stop/start both pressure and level control are disturbed making this approach less suitable. This requires the burner switches to be handled by the pressure and water level controller.

One drawback of this strategy is that when switching from high to low load the total fuel flow becomes uncertain, as the distribution of fuel between the two burners is not modelled. Burner 2 is constrained only to turn off when the fuel flow is at a minimum, in order to avoid cutting off an unknown fuel flow in future predictions.

The control problem is formulated as follows:

Problem 1. At every sample instant k, given the current state $\check{x}(k)$, minimize the following performance index over $\check{\mathbf{u}} = [\check{u}(k), \check{u}(k+1|k), \dots]$:

$$J(\check{x}(k), \check{\mathbf{u}}) = \lim_{T\to\infty} \frac{1}{T}\Big\{ \sum_{j=1}^{M(T)} h_{m_{j-1},m_j} + T_s \sum_{i=0}^{T} \big[\check{z}^T(i+k|k)\check{Q}(i)\check{z}(i+k|k) + \Delta\check{u}^T(i+k|k)\check{R}(i)\Delta\check{u}(i+k|k)\big]\Big\} \tag{9}$$

where $\Delta\check{u}(i) = \check{u}(i) - \check{u}(i-1)$, $\check{z}(i) = \check{r}(i) - \check{y}(i)$ with the reference vector $\check{r}(i)$, $m \in \{0,1,2\}$, $M(T)$ is the total number of burner switches and h_{m_{j-1},m_j} is the cost associated with a switch from burner mode m_{j-1} to mode m_j. Also $\check{x}(i)$ and $\check{y}(i)$ evolves according to (8). $\check{Q}$ and $\check{R}$ are quadratic penalties on error and input changes.

Hence the control problem poses a trade-off between output (pressure and level) setpoint deviations and control input action including costs for burner switches. It would seem natural to include a cost on the accumulated fuel use; however this is not implemented. The reason is that the performance criterion is to achieve zero steady state errors for both pressure and water level. A weight on the accumulated fuel use will urge the system to save fuel at the expense of inferior pressure performance. Further the disturbance appears as infrequent steps in the load, meaning that the fuel used in the transient response is small compared to the steady state fuel use.

An important property of the performance (9) is that, dependent on the choice of weights, there may exist constant steam flows corresponding to the gap-regions, for which the cost of allowing a constant offset in the output is larger than that of introducing a limit cycle through switching the input. This would always be the case if $\check{z}$ included the integral error of the pressure, as any possible constant input would result in the pressure approaching a constant value different from the setpoint, meaning that (9) would be infinite. When $\check{z}$ does not include the integral error there still exist steam flows and choices of weights for which the integral over one cycle of period T_p, corresponding to a switching input, will be smaller than the corresponding integral over T_p with any possible constant input and converged output. Finding the optimal limit cycle which the state trajectory converges to can be achieved by posing a relatively simple optimization problem. The period of this limit cycle is dependent on the steam flow disturbance. The reason for this is that the steady state fuel flow required to achieve zero pressure error is dependent on the steam flow. When the required steady state fuel flow is in a gap-region and close to where the steady state solution is optimal, the limit cycle period is long because the pressure error only slowly grows to a level where the cost is comparable to the cost of switching Burner 1 or Burner 2 on and off. In the middle of the gap-region the pressure error will increase and decrease faster and the limit cycle period will be shorter.

3. METHODS

In this section we describe a method for solving control problem 1. The burner switch decisions will be made at the same level as the pressure and level control. This method incorporates both the finite state automaton and the dynamical system into one mixed integer optimization problem (MIP), which is solved repeatedly in a receding horizon manner.

3.1 Finite Horizon Model Predictive Control

Recently discrete time finite horizon MPC has become a tractable tool for the control of hybrid systems (Bemporad and Morari, 1999). The reason is that the method offers a systematic design procedure for these systems. Modeling tools such as HYSDEL (hybrid system description language) (Torrisi and Bemporad, 2004) make it easy to generate MLD models suitable for implementation with an MPC control law. This is done by describing the system to be controlled as a discrete time hybrid automaton. In (Heemels et al., 2001) the equivalence between a number of classes of hybrid systems was shown. This is important since it gives methods to identify which set of equivalent classes you should use for a particular control problem. Using this framework a hybrid model of the boiler system can be put up.

Hybrid control model The boiler system (8) including the state machine of the burner described in HYSDEL can be put together in the MLD form using tools from the MPT-toolbox (Kvasnica et al., 2004):

$$x(k+1) = Ax(k) + B_1u(k) + B_2\delta(k) + B_3z(k) \quad (10a)$$
$$y(k) = Cx(k) + D_1u(k) + D_2\delta(k) + D_3z(k) \quad (10b)$$
$$E_2\delta(k) + E_3z(k) \le E_1u(k) + E_4x(k) + E_5 \quad (10c)$$

where $x \in \mathbb{R}^{n_{x_r}} \times \{0,1\}^{n_{x_b}}$, $u \in \mathbb{R}^{n_{u_r}} \times \{0,1\}^{n_{u_b}}$ and $y \in \mathbb{R}^{n_y}$. $\delta \in \{0,1\}^{n_\delta}$ and $z \in \mathbb{R}^{n_z}$ represent Boolean and continuous auxiliary variables respectively. There are many possible realizations of the boiler system using this modeling tool depending e.g. on how burner switches are described. One possibility is to use the Boolean input to set a flag signaling that the burner should switch when the conditions for a switch are satisfied. Another possibility is to let the Boolean input indicate when to initiate a sequence (maneuver) which will lead to a switch. However the most general realisation is to let the Boolean input indicate a switch, hence to be able to set this input certain conditions must be satisfied. Using this realisation a state update sequence for the model is constructed as (borrowing notation from (Torrisi and Bemporad, 2004)):

Pseudo code for state update: Given old states $x(k)$ and input $u(k)$ complete the following updates to find $x(k+1)$:

Event generator: First events are logged. These are generated according to the satisfaction of linear affine constraints

$$\delta_e(k) = f_H(x_r(k), u_r(k), k) \quad (11)$$

where $f_H : \mathcal{X}_r \times \mathcal{U}_r \times \mathbb{Z}_{\ge 0} \to \mathcal{D} \subset \{0,1\}^{n_e}$. x_r is the real part of the state vector composed of $x_r(k) = [p_s(k), V_w(k), V_b(k), \dot{m}_{fu}(k-1), \dot{m}_{fw}(k-1), d_{um,1}(k), d_{um,2}(k), t(k), m(k)]^T$ where $d_{um,1}(k)$ is an unmeasured disturbance in the direction of the steam flow and $d_{um,2}(k)$ is an unmeasured disturbance in the direction of the feed water flow. Both disturbances are modelled as integrated white noise and included to achieve offset free tracking. t is a timing variable used during burner switches, and $m \in \{0,1,2\}$ is the current burner mode — but implemented as a continuous variable. u_r is the real part of the input vector given as $u_r(k) = [\Delta\dot{m}_{fu}(k), \Delta\dot{m}_{fw}(k)]^T$.

5 events are observed: 3 time events for operating the burner sequences during start and shut down, and 2 for detecting that fuel flow constraints are satisfied such that a burner switch may occur.

Finite state machine: The update of the state machine is done according to the deterministic logic function

$$x_b(k+1) = f_B(x_b(k), u_b(k), \delta_e(k)) \quad (12)$$

where $f_B : \mathcal{X}_b \times \mathcal{U}_b \times \mathcal{D} \to \mathcal{X}_b$. x_b is the Boolean part of the state vector describing the burner finite state machine: $x_b(k) = [n_0, n_{0,1}, n_1, n_{1,2}, n_2, n_{1,0}]^T$ (Fig. 1) and u_b is the Boolean part of the input vector denoting Burner 1 and 2 on and off respectively given as $u_b(k) = [u_{b,1}, u_{b,2}]^T$.

The i-th row of the function generally has the form

$$x_b^i(k+1) = (\text{stay}_i) \vee (\text{switch}_{1i}) \vee (\text{switch}_{2i}) \vee \cdots$$

where $\vee$ is the logical *OR* operator, stay_i is a logical expression returning 1 if the next Boolean state is equal to the current and switch_{ji} is a Boolean expression returning 1 if a switch from state j to state i should occur.

Mode selector: The mode selector is usually designed to determine which dynamics govern the system at current time k. However as mentioned in section 2.2 the model dynamics do not change much with the steam load; for this reason only one set of system matrices is implemented. Here the Mode selector is used to determine when the clock (state t) should be reset and start counting. This clock requires the introduction of one auxiliary continuous variable, z.

Continuous dynamics: With appropriate A_r, B_r, C_r and D_r the update of the continuous state dynamics and the output are done according to:

$$x_r(k+1) = A_rx_r(k) + B_ru_r(k) \quad (13a)$$
$$y(k) = C_rx_r(k) + D_ru_r(k) \quad (13b)$$
$$h(k) = m(k) - m(k-1) \quad (13c)$$

where $y(k) = [p_s(k), L_w(k)]^T$ and $h(k) \neq 0$ denotes a change in burner mode. Note that this is a slight abuse of the h notation from (9).

Constraint verification: Finally constraints are added to the update to describe allowed input combinations, changing constraints as a function of burner mode and fuel constraints during switches.

The vector of auxiliary Boolean variables $\delta(k)$ is composed of the 5 variables of δ_e mentioned above and 5 variables to determine statements related to the clock reset and fuel constraints and finally 6 variables for the logic statement in the update equation for the Boolean states.

□

Summarizing, this update scheme has been implemented in HYSDEL and the dimensions in the resulting MLD model are: $n_{x_r} = 9$, $n_{x_b} = 6$, $n_{u_r} = 2$, $n_{u_b} = 2$, $n_\delta = 16$, $n_z = 1$ and $n_y = 3$. Further the number of constraints is $N_c = 109$.

It should be mentioned that this model formulation is non-unique. For instance there are numerous ways to describe the logic associated with a burner switch. Furthermore, in this framework switches can only occur at sample time instants, which restricts the choice of sample time if the burner sequences must be implemented accurately.

Predictive control setup As mentioned in (Bemporad and Morari, 1999), solving a problem like *Problem* 1 subject to the MLD model (10) is not computationally feasible, because of the infinite horizon. Hence the criterion (9) in *Problem* 1 will be approximated by a finite horizon cost:

$$J(x(0), \mathbf{v}) = (r - y(T))^T P (r - y(T)) + \sum_{i=0}^{T-1} \left[(r - y(i))^T Q (r - y(i)) + u^T(i) R u(i) + h^T(i) H h(i) \right] \quad (14)$$

where the current time $k = 0$, $\mathbf{v} = [\mathbf{u}^T, \boldsymbol{\delta}^T, \mathbf{z}^T]^T$ with $\mathbf{u} = [u(0), \dots, u(T-1)]^T$, $\boldsymbol{\delta} = [\delta(0), \dots, \delta(T)]^T$, $\mathbf{z} = [z(0), \dots, z(T)]^T$, $Q = \text{diag}([q_1, q_2])$, $R = \text{diag}([r_1, r_2, 0, 0])$, and the switching cost is equal to $H = \frac{h_{0,1}}{T_s} = \frac{h_{1,0}}{T_s} = \frac{h_{1,2}}{T_s} = \frac{h_{2,1}}{T_s}$. The terminal cost P is set equal to Q.

There is a lower bound on the horizon length T. This due to the fact that the fuel flow to Burner 1 needs to be reduced to its minimum before Burner 2 can be fired. The bound can be found by the integral inequality which says that the average energy supplied to the system over the finite horizon starting from the maximal fuel flow in state n_1 making a transition to state n_2 must be greater than what one would get had one stayed in n_1, else a switch will never occur. This also depends on the weight on the switches; however only looking at the supplied energy the bound is $T > 22$. Furthermore, it is preferable to have the horizon at least as long as the limit cycle period, in case the disturbance corresponds to a gap-region. However for disturbances close to the boundary of the gap-region the period gets very long, suggesting a long T. This is not feasible and a trade-off between performance and computational resources has to be made. To predict the fastest possible cycle in both gap-regions we need $T > 43$, and a practical limit cycle needs an even longer horizon. Here we set $T = 45$.

The weight H in the performance index is important as it expresses the cost for a switch. There are many reasons for including such a weight. The first was discussed in the introduction: too many switches can cause wear on the supply system and degrade overall combustion performance. Also too frequent burner on/off switching can cause high frequency oscillation of boiler pressure and water level. However there is also a period after a burner switch in which the system is vulnerable to disturbances. The reason for this is due to the nature of the disturbance, which is unknown but appears as steps in the load (worst case from almost 0 to 100% load). The problem is that if a step load change is applied just after a burner is shut down, it takes time to turn the burner on again due to the burner start-up sequence. For Burner 1 shut-down purging is also necessary. Increasing the weight H reduces this problem.

Controller implementation The problem of minimizing (14) can be solved using a mixed integer quadratic programing solver. There are many such solvers available, of which some of the most popular have been tested, with mixed success. The problem becomes very dependent on the available optimization software. However due to the problem size, (horizon, constraints and number of Boolean variables) solving this optimization problem on-line is computationally prohibitive. A few off-line techniques, based on multiparametric programming and dynamic programming, has been suggested in the literature for defining the explicit control law (see e.g. (Bemporad et al., 2002; Borrelli et al., 2005)); however these methods are most suitable for relatively small systems using a relatively short prediction horizon. Instead on-line computational complexity must be reduced somehow.

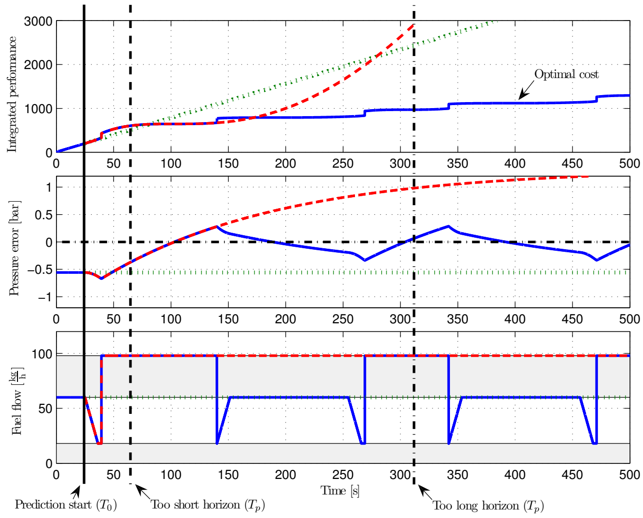

Fig. 2. Plot of the optimiser choosing to switch the input at prediction time 0 (T_0) (dash-dotted curves) or not to switch the input (dotted curves). The solid curves represent the optimal strategy according to the cost (9). The top plot shows the integrated cost without division by the time. The middle plot shows the pressure error and the bottom plot the fuel flow. Notice that the dash-dotted integrated cost becomes lower than the dotted one between 80 and 270 seconds.

The obvious way to do so is to restrict the Boolean decision variables to change only a few times in the prediction horizon, hence applying *input blocking* (Qin and Badgwell, 1997). However doing so introduces another problem. Previously the prediction horizon could be too short. But when using certain blocking schemes it can also be too long. In fact this generally occurs for systems which have interior regions of the input space which can not be reached. We shall illustrate this here using the cost (9) for the boiler, where we shall ignore water level and feed water contributions to the cost, and approximate the pressure by a first order system — since the level loop is closed. Suppose the blocking scheme is such that the Boolean input can only change at time 0; then the situation shown in Fig. 2 might occur.

From the figure it is easy to see how one can choose not only a too short, but also a too long prediction horizon. In the depicted situation what happens is that Burner 1 is *on* and Burner 2 is *off*; predicting far enough ahead, the benefit from switching Burner 2 *on*, causing the pressure to rise, will not be apparent, since the pressure will continue to rise, as Burner 2 can not be turned off again. This issue makes it very difficult to tune such algorithms and the prediction horizon must be chosen carefully considering several load disturbances.

One way to apply blocking is by introducing two new continuous input variables which represents times, T_1, T_2, at which a sequence to turn on or off Burner 1 or 2 should be initiated. Besides two new input variables, this method requires a new state variable describing absolute time over the prediction horizon, and an additional 4 mixed integer inequalities to be introduced. However this method has proven not to reduce the computational time enough to allow on-line computation.

The blocking scheme used in the final setup has full horizon for the continuous variables, whereas the Boolean variables are only allowed to change at times 0 and 1. Furthermore, the Boolean variables are defined to represent initiation of the sequence which will lead to a burner switch.

Instead of actually using model (10) as constraints in the optimization problem, and using an MIP solver, we simply implement the few optimization problems of the search tree and solve all of them at each sample time. This is necessary as introducing sequenced switches increases the model complexity to a degree where even a blocked strategy is not computationally feasible.

Regarding the feedback, a state estimator has been constructed. This estimator can operate in all modes and is hence independent of the control strategy discussed. The estimator is designed to achieve offset-free tracking of the pressure and water level. This is done by adding integrated disturbances to the process model in the direction of the steam load disturbance and the feed water flow — see e.g. (Pannocchia and Rawlings, 2003).

Remark 2. The above proposed method is suboptimal in two ways: first it solves a relaxed version of the original MIQP. Secondly the method has the inherent problem of operating over a finite horizon, which according to (Solberg et al., 2007a) is never optimal when the optimal state trajectory converges to a limit cycle, which is the case for the boiler system for certain energy requests corresponding to the gap-regions.

4. SIMULATION RESULTS

This section presents simulation results applying the controller presented in section 3 to the nonlinear simulation model of the marine boiler. Let us call this controller Design 1. The focus is directed to Gap-region 2 as this is the most interesting case regarding the sequences required to carry out a switch in Burner 2.

The simulation results are shown in Fig. 3 to the right. The figure also shows the results of applying traditional hysteresis control in combination with standard MPC, Design 2. The pressure setpoint is 8bar. The hysteresis control is given as:

$$u_{b,1} = \begin{cases} 0 \text{ for } p_s \geq 8.30\text{bar} \\ 1 \text{ for } p_s \leq 7.76\text{bar} \\ u_{b,1} \text{ otherwise} \end{cases}, u_{b,2} = \begin{cases} 0 \text{ for } p_s \geq 8.24\text{bar} \\ 1 \text{ for } p_s \leq 7.70\text{bar} \\ u_{b,2} \text{ otherwise} \end{cases} \quad (15)$$

The hysteresis bounds are asymmetric. Note that this could be avoided by e.g. defining a rule stating that a certain burner can not switch unless the estimated steady state fuel input (shown in the bottom left plot) is in a certain region of the input space. The MPC controller has the same weight matrices as the hybrid MPC controller. During burner switches the fuel flow is simply constrained to move along a predetermined trajectory.

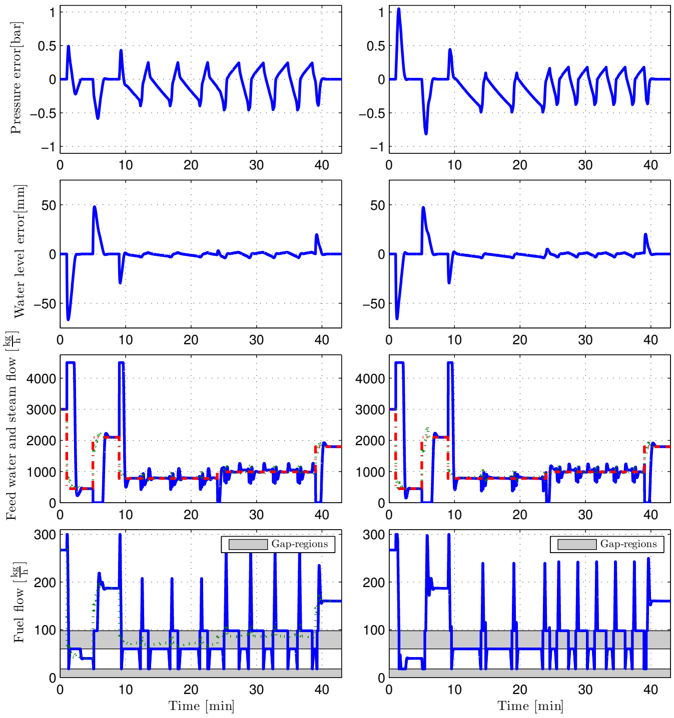

Fig. 3. Simulation results for Design 1 right and Design 2 left. From the top: the first row shows the pressure, the second row shows water level, the third row shows the feed water flow (solid), estimated (dotted) and measured (dash-dotted) disturbance both converted to represent requested steam flow, the last row shows the fuel flow and the gray fields correspond to the gap-regions, the plot on the left includes the estimated steady state fuel input. Notice the spikes in the fuel flow from 12 to 38 min. and the change in asymmetry in the pressure error oscillations in the same period for Design 1.

The disturbance profile used in the simulations is converted to represent the requested steam flow and is shown in the third row plots as dash-dotted curves. In the same plots the dotted curves represent the estimated disturbance also converted into a presumed requested steam flow.

There are a few things to notice in this figure regarding Design 1. The spikes in the fuel flow just after a burner switch from Mode 1 to Mode 2 are due to prediction mismatches. The horizon is not long enough, meaning that the algorithm cannot see the damage the choice of such a switch causes until it is too late. One could try adjusting the horizon length taking care not to make the horizon too long. In fact this method is very difficult to tune to achieve both good pressure and level control using reasonable control signals. Also it is worth noticing the asymmetry in the pressure oscillations when the disturbance corresponds to the gap-region. This stems from the maneuver which has to be performed during switches. When in Mode 1 and the maximum fuel input is injected, a switch to Mode 2 requires the fuel input first to reach the minimum level for Mode 1. As weights are put on both the pressure and input changes during these maneuvers it will naturally cost more to switch from Mode 1 to Mode 2 than the other way around. As the final performance criterion included a weight on pressure deviations and no weight on accumulated fuel use, this is not the desired performance. However this could be compensated e.g. by using a cost for the integrated pressure error, or by having asymmetric weights dependent on the current mode. However such implementations are not standard and quite cumbersome, for which reason we settle for the result presented above.

When comparing Design 1 and Design 2 there is an obvious difference in pressure behavior and hence burner switching. Design 1 can be viewed as a hysteresis controller which for some disturbances will act similar to Design 2. However, Design 1 can vary the hysteresis bounds to adapt to the current disturbance. The spikes in the fuel flow are present for both Designs (Design 2 as Design 1 does not know any better than to bring the pressure error to zero fast). Further evaluation of the performance (9) during the simulation period, for both designs, shows only small numerical differences. Also this difference is alternating in favour of Design 1 and Design 2.

Regarding the level control, only small oscillations are detected during burner on/off switching for both designs. This was consistent with an original objective, not to improve pressure performance at the expense of level regulation.

5. CONCLUSION

In this paper we described the application of the MPC/MLD method for hybrid model predictive control (Bemporad and Morari, 1999) to control of burner on/off switching in a marine boiler application. Simulation results proved adequate performance whilst indicating potential problems with the chosen strategy. The problem was seen as spikes in fuel flow after a burner switch indicating prediction mismatches in the receding horizon implementation.

This is a general shortcoming of hybrid model predictive control using a finite horizon when the state converges to a limit cycle — as was discussed in (Solberg et al., 2007a). This paper states that such systems are not rare in the industry. In particular systems including actuators which can be described as continuous in one region and discrete in another often have such properties. A well known example is valves (linear or exponential) which, to provide predictable performance, must run in on/off mode for low openings. These systems can sometimes be treated using PWM in the discrete region. However, when the on/off control has noticeable impact on the performance outputs (the switches are not filtered out by the system dynamics) or weights are assigned to switches other strategies must be applied, like the one described in the present paper.

The improvement over traditional PWM and hysteresis control is that the period and amplitude of the pressure oscillations during limit cycle behavior can adapt to the current disturbance to fulfill a desired performance criteria. The PWM and hysteresis controller can only be optimal for one disturbance and one operating point. Further the discussed method offers a systematic control design procedure though difficult to tune.

5.1 Future work

Generally focus should be directed towards developing infinite horizon predictive control strategies for hybrid systems.

In the context of marine boiler control it would be preferable to search for algorithms requiring less on-line computation. This could be some variant of hysteresis control.

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