Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Asymptotic Sensitivity Properties of Davison Type Integral Controllers for Time-Delay Plants

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Abstract: Sensitivity properties of the Davison type integral controllers for time-delay plants are discussed on the assumption that the observer gain matrix is fixes and the cheap control is used to determine the feedback gain matrix. It is shown that the Davison type integral controller can be expressed as a limit of the standard predictor-based LQG controller. The explicit representation of the asymptotic sensitivity matrix is obtained by a simple limiting procedure for the standard problem. A numerical example is presented to illustrate the difference from the standard predictor-based LQG controller.

1. INTRODUCTION

Integral control of a time-delay plant is often required in various fields of control applications especially in process control. Although various design techniques are currently available, the classical LQG technique is still attractive to design a controller with modest controller complexity.

For time-delay plants, Watanabe (1985) has proposed an integral controller which has the robust servo-mechanism structure proposed by Davison (1976). The controller is constructed based on the separation principle. The LQG theory can be used to determine the feedback gain matrix and the observer gain matrix. However, since the structure of the controller is different from the standard predictor-based LQG controller, the existing results of asymptotic properties for the predictor based LQG controllers (*e.g.*, Lee *et al*., 1988, Kwon *et al*., 1988, Wu *et al*., 1996) can not directly be applied.

The performance limitations of the Davison type controllers for lumped non-minimum phase plants have been discussed in the time domain and the frequency domain by Qiu and Davison (1993) and Ishihara *et al.* (2006), respectively. These results show that the Davison type controllers possess unique characteristics that can not be obtained as a simple special case of the LQG controllers.

The purpose of this paper is to clarify the asymptotic sensitivity properties of the Davison type integral controllers for time-delay plants. We focus our attention to the sensitivity properties at the plant output side when the cheap control is used. Our key observation is that the design of the Davison type integral controller can be regarded as a reduced order controller design for the extended plant. By applying our recent result (Ishihara and Zheng, 2005) on the relation between full order observers and reduced order observers, we can express the Davison type controller as a limit of the standard predictor-based LQG controller. By a simple limiting procedure, we can obtain the explicit representation of the asymptotic sensitivity matrix from the known LQG result. In addition, as a merit of our approach, we can easily identify the estimation problem related to the obtained asymptotic sensitivity matrix.

This paper is organized as follows: Section 2 introduces the Davison type integral controllers for time-delay plants with some preliminary observations. In Section 3, the explicit expression of the asymptotic sensitivity matrix is obtained. In Section 4, the related estimation problem is discussed. A numerical example is presented in Section 5. Concluding remarks are given in Section 6

2. PROBLEM FORMULATION

2.1 Davison Type Integral Controller

Consider a time-delay plant

$$\dot{x}(t) = Ax(t) + Bu(t-\tau), \quad y(t) = Cx(t), \tag{1}$$

where $x(t) \in R^n$ is a state vector, $u(t) \in R^m$ is an output vector, $y(t) \in R^m$ is a control input and τ is a time-delay. It is assumed that (A, B, C) is a minimal realization without zero at the origin ($s = 0$) and is minimum phase.

As a basic integral controller design, the method proposed by Watanabe (1985) is adopted with slight modifications. Assume that the plant state $x(t)$ is estimated by a full order observer

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t-\tau) + K\left[y(t) - C\hat{x}(t)\right], \tag{2}$$

where K is an observer gain matrix. Consider the extended system consisting of the plant and the integrators

$$\dot{\xi}(t) = \Phi\xi(t) + \Gamma u(t-\tau),\ \eta(t) = H\xi(t), \tag{3}$$

where $\eta(t) \in R^m$ is a state vector of the integrators and

$$\xi(t) \triangleq [\, x'(t) \quad \eta'(t)]', \\ \Phi \triangleq \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix},\ \Gamma \triangleq \begin{bmatrix} B \\ 0 \end{bmatrix},\ H \triangleq [\,0 \quad I\,]. \tag{4}$$

The output feedback regulator is constructed for the extended system using the separation principle. The control input is given by

$$u(t) = -F\hat{\xi}(t+\tau/t) \tag{5}$$

where F is a stabilizing state feedback gain matrix and $\hat{\xi}(t+\tau/t)$ is a prediction of $\xi(t+\tau)$ based on the input-output data up to time t. The prediction is given by

$$\hat{\xi}(t+\tau/t) \triangleq e^{\Phi\tau}\hat{\xi}(t) + \int_{t-\tau}^{t} e^{\Phi(t-\sigma)}\Gamma u(\sigma)d\sigma, \tag{6}$$

where

$$\hat{\xi}(t) \triangleq [\, \hat{x}'(t) \quad \eta'(t)]'. \tag{7}$$

The structure of the output feedback regulator for the extended system is shown in Fig. 1.

Define the partition of the state feedback matrix F as

$$F = [F_x \ \ F_\eta]. \tag{8}$$

Noting that the matrix exponential in (6) is written as

$$e^{\Phi\tau} = \begin{bmatrix} e^{A\tau} & 0 \\ C\left(\int_0^\tau e^{A\sigma}d\sigma\right) & I \end{bmatrix}, \tag{9}$$

we can express the Laplace transform of (5) as

$$u(s) = -M_\tau \hat{x}(s) - \frac{1}{s}F_y y(s) - N_\tau(s)u(s), \tag{10}$$

where

$$M_\tau \triangleq F_x e^{A\tau} + F_\eta C\left(\int_0^\tau e^{A\sigma}d\sigma\right), \tag{11}$$

$$N_\tau(s) \triangleq \left(F_x + \frac{1}{s}F_y C\right)\left[I - e^{-(sI-A)\tau}\right](sI-A)^{-1}B \\ -\frac{1}{s}F_\eta e^{-\tau s}C\left(\int_0^\tau e^{A\sigma}d\sigma\right)B. \tag{12}$$

Moving the integrators in the extended system to the controller part and inserting the reference input in (10), we can obtain the Davison type integral controller as Fig. 2.

2.2 Davison Type Integral Controller as a Reduced Order Controller Design

From the construction of the Davison type integral controller described in the previous section, we have the following observation.

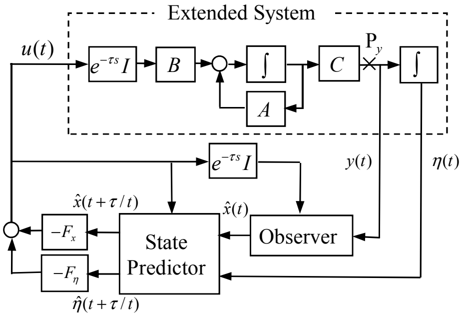

Fig. 1. Structure of the output feedback regulator for the extended system with the time-delay

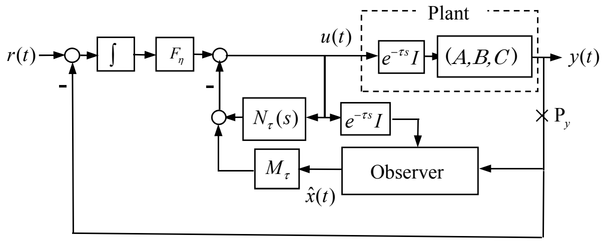

Fig. 2. Structure of the Davison type integral controller for the time-delay plant

The dimension of the lumped part of the extended system in Fig. 1 is $n+m$ while that of the controller in Fig. 1 is n which is the dimension of the lumped part of the plant. This means that the Davison type integral controller is designed as a reduced order controller for the extended system. After the design, the augmented integrator in the extended system is moved to the controller part for the implementation.

The above observation suggests that the Davison type integral controller in essence has characteristics of a reduced order controller.

2.3 Asymptotic Sensitivity by Cheap Control

Assume that the state feedback matrix F in (5) is determined by the quadratic performance index

$$J_\eta \triangleq \int_0^\infty [q\eta'(t)\eta(t) + u'(t)Ru(t)]dt, \tag{13}$$

where $q > 0$ and $R > 0$. The optimal feedback gain matrix F is given by

$$F \triangleq R^{-1}\Gamma' P_c, \tag{14}$$

where P_c is a solution of the Riccati equation

$$\Phi' P_c + P_c\Phi - P_c\Gamma R^{-1}\Gamma' P_c + qH'H = 0. \tag{15}$$

Our purpose is to clarify the asymptotic sensitivity properties at the point P_y in Fig. 2 as the weight q tends to infinity with the fixed observer gain matrix K.

Note that the sensitivity properties of the Davison type integral controller at the point P_y in Fig. 2 are equivalent to those at the point P_y in Fig. 1.

3. ASYMPTOTIC SENSITIVITY PROPERTIES

3.1 Fictitious Integral Controller

Consider the control system in Fig. 1. The controller transfer function matrix from the output $y(t)$ to the control input $u(t)$ can be expressed as

$$C(s) \triangleq -\left[I + N_\tau(s) + M_\tau(sI - A + KC)^{-1} Be^{-\tau s} \right]^{-1} \left[M_\tau(sI - A + KC)^{-1} K + (1/s)F_\eta \right], \quad (16)$$

where M_τ and $N_\tau(s)$ are defined in (11) and (12), respectively.

Recently, Ishihara and Zheng (2005) have shown that, for a given reduced order observer based controller, we can always find an equivalent controller including high gain full order observer. A brief summary of the result is given in the Appendix A. Applying this result to the observation given in 2.2, we reveal the asymptotic sensitivity properties.

First, we construct a controller including a high gain full order observer for the extended system.

For the extended system described by (3) and (4), we consider the standard predictor-based regulator including a full order observer for the extended state $\xi(t)$:

$$\dot{\hat{\xi}}_f(t) = \Phi \hat{\xi}_f(t) + \Gamma u(t-\tau) + K_f[\eta(t) - H\hat{\xi}_f(t)], \quad (17)$$

where K_f is an observer gain matrix. Note that $\hat{\xi}_f(t)$ includes the estimate of $\eta(t)$ as well as that of $x(t)$.

The control input for the fictitious controller is given by

$$u(t) = -F\hat{\xi}_f(t+\tau/t), \quad (18)$$

where $\hat{\xi}_f(t+\tau/t)$ is the prediction

$$\hat{\xi}_f(t+\tau/t) \triangleq e^{\Phi\tau}\hat{\xi}_f(t) + \int_{t-\tau}^{t} e^{\Phi(t-\sigma)}\Gamma u(\sigma)d\sigma. \quad (19)$$

The structure of the above controller is shown in Fig. 3.

For the control system in Fig. 3, the transfer function matrix from $\eta(t)$ to $u(t)$ can be expressed as

$$C_f(s) \triangleq -\left\{ I + F\left[I - e^{-(sI-\Phi)\tau} \right](sI-\Phi)^{-1}\Gamma + Fe^{-(sI-\Phi)\tau}(sI - \Phi + K_f H)^{-1}\Gamma \right\}^{-1} Fe^{\Phi\tau}(sI - \Phi + K_f H)^{-1}K_f. \quad (20)$$

Now we give an explicit procedure for constructing the fictitious integral controller equivalent to the controller (16).

Proposition 1: Consider the controller (20) with the fixed feedback gain matrix F and the special choice of the observer gain matrix

$$K_f(\lambda) \triangleq \begin{bmatrix} (A+\lambda I)K \\ CK + \lambda I \end{bmatrix}, \quad (21)$$

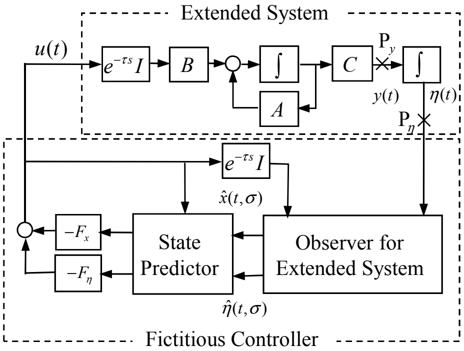

Fig. 3. Structure of the fictitious controller

where λ is a positive scalar and K is the observer gain matrix used in the controller (16). Let $C_f(s,\lambda)$ denote the controller transfer function matrix (20) with (21). Define the controller given by $C_f(s,\lambda)/s$ as the fictitious integral controller. Then

$$\frac{1}{s}C_f(s,\lambda) \to C(s), \quad (22)$$

as $\lambda \to \infty$ pointwise in s. □

The matrix (21) is obtained from the Proposition A in the Appendix A by choosing $A_{11} = A$, $A_{12} = 0$, $A_{21} = C$ and $A_{22} = 0$ with $L = K$ in (A.6). The proof is almost similar to the Proposition A but requires somewhat different matrix calculation due to the structural difference. The outline of the proof is given in the Appendix B.

Note that the fictitious integral controller consists of the integrators in the extended system and the controller (20) with (21).

3.2 Asymptotic Sensitivity Matrix

Consider two limiting operations $q \to \infty$ and $\lambda \to \infty$ for the fictitious integral controller. It is known that the feedback gain matrix F depends linearly on q for sufficiently large q. In addition, by the definition (21), λ appears in matrix bilinear form in the denominator and numerator matrices in the controller transfer function matrix (20) with (21). It can be shown that the two operations are interchangeable for a fixed s so that identical result is obtained irrespective of the order of the operations.

Proposition 1 guarantees that the desired asymptotic sensitivity matrix is obtained by applying $\lambda \to \infty$ followed by $q \to \infty$ since it is equivalent to the direct approach using the transfer function matrix (16) with $q \to \infty$.

We take a reverse approach. Fixing λ in the controller (20) with (21), we apply $q \to \infty$ as a first step followed by $\lambda \to \infty$. For the first step, we can use the result of the standard predictor-based LQG controller (Wu *et al.*, 1996) as follows,

Lemma 1: Consider the predictor-based LQG controller given by (17)-(19) with the observer gain matrix (21) for the fixed λ. Assume that the state feedback gain matrix F is determined by the quadratic performance index (13). As $q \to \infty$, the sensitivity matrix at the point P_η in Fig. 3 approaches

$$\Sigma_\tau(s) \triangleq \left\{ I + H\left[I - e^{-(sI-\Phi)\tau}\right](sI-\Phi)^{-1}K_f(\lambda)\right\}\Sigma_0(s), \quad (23)$$

where

$$\Sigma_0(s) \triangleq [I + H(sI-\Phi)^{-1}K_f(\lambda)]^{-1} \quad (24)$$

is the sensitivity matrix of the observer (17). □

As the second step, we can easily show the following result by letting $\lambda \to \infty$ in (23) and (24).

Proposition 2: Consider the Davison type integral controller shown in Fig. 2 with the full order observer gain matrix K and the state feedback gain matrix F determined by the quadratic performance index (13). As $q \to \infty$, the sensitivity matrix at the point P_y in Fig. 2 approaches

$$\Sigma_\tau^*(s) \triangleq V_\tau(s)\Theta(s), \quad (25)$$

where

$$V_\tau(s) \triangleq I + C\left[I - e^{-(sI-A)\tau}\right](sI-A)^{-1}K - e^{-\tau s}\left[I + C\left(\int_0^\tau e^{A\sigma}d\sigma\right)K\right]. \quad (26)$$

and

$$\Theta(s) \triangleq \left[I + C(sI-A)^{-1}K\right]^{-1} \quad (27)$$

is the sensitivity matrix of the observer (2). □

From the above result, we readily have the following approximations in high frequency region.

Corollary 1: For sufficiently large ω,

$$\Sigma_\tau^*(j\omega) \approx I - e^{-j\omega\tau}\left[I + C\left(\int_0^\tau e^{A\sigma}d\sigma\right)K\right], \quad (28)$$

$$\Pi_\tau^*(j\omega) \triangleq I - \Sigma_\tau^*(j\omega) \approx e^{-j\omega\tau}\left[I + C\left(\int_0^\tau e^{A\sigma}d\sigma\right)K\right], \quad (29)$$

where $\Pi_\tau^*(j\omega)$ is the complementary sensitivity matrix.

Remark 1: It can easily be checked that $V_\tau(s)$ defined in (27) satisfies $V_\tau(0)=0$ for all $\tau \geq 0$, which guarantees the integral action in the controller.

Remark 2: For the delay-free case ($\tau = 0$), it follows from (27) that $V_0(s)=0$ for all s, which implies that the zero sensitivity for all frequencies is asymptotically achieved for arbitrary choice of the observer gain matrix K. In our view, this result is plausible from the fact that the Davison type integral controller is equivalent to the fictitious integral controller including a high gain full order observer. This property has been pointed out by Ishihara *et al*. (2006).

Remark 3: For the standard LQG controller including full order observer with finite observer gain, the asymptotic complementary sensitivity always has roll-off in high frequency region. The expression (29) shows that the Davison type integral controller does not have this property. The difference arises from the fact that the Davison type integral controller is equivalent to the fictitious integral controller including a high gain full order observer.

4. RELATED ESTIMATION PROBLEM

It is well known that, for the standard LQG controller, the asymptotic sensitivity properties achieved by the cheap control is related to the estimation error dynamics of the observer (*e.g.*, Stein and Athans, 1987). For the predictor-based LQG controller, Wu *et al.* (1996) have discussed related estimation problem. For our problem, their result can be stated as follows.

Consider the estimation problem with the observation delay:

$$\dot{\xi}(t) = \Phi\xi(t), \quad \mu(t+\tau) = H\xi(t), \quad (30)$$

where $\xi(t)$, Φ and H are defined in (4) and $\mu(t+\tau)$ is the observation vector with the observation delay τ. Using the observer gain matrix $K_f(\lambda)$ used in (23), we can construct a filtering type observer for (28) as

$$\dot{\hat{\xi}}(t) = \Phi\hat{\xi}(t) + K_f(\lambda)\left[\mu(t+\tau) - H\hat{\xi}(t)\right], \quad (31)$$

where $\hat{\xi}(t)$ is an estimate of $\xi(t)$ based on the data up to time $t+\tau$. Define the prediction values of $\xi(t)$ and $\mu(t)$ based on the observation up to time t as

$$\hat{\xi}_p(t) \triangleq e^{\Phi\tau}\hat{\xi}(t-\tau), \quad \hat{\mu}_p(t) \triangleq H\hat{\xi}_p(t-\tau), \quad (32)$$

respectively.

The dynamic system generating the prediction error has the following feedback properties.

Lemma 2: Define the prediction errors of $\hat{\xi}_p(t)$ and $\hat{\mu}_p(t)$ for the estimation problem (30) as

$$\tilde{\xi}_p(s) \triangleq \xi(s) - \hat{\xi}_p(s), \quad \tilde{\mu}_p(t) \triangleq \mu(t) - \hat{\mu}_p(t), \quad (33)$$

respectively. Then the prediction errors are generated by the feedback system consisting of two subsystems

$$\dot{\tilde{\xi}}_p(t) = \Phi\tilde{\xi}_p(t) + \nu(t), \quad \tilde{\mu}_p(t+\tau) = H\tilde{\xi}_p(s), \quad (34)$$

$$\nu(t) = -\Xi_\tau(s)\tilde{\mu}_p(t) \quad (35)$$

where

$$\Xi_\tau(s) \triangleq e^{\Phi\tau}K_f(\lambda)\left\{I + H\left[I - e^{-(sI-\Phi)\tau}\right](sI-\Phi)^{-1}K_f(\lambda)\right\}^{-1}. \quad (36)$$

The subsystem (34) corresponds to an open loop error dynamics while the subsystem (35) is an error compensator with the infinite-dimensional frequency-shaped feedback gain matrix (36). For the feedback system, the sensitivity matrix at the output of the system (34) is given by (23) and (24). □

Letting $\lambda \to \infty$ in the expression (36), we can identify the prediction error dynamics corresponding the asymptotic sensitivity matrix (25).

Proposition 3: Consider the estimation problem where the prediction errors are generated by the two subsystems

$$\begin{cases} \dot{\tilde{x}}_p(t) = A\tilde{x}_p(t) + v_x(t), \ \ \dot{\tilde{\eta}}_p(t) = C\tilde{x}_p(t) + v_\eta(t), \\ \tilde{\mu}_p(t+\tau) = \tilde{\eta}_p(t), \end{cases} \quad (37)$$

$$\begin{bmatrix} v_x(t) \\ v_\eta(t) \end{bmatrix} = -\begin{bmatrix} e^{A\tau}K \\ I + C\left(\int_0^\tau e^{A\sigma} d\sigma\right) K \end{bmatrix} \bar{V}_\tau^{-1}(s)\tilde{\mu}_p(t) \quad (38)$$

where $\bar{V}_\tau(s)$ is defined by use of (26) as

$$\bar{V}_\tau(s) = \frac{1}{s} V_\tau(s). \quad (39)$$

Then, the sensitivity matrix at the output of the subsystem (37) coincides with (25). □

The above result gives a simpler feedback system which provides the same sensitivity properties as the asymptotic sensitivity properties of the Davison type controller. However, the prediction error dynamics is rather complex compared with the standard LQG controller.

Remark 4: As pointed out in Remark 1, $V_\tau(s)$ defined in (26) has always has a zero at $s = 0$ so that s in the denominator of (39) is cancelled by the zero of $V_\tau(s)$ at $s = 0$.

Remark 5: In the case that the matrix A is non-singular, it is easy to show that the matrix $\bar{V}_\tau(s)$ defined in (39) can explicitly be given as

$$\begin{aligned} \bar{V}_\tau(s) &= h_\tau(s)[I + C(sI - A)^{-1}K] \\ &\quad + e^{-\tau s}C(sI - A)^{-1}(I - e^{A\tau})A^{-1}K, \end{aligned} \quad (40)$$

where

$$h_\tau(s) \triangleq (1 - e^{-\tau s})/s. \quad (41)$$

5. NUMERICAL EXAMPLE

Consider a simple time-delay plant given by

$$G(s) = \frac{e^{-5s}}{(1+10s)(1+60s)}. \quad (42)$$

As a realization of the lumped part of (42), we choose

$$A = \begin{bmatrix} -0.017 & -0.017 \\ 0 & -0.1 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \ C = \begin{bmatrix} 1 & 0 \end{bmatrix}. \quad (43)$$

The observer gain matrix K is determined as the Kalman filter gain for the dynamic noise covariance matrix $W = \rho BB'$ with the positive parameter ρ and the observation noise variance $V = 1$.

The magnitude characteristics of the asymptotic sensitivity (25) and the corresponding complementary sensitivity are shown in Fig. 4 for three Kalman filter gain matrices corresponding to $\rho = 1, 10$ and 100. As ρ increases, the sensitivity at low frequency region decreases. The peaks and dips of the sensitivity in high frequency region can be explained by the approximation (28). The complementary sensitivity becomes flat without roll-off at high frequency region as expected from (29), which has been pointed out in Remark 3.

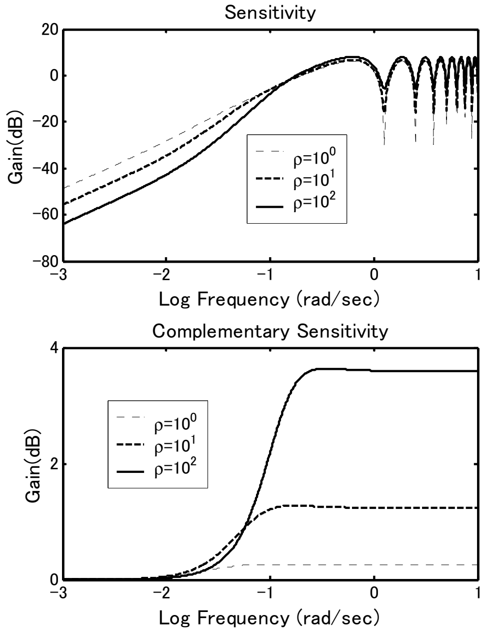

Fig. 4. Asymptotic feedback properties for the example

For a finite q, the complementary sensitivity has 40dB/dec roll-off in the frequency region higher than a frequency determined by q. Note that the convergence to the asymptotic sensitivity function or the complementary sensitivity function occurs frequency-wise. We confirm numerically that the convergence as $q \to \infty$ occurs from low frequency with significant differences in high frequency region.

Although care should be taken in interpreting the asymptotic properties in high frequency region, the asymptotic sensitivity and the complementary sensitivity provide useful information for the choice of the observer gain matrix K.

6. CONCLUSIONS

Our recent result (Ishihara and Zheng, 2005) has been used to reveal the asymptotic sensitivity properties of the Davison type integral controllers for time-delay plants.

It is an interesting future problem to compare the result with the other type of integral controller designs (*e.g.*, Ishihara *et al.*, 2005, Ishihara and Guo, 2007, Wu *et al.*, 2007).

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APPENDIX A. REDUCED ORDER OBSERVER AS A HIGH GAIN FULL ORDER OBSERVER

Consider the state estimation problem for the plant

$$\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t), \tag{A.1}$$

where $x(t) \in R^n$, $u(t) \in R^m$ and $y(t) \in R^m$. Assume that the state vector and the matrices are given in the partitioned form

$$\begin{gathered} x(t) \triangleq \begin{bmatrix} x_1'(t) & x_2'(t) \end{bmatrix}', \\ A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & I \end{bmatrix}. \end{gathered} \tag{A.2}$$

For the state space representation (A,1), we can easily construct a reduced order observer as

$$\dot{z}(t) = \Phi_L z(t) + \Theta_L y(t) + \Gamma_L u(t) , \tag{A.3}$$

$$\hat{x}(t) = \begin{bmatrix} \hat{x}_1(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix} z(t) + \begin{bmatrix} L \\ I \end{bmatrix} y(t) , \tag{A.4}$$

where $z(t) \in R^{(n-m)}$ is a state vector of the observer, L is an $(n-m)\times m$ matrix and

$$\begin{gathered} \Phi_L \triangleq A_{11} - LA_{21}, \quad \Gamma_L \triangleq B_1 - LB_2, \\ \Theta_L \triangleq A_{11}L + A_{12} - LA_{22} - LA_{21}L. \end{gathered} \tag{A.5}$$

Ishihara and Zheng (2005) have shown that, for a given reduced order observer based controller, we can always find an equivalent controller including a high gain full order observer. The result is formally stated as follows.

Proposition A: Assume that the reduced order observer with the matrix L is given. Consider the controller generating the control input $u(t) = -F\hat{x}(t)$, where $\hat{x}(t)$ is the state estimate given by the reduced order observer. Let $C(s)$ denote the controller transfer function matrix of the reduced order observer based controller. Construct the full order observer with the observer gain matrix

$$K_L(\lambda) \triangleq \begin{bmatrix} K_{L1}(\lambda) \\ K_{L2}(\lambda) \end{bmatrix} = \begin{bmatrix} (\lambda I + A_{11}) L + A_{12} \\ \lambda I + A_{21}L + A_{22} \end{bmatrix}, \tag{A.6}$$

where L is a design parameter of the given reduced order observer and λ is a positive scalar. Consider the controller given by $u(t) = -F\hat{x}_f(t)$, where $\hat{x}_f(t)$ is a state estimate obtained by the full order observer with the observer gain $K_L(\lambda)$. Let $C_f(s,\lambda)$ denote the controller transfer function matrix of the full order observe based controller. Then

$$C_f(s,\lambda) \to C(s) , \tag{A.7}$$

as $\lambda \to \infty$ pointwise in s. □

APPENDIX B. OUTLINE OF THE PROOF OF PROPOSITION 1

Define

$$T_K \triangleq \begin{bmatrix} I & K \\ 0 & I \end{bmatrix}. \tag{B.1}$$

Then it can easily be checked that

$$\Phi - K_f H = T_K \begin{bmatrix} A - KC & 0 \\ C & -\lambda I \end{bmatrix} T_K^{-1}, \tag{B.2}$$

$$\begin{aligned} &\left(sI - \Phi + K_f H\right)^{-1} \\ &= T_K \begin{bmatrix} (sI - A + KC)^{-1} & 0 \\ (s+\lambda)^{-1} C (sI - A + KC)^{-1} & (s+\lambda)^{-1} I \end{bmatrix} T_K^{-1}. \end{aligned} \tag{B.3}$$

Using the above matrix identities and the expression (9) for the matrix exponential, we can easily show that the numerator matrix in (20) satisfies

$$\begin{aligned} &F e^{\Phi\tau} (sI - \Phi + K_f H)^{-1} K_f \\ &\quad \to sM_\tau (sI - A + KC)^{-1} K + F_\eta, \end{aligned} \tag{B.4}$$

as $\lambda \to \infty$ pointwise in s. By similar straightforward matrix calculation, we can show that the denominator matrix in (20) converges to that of (16). Consequently, the relation (22) can be proved.