Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

TEAM BUILDING UNDER PARETO UNCERTAINTY

Dmitry A. Novikov

Institute of Control Sciences, Russian Academy of Sciences
Profsojuznaya st, 65, Moscow, 117997, Russia
novikov@ipu.ru

Game-theoretical models of team-building and team incentive problems are described. Solutions of these problems are obtained for the case of probabilistic uncertainty of the team members abilities. Conditions of team «vitality» are formulated in terms of the distribution and the reservation utility properties.

Keywords – Social Systems, Agents, Game Theory, Uncertainty, Team Building

1. INTRODUCTION

During the last decades more and more attention in management, project management, social psychology, etc., is paid to the team activity of the organization personnel. A team is understood as the collective (the community of people, who implement joint activity and possess common interests) that is able to achieve goals autonomously and coordinately under minimal control.

There are two separate mainstreams in team mathematical model: the incentive problems under uncertainty (which are the subject of investigation in the contract theory) – see Hart and Holmstrom, 1987, Mas-Collel, *et al*. (1995) and the team theory – see Holmstrom, 1982, Marshak and Radner, 1976, Novikov, 2001, 2005.

This paper develops the team-building models under uncertainty of agents abilities described by Pareto distribution, and is organized as follows. At first, some properties of Pareto distribution required for further presentation are briefly discussed in Section 2. Then the solution of the team incentive problem is given in Section 3. Section 4 presents the team-building problem statement. Sections 5 and 6 are devoted to the problem solution for several rather general cases. The description of practical application and interpretation of formal results are missed, so the trivial numerical examples are omitted for the sake of brevity. The conclusion contains the discussion of several possible generalizations and challenges.

2. PARETO LAW AND PARETO DISTRIBUTION

The so-called Pareto law sometimes called «80/20 law» or, in professional cant, «beer law» (20 % of persons drink 80 % of beer) is widely known. This law reflects the distribution heterogeneousness for characteristics of economic and social phenomena and processes:
- 20 % of population own 80 % of assets (initial formulation by V. Pareto himself – Pareto, 1897; see also the review of Levy, 2001;
- 80 % of entire stored stock cost falls to 20 % of stock nomenclature;
- 80 % of sale income is brought by 20 % of buyers;
- 20 % of efforts bring 80 % of result;
- 80 % of problems are due to 20 % of causes;

- employees perform 80 % of work for 20 % of their working time;
- 80 % of work is done by 20 % of employees, and so on.

The Pareto law formalization is represented by the Pareto distribution of random variable $z \geq y > 0$ characterized by two parameters such as the minimum possible value of y and exponential quantity $\alpha > 0$:

$$p(\alpha, y, z) = \frac{\alpha}{y}\left(\frac{y}{z}\right)^{1+\alpha}. \quad (1)$$

The integral distribution function

$$F(\alpha, y, z) = 1 - \left(\frac{y}{z}\right)^{\alpha} \quad (2)$$

corresponds to distribution density (1).

The Pareto distribution is self-similar: the distribution of values exceeding $z^0 \geq y$ also is the Pareto distribution:

$$\forall\, z^0 \geq y \;\; p(\alpha, z^0, z) = p(\alpha, y, z) / (1 - F(\alpha, y, z^0)) = \frac{\alpha}{z^0}\left(\frac{z^0}{z}\right)^{1+\alpha}. \quad (3)$$

For the Pareto distribution, there exist only the moments that are smaller than the degree α. For example, the expectation of the random variable z with distribution (1) exists for $\alpha > 1$ and equals to (E denotes the expectation operator)

$$E\,z = \frac{\alpha}{\alpha - 1}\, y, \quad (4)$$

If a random variable is assumed to have the Pareto distribution, then with a knowledge of the expectation Ez and minimum value of y, one can directly calculate the distribution parameter α (see (4)):

$$\alpha = \frac{Ez}{Ez - y}. \quad (5)$$

Let us present a formal interpretation of «80/20 law». Suppose that z characterizes agent efficiency, while the Pareto distribution defines the amount of agents with certain efficiency. Define $\tilde{z}$ such that Prob $\{z \leq \tilde{z}\} = 0.8$: $\tilde{z} = (0.2)^{\frac{1}{\alpha}} z_0$, where z_0 is the minimum efficiency. Then define the summarized efficiency of «top-agents»: $\int_{\tilde{z}}^{+\infty} z p(z_0, \alpha) \mathrm{d}z = (0.2)^{\frac{\alpha-1}{\alpha}} \frac{\alpha}{\alpha - 1} z_0$ that must be equal to 80 % of total efficiency: $(0.2)^{\frac{\alpha-1}{\alpha}} \frac{\alpha}{\alpha - 1} z_0 = 0.8 \frac{\alpha}{\alpha - 1} z_0$.
Thus, if $\alpha = 1.161$, the Pareto distribution describes «80/20 law».

Below the Pareto distribution is applied to the uncertainty description for abilities of agents considered as potential team members.

3. TEAM INCENTIVE PROBLEM

Consider the team $N = \{1, 2, \ldots, n\}$, consisting of n agents. The efficiency function of i-th agent is given by

$$f_i(y_i, r_i) = \sigma_i(y_i) - c_i(y_i, r_i),\; i \in N, \quad (6)$$

where $y_i \geq 0$ is the action of i-th agent, r_i is its type (reflecting qualification, abilities, activity efficiency, etc.), $c_i(\cdot)$ is the cost function, $\sigma_i(\cdot)$ is the incentive function associating agent action with nonnegative compensation – see Novikov (2007).

Assume that the cost functions of agents are the same, and the agents are differ only in their types $c_i(y, r) = c(y, r)$, $i \in N$. Also assume that the cost function $c(y, r)$ is a smooth, convex function non-decreasing in the action y, non-increasing in the type r, i.e. $c_y > 0$, $c_y(0, r) = 0$, $c_{yy} \geq 0$, $c_r < 0$, and $c_{ry} \leq 0$. The power cost function $c(y, r) = \frac{1}{\gamma} y^{\gamma} r^{1-\gamma}$, $\gamma > 1$, used below for illustration, is an example of such a function.

As a criterion of team performance effectiveness, consider the difference between the sum of the agents' actions and their incentive cost:

$$G(\boldsymbol{y}, \boldsymbol{r}) = \sum_{i \in N} y_i - \sum_{i \in N} \sigma_i(y_i). \quad (7)$$

The behavior rationality of each agent consists in striving for maximization of its cost function (6) by means of independent choice of action: $y_i^*(\sigma_i(\cdot)) = \arg\max_{y_i \geq 0} [\sigma_i(y_i) - c_i(y_i, r_i)]$, $i \in N$.

If reservation utilities of the agents (guaranteed values of their cost functions) equal to zero, then the maximum of effectiveness (7) will be reached with guaranteed compensation of agents costs under condition that they choose the following actions: $x_i^*(r_i) = \arg\max_{y_i \geq 0} [y_i - c(y_i, r_i)]$. At that the team effectiveness equals to $\Phi(\boldsymbol{r}) = G(x^*(r), r) = \sum_{i \in N} [x_i^*(r_i) - c(x_i^*(r_i), r_i)]$. If the agents are of independent types and distributed equally then the team effectiveness equals to $E\Phi(\boldsymbol{r}) = n\,[Ex^*(\boldsymbol{r}) - Ec(\boldsymbol{x}^*(\boldsymbol{r}), \boldsymbol{r})]$, where $n = |N|$.

Assertion 1. If the agents types are given by the Pareto distribution $p(\alpha, r_0, r)$, where $\alpha > 1$, then the team effectiveness is given by

$$E\Phi(\boldsymbol{r}) = n\, r_0\, \frac{\alpha}{\gamma}\, \frac{\gamma - 1}{\alpha - 1}. \quad (8)$$

At the right-hand part of expression (8), among others, the team size appears (the quantity of its members n). It takes the opportunity stating and solving the problems of its composition optimization.

4. TEAM BUILDING PROBLEM

Consider the set of agents N_0, $|N_0| = n_0$, with types described by the Pareto distribution $p_r(\alpha, r_0, r)$. The problem consists in finding the set of agents $N \subseteq N_0$ that should be included into the team. Let us find the solution in the form $n = |N|$, where

$$n = n(r_{\min}) = |\{i \in N_0 \mid r_i \geq r_{\min}\}|, \quad (9)$$

that is find the minimum value of agent type that should be included into the team. Neglecting discontinuity here and further, from the Pareto distribution properties we obtain

$$n(r_{\min}) = n_0 \left(\frac{r_0}{r_{\min}} \right)^{\alpha}. \quad (10)$$

Substituting (10) into expression (8), with expression (4) in mind, we obtain:

$$E\Phi(\mathrm{r}_{\min}) = n_0 \left(\frac{r_0}{r_{\min}} \right)^{\alpha} r_{\min}\, \frac{\alpha}{\gamma}\, \frac{\gamma - 1}{\alpha - 1}. \quad (11)$$

Assertion 2. If the reservation utilities of the agents equal to zero, and the types of agents are described by the Pareto distribution $p_r(\alpha, r_0, r)$, where $\alpha > 1$, then the maximum team composition

$$n^* = n_0,\; r_{\min} = r_0, \quad (12)$$

is the optimum one, and the team effectiveness equals to $n_0\, r_0\, \frac{\alpha}{\gamma}\, \frac{\gamma - 1}{\alpha - 1}$.

Substantially, Assertion 2 states that the team effectiveness grows with entering any agents even with very small types. It is conditioned by the fact that by virtue of accepted assumptions the limiting agent cost in zero equals to zero, and the limiting agent efficiency equals to unity. In order to go away from «trivial» solution proposed by Assertion 2, it is necessary either changing the cost function, or introducing nonzero reservation utility of agents both entered and not entered into the team.

5. TEAM OPTIMIZATION PROBLEM

The optimal team composition synthesis problem was solved from zero point. In general, the team composition optimization problem consists in the most effective change of current composition, i.e. finding new agents that should be included into the team (hire problem) and the team members that must be excepted from the team (reduction problem or resources use effectiveness increase problem). Consider the latter problem for the case when the agent individual characteristics have the Pareto distribution.

Let observable distribution of the agent activity (selected actions) results be $p_y(\alpha, y_0, y)$. Define the effectiveness $K(N_0)$ of the team N_0 activity as the ratio of their expected activity result (expected sum of actions) to incentive cost.

Case 1. Let the resource divide into equal parts between the agents. Then $K(n_0) = n_0\, Ey / R$. We obtain

$$K(n_0) = n_0\, y_0\, \frac{\alpha}{\alpha - 1} / R. \quad (13)$$

Now let us use the principle similar to (9), i.e. enter into the team only the agents with actions not less than $y_{\min}$. We have

$$K(y_{\min}) = n_0\, y_{\min}\, \frac{\alpha}{\alpha - 1} / R. \quad (14)$$

From (14) it follows that the effectiveness is proportional to the «cutting point» $y_{\min}$. This conclusion is quite evident: more effective agents remain in the team, higher effectiveness with «leveling» payment system. But expected integral result of activity Y of all agents entered into the team for $\alpha > 1$ decreases with increase of the "cutting point", generally nonlinear:

$$Y(y_{\min}) = n_0 \left(\frac{y_0}{y_{\min}} \right)^{\alpha} y_{\min}\, \frac{\alpha}{\alpha - 1}. \quad (15)$$

To reach rational balance between the effectiveness growth and decrease of total result (with increase of $y_{\min}$), one need enter additional criteria. However it should be noted that the assumption on equal payments to all the agents demonstrating essentially different results seems to be not very realistic (it is impossible to use «saving» of resources obtained due to reduction of ineffective agents). Therefore consider the case of linear incentive system.

Case 2. Let the proportional payment system $\sigma_L(y) = \lambda y + \lambda_0$, where $\lambda > 0$, be used, and the agents cost functions are given by

$$c(y, r) = c_0 + r\,\varphi(y / r), \tag{16}$$

where $\varphi(\cdot)$ is the smooth non-decreasing function. The cost function of Cobb-Douglas type is the particular case of cost function (16). Then the action $y^*(r, \lambda)$ selected by the agent (maximizing its objective function for give incentive system) is $y^*(r, \lambda) = r\,\varphi^{'\,-1}(\lambda)$, i.e. it is proportional to the agent type, and hence, described by the distribution $p_y(\alpha, y_0, y)$, where $y_0 = r_0\,\varphi^{'\,-1}(\lambda)$.

At that, the expected sum of the agent actions is given by

$$Y_0 = n_0\, r_0\, \frac{\alpha}{\alpha-1}\, \varphi^{'\,-1}(\lambda). \tag{17}$$

The total expected agent costs must be compensated, i.e. the total expected incentive costs equal to

$$S(n_0, \lambda) = n_0\,[c_0 + r_0\, \frac{\alpha}{\alpha-1}\, \varphi(\varphi^{'\,-1}(\lambda))]. \tag{18}$$

If the amount of resource used for provision of incentive for the team members is fixed and equal to R then, equating $S(n_0, \lambda) = R$, we obtain

$$Y_0 = n_0\,[c_0 + r_0\, \frac{\alpha}{\alpha-1}\, \varphi^{-1}(\frac{(R - n_0 c_0)(\alpha-1)}{\alpha\, n_0 r_0})]. \tag{19}$$

Dividing (19) by R, derive the estimate of effectiveness

$$K(n_0, R) = \frac{n_0}{R}\,[c_0 + r_0\, \frac{\alpha}{\alpha-1}\, \varphi^{-1}(\frac{(R - n_0 c_0)(\alpha-1)}{\alpha\, n_0 r_0})]. \tag{20}$$

With use of principle (9), the expected sum of agents actions is

$$Y(r_{\min}) = n_0 \left(\frac{r_0}{r_{\min}}\right)^{\alpha} r_{\min}\, \frac{\alpha}{\alpha-1}\, \varphi^{'\,-1}(\lambda). \tag{21}$$

The total expected costs of agents must be compensated, i.e. the total expected incentive costs are

$$S(r_{\min}, \lambda) = n_0 \left(\frac{r_0}{r_{\min}}\right)^{\alpha} [c_0 + r_{\min}\, \frac{\alpha}{\alpha-1}\, \varphi(\varphi^{'\,-1}(\lambda))]. \tag{22}$$

Equating $S(r_{\min}, \lambda) = R$, we have

$$Y(r_{\min}) = n_0 \left(\frac{r_0}{r_{\min}}\right)^{\alpha} r_{\min}\, \frac{\alpha}{\alpha-1}\, \varphi^{-1}\left(\frac{[R - n_0 c_0 \left(\frac{r_0}{r_{\min}}\right)^{\alpha}](\alpha-1)}{\alpha\, n_0 \left(\frac{r_0}{r_{\min}}\right)^{\alpha} r_{\min}}\right). \tag{23}$$

Dividing (23) by R, obtain the effectiveness estimation

$$K(r_{\min}) = \frac{n_0}{R} \left(\frac{r_0}{r_{\min}}\right)^{\alpha} r_{\min}\, \frac{\alpha}{\alpha-1}\, \varphi^{-1}\left(\frac{[R - n_0 c_0 \left(\frac{r_0}{r_{\min}}\right)^{\alpha}](\alpha-1)}{\alpha\, n_0 \left(\frac{r_0}{r_{\min}}\right)^{\alpha} r_{\min}}\right). \tag{24}$$

Let us denote $r^*_{\min} = \arg \max_{r_{\min} \geq r_0} Y(r_{\min})$. Comparing (20) and (24), we come to the following assertion.

Assertion 3. Let the bonus pool be fixed. Then with use of unified linear incentive system the value of «cutting point» maximizing the incentive effectiveness $K(r_{\min})$ coincides with the value of «cutting point» $r^*_{\min}$ maximizing the total expected result of agents activity.

6. ROLE OF RESERVATION UTILITY

The common qualitative conclusion following from the above results on considering the team composition formation problem consists in the following. If the linear incentive system is used for agent cost functions (11), the maximum composition is optimal under condition that the agent costs for choosing of zero actions (constant costs) equal to zero, and the reservation utility is also zero. If at least one of these parameters is nonzero, we obtain nontrivial solution. The case of nonzero constant costs c_0 is described above, therefore let us analyze the role of reservation utility.

Assume that all the agents entered into the composition of organizational system (i.e. the agents with the type exceeding $r_{\min}$) are required to be provided with the reservation utility u. Then expression (11) becomes

$$E\Phi(r_{\min}) = n_0 \left(\frac{r_0}{r_{\min}}\right)^{\alpha} [r_{\min}\, \frac{\alpha}{\gamma}\, \frac{\gamma-1}{\alpha-1} - u]. \tag{25}$$

Find the maximum of expression (25) in $r_{min} \geq r_0$. It is reached with

$$r^*_{min}(u) = u\ \frac{\gamma}{\gamma - 1}. \quad (26)$$

One can see that the higher reservation utility, the more strict qualification requirements to the agents entered into the team. It is interesting that the value of «cutting point» (26) does not depend on α, the Pareto distribution exponent.

Another problem statement is also possible. Assume that the team in addition has to provide the reservation utility u_0 for that agents which did not entered the team. Then expression (25) becomes

$$E\Phi(r_{min}) = n_0 \left(\frac{r_0}{r_{min}}\right)^{\alpha} [r_{min}\ \frac{\alpha}{\gamma}\ \frac{\gamma-1}{\alpha-1} - u] -$$

$$- n_0\, u_0\, (1 - \left(\frac{r_0}{r_{min}}\right)^{\alpha}). \quad (27)$$

If $u_0 = 0$, then (27) turns to (25). If $u = 0$ and $u_0 > 0$, then the maximum composition is optimum. Consider the intermediate case $u > 0$, $u_0 > 0$. Find maximum of expression (27) in $r_{min} \geq r_0$. It is reached with

$$\hat{r}_{min}(u_0, u) = (u - u_0)\ \frac{\gamma}{\gamma - 1}. \quad (28)$$

From (26) and (28) it follows that the team composition optimization is not unreasonable if

$$u - u_0 \geq \frac{\gamma - 1}{\gamma}\, r_0. \quad (29)$$

In other words, the difference between the reservation utilities of the agents entered into the team and the agents not entered into the team must be sufficiently large (see expression (29)).

Furthermore, with sufficiently large values of the reservation utilities, the team effectiveness may become negative as it may be unable satisfying the agents needs (reflected by their reservation utility). It means that this group of agents, in principle, cannot operate effectively with any incentive system and any optimization of its composition.

We obtain that the expectation of the team effectiveness for optimum composition and optimum agent incentive system is nonnegative if (29) and

$$\left(r_0\, \frac{\gamma - 1}{\gamma}\right)^{\alpha} \geq (\alpha - 1)\, u_0\, (u - u_0)^{\alpha - 1} \quad (30)$$

hold true.

In other words, condition (30) is the criterion of the team «vitality». Let us join together the obtained results into the following assertion.

Assertion 4. Let the team includes only the agents with type not less than $\hat{r}_{min}(u_0, u)$. Then the team composition is optimum.

7. CONCLUSION

The considered models can be extended to the case when each of the reservation utilities depends on the agent type. The substantial interpretation of this is rather clear. For example, the higher agent qualification, the higher reservation utility. The analysis technique remains the former: given function $u(r_{min})$, the maximum of expression (27) in $r_{min} \geq r_0$ must be found.

It should be emphasized that in the most models considered in this paper we assume that the quantity of resource R assigned for provision of incentive for the team members is fixed. If both fixed resource quantity the analytical solution of incentive problem and team composition problem is found, the problem of finding optimum resource quantity is the standard optimization problem (maximization of the team effectiveness in $R \geq 0$) with solution usually creating no problems.

Also denote that stating and solving the team composition optimization problem we direct attention to economic indexes and implicitly have in mind that the agents types (effectiveness of their activity) do not depend on dimension and composition of the team that are working in. In a number of cases this assumption is justified. However, sometimes it is not so. From research of psychologists and sociologists it can be seen that effectiveness and productivity of an individual activity can depend on observable and/or actually achieved results of his colleagues activity. That is both the composition and dimension of the team become essential meaning that the team reaches some «critical mass», especially in creative and well-qualified kinds of activity. Scientific groups can be the most striking example. As Norbert Viener said, «It is quite likely, that 95 % of original scientific works belong to less than 5 % of professional scientists, but their most part would never be written if remaining 95 % of scientists did not assist to creation of common sufficiently high level of science».

Furthermore, it must be taken into account that the statistical models considered above do not reflect the dynamics of agent types, for example, their growth in training and education. The dependence of productivity and efficiency on working experience of agent must be taken into account by long-sighted organization since acting locally optimal one can dismiss ineffective (for example, inexperienced) employees today, but in the nearest future suffer «age collapse» among mean-age employees.

The construction of formal models describing denoted effects seems to be the promising area of further research.

REFERENCES

Hart, O.D. and B. Holmstrom (1987). *Theory of Contracts,* Proceedings of the 5-th World Congress «Advances in Economic Theory», Cambridge University Press, Cambridge, 71-155.

Holmstrom, B. (1982). *Moral hazard in teams*, Bell Journal of Economics, **13,** 324-340.

Levy, M. (2001). *Market Efficiency, the Pareto Wealth Distribution and the Levy Distribution of Stock Returns,* Hebrew University, Jerusalem.

Marshak, J. and R. Radner (1976). *Economic Theory of Teams*, New Haven, London.

Mas-Collel, A., M.D. Whinston and J.R. Green (1995). *Microeconomic theory*, Oxford Univ. Press, N.Y.

Novikov, D.A. (2005). *Game-theoretical Models of Team Building*, Proc. of 16-th IFAC World Congress, Prague.

Novikov, D.A. (2007). *Incentive Problem for Risk-neutral Pareto-agent,* Automation and Remote Control, **1**, 78-85.

Novikov, D.A. (2001). *Management of Active Systems: Stability or Efficiency,* Systems Science, **2**, 85-93.

Pareto, V. (1897). *Cours d'Economique Politique*, Touluse.