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On-Line Estimation of Wind Turbine Power Coefficients Using Unknown Input Observers

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Abstract: As installed wind turbine energy generation capacity increases, the interest in optimizing these wind turbines increases as well. The optimal operating points for the power and speed control of the turbines depends on a mapping to the power conversion ratio (C_p) from tip speed ratio and blade pitch angles. This mapping changes slowly with time, which can lead to a non-optimal operation of the turbine with time. Another issue is quality of the initial mapping. It might be correct but it can be uncertain. This paper introduces a scheme to estimate this power conversion ratio. The estimated values can subsequently be used to calculate a new operating point. The estimation is based on an optimal unknown input observer.

1. INTRODUCTION

In the recent years the focus on renewable energy source have dramatically increased, due to a number of factors such as limitation of fossil-based fuels and environmental consequences of the usages of these fuels. One of the technologies with an explosive increase in installed energy generation capacity is the wind turbine technology.

A wind turbine often consists of a tower on which a nacelle is placed in which a generator is placed. The generator is driven by a main shaft at which the turbine's blades are fixed, for the turbine in question 3 blades are used, and these are pitch-able meaning that their angle towards the wind can be controlled. A gearbox, enabling the possibility of different rotational speeds, divides the main shaft.

One of the consequences of the increased interest in wind power is an optimization of the generated power per wind turbine, both in terms of an increased turbine size but also in terms of more efficient energy production, see for example (Johnson et al., 2006; Song et al., 2000). The increased size of the wind turbines have also increased the interest of turbines with variable speed and active pitch of the blades, which are used to keep the turbine at rated power when the rated wind speed is exceeded. Until the rated power is achieved the power optimum is obtained by requiring the optimal reference torque at the generator, see (Johnson et al., 2006). It is assumed that there is no limit on the allowed rotor speed.

The optimal torque and pitch references are obtained by a mapping between power generation ratio, pitch angle and a ratio between wind speed and the speed of the blade tip, (the rotation speed of the rotor can be controlled by the torque reference). This mapping is denoted the C_p - surface and could be obtained by: finite element simulations, wind tunnel experiments etc.

A problem in achieving optimal power and speed control of a wind turbine is that the power coefficient C_p - surface is not well known. Initially these values are most often not actually measured but computed, and if measured, only a few blades in an entire series, no measurements are performed on the actual turbine. Secondly these values can be assumed to change slowly with time, though only a few percent per year see (Johnson et al., 2006). These changes are due to wear and tear of the blades as well as debrief build up on them.

Another problem often encountered is the difficulty measuring the actual wind speed acting on the turbine. This might cause problems since the optimality of a power control scheme depends on the actual wind speed. In (Zhang et al., 2004) a Kalman estimator is designed to estimate the wind speed.

Some work has been published regarding adapting the power controller to the specific C_p - surface. In (Johnson et al., 2006) a scheme is proposed which use a Newton like scheme to find the power optimum online. A non-linear controller is proposed in (Song et al., 2000), which assumes that the wind speed is well known. (Sakamoto, 2004) presents an adaptive scheme which uses a least square method to online identify the system parameters; the controller is designed using a minimum variance controller, which is impractical. (Dadone & Dambrosio, 2003) presents a fuzzy control to adapt the power controller.

Another solution dealing with this problem could be to estimate the C_p -value and the wind speed; these can be assumed to be separated in frequencies, due the slow change in the C_p values. The estimated C_p-values can subsequently be used in an update of the C_p-surface, for example once per month or even with a lower frequency. Based on the updated C_p-surface a new power optimum is found. This means that the power control is adapted using an adaptive C_p-surface. A large advantage of this scheme is that the existing control structure is not influences by this scheme, it do only provide updated power references when present.

The variations in C_p values can be assumed to be very slow, while the wind speed variations on the other hand are relatively high frequently in content. This problem seems similar to a problem from another area of the energy generation industry. Estimation of moisture content and fault detection in coal mills in coal-fired power plants, see (Odgaard & Mataji, 2008; Odgaard & Mataji, 2006a; Odgaard & Mataji 2006b; Odgaard et al., 2006) In which an optimal unknown input observer, see (Chen & Patton, 1999), is used to estimate these variables as cause of energy imbalances. The variation in C_p values and wind speed can be viewed as additional unknown inputs.

The model of the wind turbine is subsequently introduced, which leads to an estimator design using the optimal unknown input observer scheme. In the end the scheme is tested on simulated data based on standard wind turbine models.

2. WIND TURBINE MODEL

In a typical variable speed wind turbine the generated power is controlled by two modes power and speed control. In power controlling the generator torque maximizes control the generated power, such that a specific tip speed ratio is obtained in relation to the optimum on the C_p-surface. The produced power of these two control modes is mapped as a function of the wind speed in Fig.1. In speed control mode the blades are pitched such that the rated power is obtained and the generator torque chosen such that the rotational speed of the rotor is minimized. In order to obtain the optimum for both control modes the C_p-surface is highly important, e.g. the turbine will not be controlled optimally if the optimum on C_p-surface is moved from the assumed.

Inspecting the problem deeper, the torque balance model of the wind turbine is considered.

$$\dot{\omega}(t) = \frac{1}{J}(\tau_{aero}(t) - \tau_{ref}(t)), \qquad (1)$$

and

$$\tau_{aero}(t) = \frac{\rho A C_p(\theta(t), \lambda(t)) v(t)^3}{2\omega(t)}, \qquad (2)$$

where $\omega(t)$ is the rotational velocity, $\tau_{ref}(t)$ is the reference torque to the generator, $\theta(t)$ is the pitch angle illustrated by Fig. 2, $\lambda(t)$ is the tip speed ratio, $v(t)$ is the wind speed, J is the moment of inertia of blades shaft etc, ρ is the density of the air, A is the area covered by the blades in the rotation. $\omega(t)$ and $\tau_{ref}(t)$ are measurable. The wind denoted $v(t)$ is measured as well, but is very non-reliable, since it should be the average over the entire swept area, and not a point measurement.

If this model is linearized all changes in $C_p(\theta(t), \lambda(t))$ is replaced by $\Delta C_p(t)$.

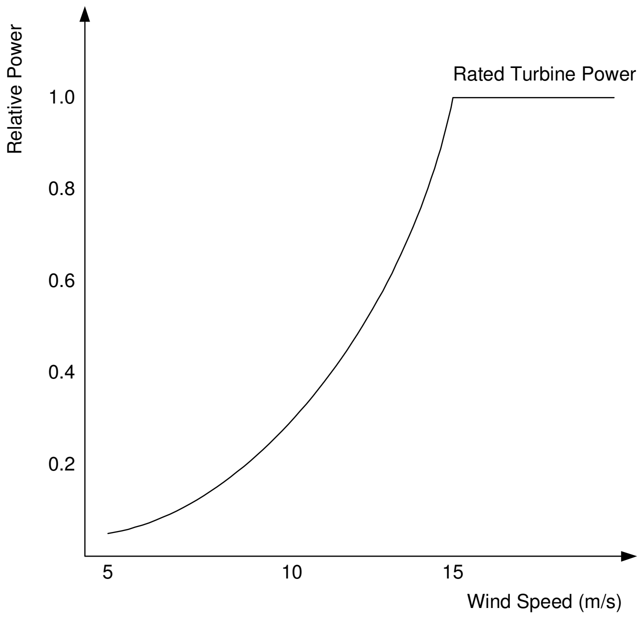

Fig. 1 *Illustration of the produced power until the rated power is reached at a wind speed of 15 m/s, the power is optimized and above this speed it is limited by blade pitching.*

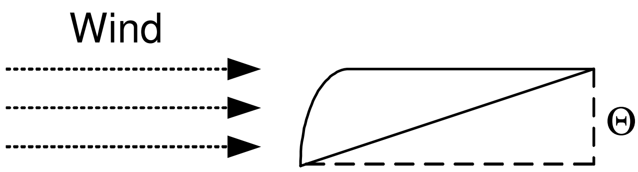

Fig. 2 *The pitch angle* θ *represent the angle the blade is pitched in relation to the wind direction.*

$$\Delta\tau_{aero}(t) = -\frac{\rho A C_{p,0} v_0^3}{2\omega_0^2}\Delta\omega(t) + \frac{\rho A v_0^3}{2\omega_0}\Delta C_p(t) + \frac{3\rho A C_{p,0} v_0^2}{2\omega_0^2}\Delta v(t) \qquad (3)$$

In this content the power coefficient and the wind speed is assumed to be unknown variables, which varieties around a well-known working point. Changes in C_p would be very slowly, meaning that the frequency content of C_p is in the region close to 0 rad/s. Compared to this the changes in the wind speed will be placed in a region with much higher frequency content, meaning that frequency separation can be assumed.

The anemometer, introduces a risk of a DC-error on the measurement. This is much more critical than high uncertainties at higher frequencies since these can be decoupled by the frequency information. If a DC calibration of the anemometer is not performed, an offset will be introduced on the C_p estimates. It will be a constant offset on the entire C_p table, and consequently not resulting in a false optimal C_p value, but the absolute value cannot be determined.

The existence of the unknown inputs in the system, points at the usage of a specific scheme to estimate the C_p values, this is the optimal unknown input observer,

3. OBSERVER DESIGN

The estimator is based on a simple torque balance model presented in (1-2) and in the linearized version in (3).

Since the system in mind contains some unknown inputs the idea is to use an unknown input observer in its optimal version., see (Chen & Patton, 1999). The structure is:

$$\begin{aligned} z[n+1] &= F_{n+1}z[n] + T_{n+1}B_n u[n] + K_{n+1}y[n], \\ \hat{x}[n+1] &= z[n+1] + H_{n+1}y[n+1], \end{aligned} \tag{4}$$

where $F_{n+1}, T_{n+1}, K_{n+1}$ and H_{n+1} are matrices designed to achieve decoupling from the unknown input and as well obtain an optimal observer. $\hat{x}$ is a vector of the states of the wind turbine model and wind speed and power coefficient. The matrices in the unknown input observer are found using the following equations.

$$E_n = H_{n+1}C_{n+1}E_n, \tag{5}$$

$$T_{n+1} = I + H_{n+1}C_{n+1,}, \tag{6}$$

$$F_{n+1} = A_n - H_{n+1}C_{n+1}A_n - K^1_{n+1}C_n, \tag{7}$$

$$K^2_{n+1} = F_{n+1}H_n \tag{8}$$

$$K^1_{n+1} = A^1_{n+1}P_n C^T_{n+1}(C_n P_n C^T_n + R_k)^{-1} \tag{9}$$

$$A^1_{n+1} = A_n - H_{n+1}C_{n+1}A_n, \tag{10}$$

$$\begin{aligned} P_{n+1} = {} & A^1_{n+1}P'_{n+1}(A^1_{n+1})^T + T_{n+1}Q_n T^T_{n+1} \\ & + H_{n+1}R_{n+1}H^T_{n+1}, \end{aligned} \tag{11}$$

$$P'_{n+1} = P_n - K^1_{n+1}C_n P_n (A^1_{n+1})^T \tag{12}$$

$$H_{n+1} = E_n(C_n E_n)^+ \tag{13}$$

The observer design procedure can be described by the following algorithm:

1) Set Initial values:

$$\begin{aligned} &P_0 = P(0), \\ &z[0] = x[0] - C_0E_0(C_0E_0)^+ y[0], \\ &H_0 = 0, \\ &k = 0. \end{aligned}$$

3) Compute H_{n+1} using (13).

4) Compute K^1_{n+1} and P'_{n+1} using (9) and (12).

5) Compute T_{n+1}, F_{n+1}, K^2_{n+1} and K_{n+1} by (6), (7), (8) and $K_{n+1} = K^1_{n+1} + K^2_{n+1}$.

6) Compute the state estimate $\hat{x}[n+1]$ and $z[n+1]$ using (4).

7) Compute P_{n+1} using (11) & (12).

8) Set $k = k+1$ go to step 2).

As mentioned in Section 2, the power coefficient and the wind speed is assumed to be unknown variables, but if the optimal unknown input observer is used directly only the system states are estimated decoupled the uncertain inputs, which is of no interest since the only state in this model, $\omega(t)$ is already relatively well known. Instead internal models are used to represent the two unknown inputs, using that the variations of C_p is very slowly i.e. low frequently and that the changes in the wind speed is much faster. This means that C_p can be modelled by a low pass filter. $v(t)$ has high frequency content and can thereby be modelled by a high pass/band pass filter. A DC-error on the wind measurement will result in an offset on the estimated C_p values, but it would be the same value on all C_p values meaning that it will not change the power maximum location but only the value of the optimum. The change in the C_p curve will change the location of the optimum of it. A measurement of the ambient air temperature will be useful as well in order to correct the estimations due to changes in the air density, which is highly depending on the air temperature.

Next it is assumed that the difference between the measured $\omega(t)$ and the one computed by the model can be explained by the variations in C_p and the wind speed. This gives the following linear model of the torque balance and internal models.

$$\dot{x}_c(t) = A_c x_c(t) + B_c u_c(t) + E_c d_d(t),$$
$$y_c(t) = C_c x_c(t), \qquad (14)$$

Where

$$x_c(t) = \begin{bmatrix} \omega(t) \\ x_{c_p}(t) \\ x_v(t) \end{bmatrix},$$

$$y_c(t) = \begin{bmatrix} \tau_{aero}(t) \\ \omega(t) \\ v(t) \end{bmatrix}, \qquad (15)$$

$$u_c(t) = \tau_{ref}(t), \qquad (16)$$

$x_{c_p}(t)$ is a state representing the internal model of the C_p coefficient, $x_v(t)$ is the state vector representing the internal model of the wind speed, $d_d(t)$ is a signal representing the unknown input. The internal model of the C_p coefficient is in state space form ($A_{c_p}, B_{c_p}, C_{c_p}, D_{c_p}$), and the internal model of $v(t)$ is in state space form (A_v, B_v, C_v, D_v), the merged system matrices are defined by:

$$A_c = \begin{bmatrix} -\frac{\rho A C_{p,0} v_0^3}{2\omega_0^2 J} & \frac{\rho A v_0^3}{2\omega_0 J} C_{c_p} & \frac{3\rho A C_{p,0} v_0^2}{2\omega_0^2 J} C_v \\ 0 & A_{c_p} & 0 \\ 0 & 0 & A_v \end{bmatrix}, \qquad (17)$$

$$B_c = \begin{bmatrix} \frac{-1}{J} \\ 0 \\ 0 \end{bmatrix}, \qquad (18)$$

$$C_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & v_0^3 C_{c_p} & 3C_{p,0} v_0^2 C_v \\ 0 & 0 & C_v \end{bmatrix}, \qquad (19)$$

$$E_c = \begin{bmatrix} 1 \cdot 10^{-3} \\ B_{c_p} \\ B_v \end{bmatrix}, \qquad (20)$$

The first element in E_c is non-zero even though that $\omega(t)$ is not assumed to be driven by the uncertain input. The reason is that this small but non-zero elements in the matrix introduce some robustness towards model uncertainties, which could be due to linearization of the nonlinear model before it is used to design the observer.

The system is subsequently descretized with sampling frequency at 10 Hz. Resulting in a state space system defined as in (21), where stochastic disturbances and measurement noises are added.

$$x_c[n+1] = A_d x_c[n] + B_d u_c[n] + E_d d_d[n,] + \zeta[n]$$
$$y_c[n] = C_d x_c[n] + \eta[n] \qquad (21)$$

Two filters represent the internal models of the C_p coefficients and wind speed. In addition to these filters the covariance matrices *Q* and *R* introduces design flexibility into the system as a couple of design parameters. The filters representing C_p and $v(t)$ have to be designed, the point is that A_{C_p}, B_{C_p} and C_{C_p} are found such that it is a low pass filter with a time constant of days, and A_v, B_v and C_v such that they form a high pass filter/ band pass filter such that its pass region is placed in the much higher region.

Due to the non-linearity of the model, especially the cubic dependency on the wind speed, a number of operating points are used in practice such that an observer is designed for each, and bump less transfer is used to switch between them. However, for simplicity this is left out in this paper, and only a single observer is designed to one point of operation. However, the introduction of these multiple observers would increase the performance of the estimated.

4. EXPERIMENTAL TEST

The observer is subsequently tested on a simulation of a wind turbine fed with some measured wind data. A simulation based on measured wind speed is used instead of real data, since the C_p values cannot be measured and thereby not be used to verify the designed observer. In the simulation the actual C_p-values are computed, and can consequently be used for comparison.

The wind turbine is modelled by the nonlinear model in (1-2), and the following model parameters are used:

$$J = 7.794e6\ kg * m^2,\ R = 26.2\ m,\ \rho = 1.225\ kg/m^3$$

For the linear model the points of operation are chosen such that the entire range of the wind speed in the data set is covered as well as possible. The values are found to be $C_{p,0} = 0.46$, $v_0 = 15\ m/s$, $\omega_0 = 1\ rad/s$ and $\tau_{ref,0} = 1.35 \cdot 10^6\ Nm$.

The linear observer model matrices of the two internal models are found iteratively to:

($A_{c_p} = -1 \cdot 10^{-7}$, $B_{c_p} = 1$, $C_{c_p} = 1 \cdot 10^{-7}$, $D_{c_p} = 0$) and

($A_v = \begin{bmatrix} -1.01 & -0.10 \\ 0.10 & 0 \end{bmatrix}, B_v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$,
$C_v = \begin{bmatrix} 1.00 & 0.001 \end{bmatrix}$, $D_v = 0$).

The two cross correlation matrices are found iteratively as:

$$Q = I_{4x4}\ ,\ R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \cdot 10^5 \end{bmatrix}.$$

The simulation is preformed using the nonlinear model structure with a stochastic noise on the wind speed, with a variance of 10^4.

A simple PI-controller is formed to control $\tau_{ref}(t)$. The performance of this controller is not relevant, since the interesting issue is to determine if the observer can estimate $v(t)$ and $C_p(t)$.

The model and observer are sampled with a frequency of 10 Hz.

In the "real" simulated $v(t)$ is compared with the estimated $v(t)$. The estimated value follows the real values relatively well, better performance could be obtained by choosing better points of operation, or eventually introduce additional points of operations.

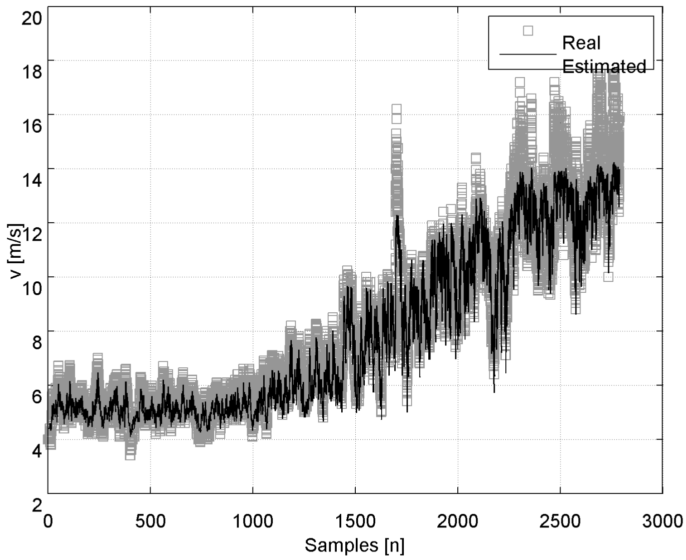

Fig. 3 Simulated wind speed compared with the estimated.

In Fig. 4 the estimated C_p values are compared with these obtained from the simulation model. Again the C_p value is relatively well estimated by the proposed scheme.

The fluctuations in C_p are due to the fact that the controller cannot track the optimum curve perfectly. After 1000 to 1500 samples the turbine starts pitching which changes the C_p value.

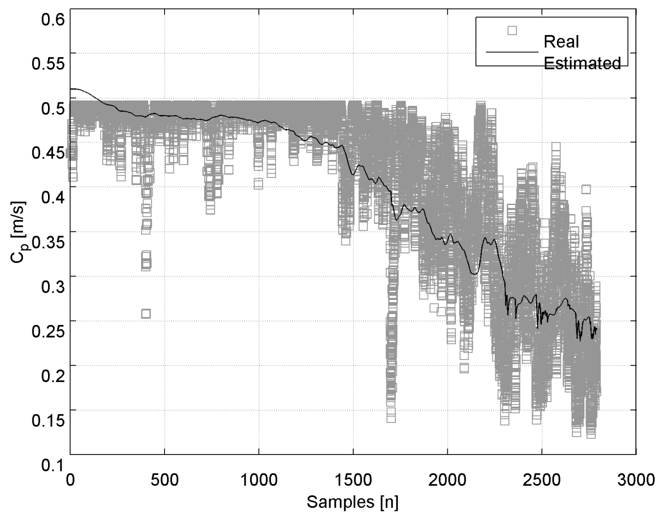

Fig. 4 Simulated C_p compared with the estimated one.

The bump in the beginning of the estimated C_p is caused by initialization effects on the estimator, and can consequently be dealt with by more efficient initialization schemes.

Even though better performance could be obtained if more than one point of operation were used, the performance of the estimated values seems quite good taken the system nonlinearities into account. The wind velocity increased from 5 m/s to around 15 m/s, and the C_p decreases from approximately .47 to 0.25. This means that the number of required operational points might be a low number.

Another way to increase the performance could be to introduce a model of the drive train (gearbox) into the turbine model, instead of assuming it to be stiff.

5. CONCLUSION

For optimizing the power generated by a wind turbine it is important to have correct information of the power coefficient. This will change with time and/or can initial be uncertain. In this paper an observer-based scheme is suggested to estimate this power coefficient as well as the wind speed using a torque balance model of the wind turbine. By simulations it is verified that the scheme estimates the wind speed and power coefficient well, even though only one point of operation is used, and the wind speed is tripled and the power coefficient is decreased with a factor of two.

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