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Robust Decentralized Data Fusion Based on Internal Ellipsoid Approximation

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Abstract: Based on M-estimate, the problem of robust estimation fusion in decentralized architecture when the sensor noises are contaminated by outliers is considered. A simple robust Kalman filtering (RKF) scheme with weighted matrices of innovation sequences is introduced for local state estimation. Then, to avoid both the inconsistency of the Kalman filter and the performance conservation of the covariance intersection method, an internal ellipsoid approximation method (IEA) is proposed to fuse the local estimation in the fusion center. Finally, to demonstrate robustness of the proposed RKF and the effectiveness of IEA strategy, a simple tracking example in the presence of outliers is introduced.

1. INTRODUCTION

Estimation fusion, or data fusion for estimation, is the problem of how to best utilize useful information contained in multiple sets of data for the purpose of estimation an unknown quantity—a parameter or process (Li, Zhu, & Han, 2000). It has wide-spread applications since many practical problems involve data from multiple sources, including guidance, defence, robotics, integrated navigation, target tracking, and GPS positioning. It has been realized for many years following the original work of Bar-Shalom (1981) that local estimates have correlated errors. How to counter this cross correlation has been a central topic in distributed fusion. One problem with the Kalman Filtering (KF) is that it requires either that the measurements are independent or that the cross-covariance is known (See Smith & Sameer, 2006). In this case that the cross-covariance information is available, the optimal KF-based approach that the KF maintains cross-covariance information between updates is proposed by *Bar-shalom* (1981), and *Sun* (2004).

Unfortunately, even though under the assumption that the cross-covariance is known, the optimal KF-based approach scales quadratically with the number of updates, which makes it impractical (See Drummond, 1997). A common simplification is to assume the cross-covariance to be zero, i.e. the measurements are independent, though, in this situation, the KF produces nonconservative covariance. This leads to an artificially high confidence value, which can lead to filter divergence (Julier & Uhlmann, 1997). Recently proposed covariance intersection filtering (See Julier & Uhlmann, 1997) is based on convex combination of information matrices, i.e., inverse covariance matrices and the corresponding information states. The algorithm provides a general framework for information fusion with incomplete knowledge about the signal sources since it yields consistent estimates for any degree of cross correlation. Since covariance intersection filtering requires optimization of a nonlinear cost function and instead of underestimation of the actual covariance matrix, the covariance intersection method overestimates it, which obviously results in a significant decrease in performance. To avoid both the inconsistency of the basic convex combination and the lack of performance of the covariance intersection method, largest ellipsoid algorithm has been proposed by Benaskeur (2002). The algorithm provided in (Benaskeur, 2002) solved the matrices orientation incompatibility problem in the case of two sensors, and the largest ellipsoid within the intersection of two ellipsoids can be computed. Unfortunately, it did not derive the formulation of the center of the largest ellipsoid (it presents the resulting fusion estimate) correctly, and the estimation performance may degrade severely, which will be shown in Section 4 & 5. Therefore, an Internal Ellipsoid Approximation (IEA) method is proposed to obtain the largest volume ellipsoid within the intersection of covariance ellipsoids in this paper.

On the other hand, the distribution of noise arising in application deviates frequently from assume Gaussian model, often being characterized by skewed (asymmetric) or heavier tails generating the outlier. Since in the presence of outliers, even a single outlier –if located sufficiently far away- can completely spoil the Least Squares estimate or the Kalman filter, causing it to break down (See Huber, 1981; Djurovic & Kovacevic, 1995), which will degrade the fusion performance greatly. Therefore it is of practical interest to consider filters which are robust to perform fairly well in non-Gaussian environment especially in the presence of outliers, and some results have been obtained during the last decade. Robust statistical procedures provide formal methods to spot the outlying data points and reduce their influence. Most of the contributions in this area have been directed toward censoring data, namely, if an observation differs sufficiently from its predicted value then it is discarded. For example, an M-estimate filter for robust adaptive filtering in impulse noise is proposed in (Djurovic & Kovacevic, 1999), a recursive adaptive algorithm and a robust threshold estimation method are derived employing an M-estimate cost function. Recently, *Djurovic* and *Kovacevic* established the equivalence between the Kalman filter algorithm and a particular leasat-squares regression problem. Based on the equivalence it solved the robust estimation with unknown noise statistics with the help of M-estimate method, and the equivalence between the Kalman filter and proposed technique is established (Phan & Zoubir, 2005). Very recently, in (Muth, Wang, & Conn, 2006) a sequential M-estimation algorithm is proposed as an alternative to sequential least squares. Being an approximation to exact M-estimate, the proposed technique is robust to non-Gaussian noise and outperforms sequential least squares. To the best of the authors' knowledge, however, there is no result discussing how to eliminate or reduce the influence of outliers on the fusion performance, which remains an open and challenging problem.

In this paper, a novel and straightforward robust Kalman filter is developed by robustifying the conventional Kalman filter with weighted innovation, which is obtained from the influence function of M-estimate. It is shown that the proposed robust Kalman filtering takes conventional Kalman filter as a special case when the noises are normal ones. Then, an Internal Ellipsoid Approximation method is proposed to obtain the largest volume ellipsoid within the intersection of the covariance ellipsoids. Finally, in order to demonstrate the robustness of the robust Kalman filter, the high fusion accuracy of IEA, and the efficiency of suppressing the influence of outliers to the fusion performance, a simulation example is introduced.

Notations: For real symmetric matrices X and Y, the notation $X > Y$ (or $X \geq Y$) means that the matrix $X - Y$ is positive define (or positive semi-define). I is the identity matrix with appropriate dimension. $E(x)$ is the expectation of x, the superscript T, + denote the transpose and Moore-Penrose inverse respectively; I_n and 0 denote the $n\times n$ unit matrix and the zero matrix with compatible dimension, respectively. tr(P) stands for the trace of matrix P. δ_{tk} is the Kronecker's delta function($\delta_{tk} = 1$ for $t = k$, and $\delta_{tk} = 0$ for $t \neq k$). Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. PROBLEM STATEMENT AND LEMMAS

Consider the discrete linear stochastic system with N sensors

$$x(t+1) = Fx(t) + G\omega(t) \tag{1}$$

$$y_i(t) = H_i x(t) + \upsilon_i(t), \qquad i = 1,\ 2,\ 3, \cdots, N \tag{2}$$

where $x(t) \in \Re^n$ is the state vector, $y_i(t) \in \Re^{m_i}$ is the ith measurement in the sampling period tT; $\omega(t) \in \Re^p$ is the disturbance input or system white noise with zero mean and variance matrix Q, $\upsilon_i(t) \in \Re^{m_i}$, $i = 1, 2, 3, \cdots$, N are the contaminated Gaussian noise vectors noises. The matrices F, G, and H_i are known real constant matrices with appropriate dimensions.

Assumption 1. $\omega(t)$ and $\upsilon_i(t), i = 1,\ 2,\ 3, \cdots, N$ are independent, and $\upsilon_i(t)$ are with non-Gaussian density function $f(e)$ described by

$$F_i = (1-\alpha)G_i + \alpha\Delta G_i \tag{3}$$

where G_i is the zero-mean Gaussian density, and ΔG_i is some unknown symmetric function representing the impulsive part of the noise density or outliers. This problem is of practical importance in a target tracking system using multiple sensors (radar or infrared) (See Muth, Wang, & Conn, 2006), and communication applications where non-Gaussian (heavy tailed) noise occurs, such as in underwater acoustics, and satellite communications through the ionosphere (See Wang & Poor, 1999) et al.

Assumption 2. The initial state $x(0)$ is independent of $\omega(t)$ and $\upsilon_i(t), i = 1,\ 2,\ 3, \cdots, N$, and

$$Ex(0) = x_0$$

$$E[(x(0) - x_0)(x(0) - x_0)^T] = P_0$$

The problem is to find the optimal decentralized robust estimation fusion $\hat{x}_0(t)$ of the state $x(t)$ in terms of local robust Kalman filter based on the measurements $(y_i(t), \cdots y_i(1))$, which are contaminated by outliers. The estimation should have the desirable properties of efficiency and robustness, i.e. it (a) yields the estimation fusion with a high accuracy for normal distributed observation while keep the solution in high efficiency; (b) reduces the bad effect of moderate errors on filtering and fusion in the way of weighting innovation; (c) is robust in the sense that heavy-tailed errors or outliers do not affect the solution by setting the weighted matrix of innovation to be zeros, and further suppress the influence of outliers to the performance of fusion. We start with some lemmas.

Lemma 1. Checking if two ellipsoids $\varepsilon(x_{0_1}, P_1)$, $\varepsilon(x_{0_2}, P_2)$ have nonempty intersection, can be cast as to the following Quadratic Programming (QP) problem with quadratic constraints (Vazhentsev, 2000; Kurzhanski, 1991)

$$\beta_1 = \min_{<x, P_2^{-1}x>=1} < x, P_1^{-1}x >= \min_{x^T P_2^{-1} x = 1} x^T P_1^{-1} x \tag{4}$$

$$\beta_2 = \min_{<x, P_1^{-1}x>=1} < x, P_2^{-1}x >= \min_{x^T P_1^{-1} x = 1} x^T P_2^{-1} x \tag{5}$$

where β_1 and β_2 are invariant with respect to affine coordinate transformation and describe the position of ellipsoids $\varepsilon(x_{0_1}, P_1)$, $\varepsilon(x_{0_2}, P_2)$ with respect to each other:

(1) If $\beta_1 \geq 1, \beta_2 \leq 1$, then $\varepsilon(x_{0_1}, P_1) \subseteq \varepsilon(x_{0_2}, P_2)$

(2) If $\beta_1 \leq 1, \beta_2 \geq 1$, then $\varepsilon(x_{0_1}, P_1) \supseteq \varepsilon(x_{0_2}, P_2)$

(3) If $\beta_1 < 1, \beta_2 < 1$, then $\varepsilon(x_{0_1}, P_1) \cap \varepsilon(x_{0_2}, P_2) \neq \phi$.

3. A NOVEL ROBUST KALMAN FILTERING SCHEME

As can be seen from the traditional KF update equations, at the sampling period of (t+1), the ith filtering estimate $\hat{x}_i(t+1|t+1)$ is corrected by the linear combination of innovation $e_i(t+1)$. Therefore, when the measurements $y_i(t+1)$ are contaminated by outliers, $e_i(t+1)$ will correct $\hat{x}_i(t+1|t+1)$ in the wrong way, which should make Kalman filter performs poorly or even divergently (See *Sun* & Deng, 2004).

In another point of view, the conventional Kalman filter can also be thought of as the solution to a particular weighted least squares problem (See Djurovic & Kovacevic, 1999), unfortunately, it is nonrobust because extreme outliers with arbitrarily large residuals can have an infinitely large influence on the resulting estimate. Robust statistics is concerned with the fact that many assumptions commonly made in classical statistics, such as normality, are at most approximations to reality and that deviations from the assumptions due to, for instance, gross errors are 'dangerous'. An estimator is said to be robust if it is insensitive to deviations from certain assumptions about the measurements and is able to provide a good solution even with measurements containing outliers. The M-estimator is one of the most sophisticated approaches to this problem among the robust statistics approaches. Further, the M-estimator has an advantage of less computational effort as it can be computed by a standard least squares algorithm with minor modifications (Hampel, Ronchetti, Rousseeuw, & Stahel, 1986) .

M-estimators attempt to suppress the influence of outliers by replacing the square of the residuals with a less rapidly increasing loss function, which is

$$J = \sum_{j=1}^{m_i} \rho(y_{ij}(t) - H_{ij}x(t)) = \sum_{j=1}^{m_i} \rho(e_{ij}(t)) \qquad (6)$$

where $y_{ij}(t)$, H_{ij} stand for the j-th row of $y_i(t)$ and H_i ,respectively. $\rho(\cdot)$ is a usually nonnegative and symmetric scalar robust convex function that has to cut off the outliers. Particularly, if one chooses $\rho(\cdot)$ to be a quadratic function, the solution of (7) will reduce to the least squares or Kalman filter.

Equating the first partial derivatives to zero with respect to the state to be estimated $x(t)$ leads to the following M-estimator

$$\sum_{j=1}^{m_i} \psi(y_{ij}(t) - h_{ij}\hat{x}(t))h_{ij} = \sum_{j=1}^{m_i} \psi(e_{ij}(t))h_{ij} = 0 \qquad (7)$$

where $\psi(\cdot)$, the derivative of $\rho(\cdot)$, is often called the influence (score) function, since it describes the influence of the measurement errors on the solutions. Now, (8) can be rewritten as

$$\sum_{j=1}^{m_i} h_{ij}^T \frac{\psi(e_{ij})}{e_{ij}} e_{ij} = 0 \qquad (8)$$

Letting $d(e_j) = \dfrac{\psi(e_{ij})}{e_{ij}}$, then (8) can be reformulated as the matrix form

$$H_i^T D_i(e_i)e_i = 0 \qquad (9)$$

where $D(e_i) = diag(d(e_{i1}), d(e_{i2}), \cdots d(e_{in}))$.

In the light of the above comparison and analysis of conventional Kalman filtering and M-estimator, the proposed RKF is given in Theorem 1 as follows.

Theorem 1. Under the assumption 1 and 2, for the i-th estimation subsystem with the dynamic and measure equations (1)-(2), we have the robust Kalman filter steps

$$\hat{x}_i(t+1|t+1) = \hat{x}_i(t+1|t) + K_i(t+1)D_i(t)e_i(t+1) \qquad (10)$$

$$K_i(t+1) = P_i(t+1|t)H_i^T[H_iP_i(t+1|t)H_i^T + D_i(t)R_iD_i^T(t)]^{-1} \qquad (11)$$

$$P_i(t+1|t+1) = [I_n - K_i(t+1)D_i(t)H_i]P_i(t+1|t) \qquad (12)$$

Other recursive steps are just same as traditional KF.

Proof. The update formula (10) can be derived from the above analysis directly, and the covariance of weighted innovation is

$$E[(D_i(t)e_i(t+1))(D_i(t)e_i(t+1))^T] = D_i(t)R_iD_i^T(t) \qquad (13)$$

from which we have the robust Kalman gain matrix as (11).

Substituting (11) into the filtering error equation

$$\tilde{x}_i(t+1|t+1) = x_i(t+1) - \hat{x}_i(t+1|t+1) =$$
$$[I_n - K_i(t+1)D_i(t)H_i]\tilde{x}_i(t+1|t) - K_i(t+1)D_i(t)\upsilon_i(t) \qquad (14)$$

where $\tilde{x}_i(t+1|t)$ is the one step prediction residual, and after mathematical manipulation, the robust filter covariance can be computed as

$$P_i(t+1|t+1) = E[\tilde{x}_i(t+1|t+1)\tilde{x}_i^T(t+1|t+1)]$$
$$= [I_n - K_i(t+1)D_i(t)H_i]P_i(t+1|t)[I_n -$$
$$K_i(t+1)D_i(t)H_i]^T + K_i(t+1)D_i(t)R_iD_i^T K_i^T(t+1)$$
$$= [I_n - K_i(t+1)D_i(t)H_i]P_i(t+1|t) \quad (15)$$

This completes the proof. □

Remark 1. $\rho(\cdot)$ is a robust M-estimate function for suppressing the outliers and is important for the estimation performance. Different $\rho(\cdot)$ will result different M-estimate and fusion performance. Say, for a give density *f*, the choice $\rho(\upsilon) = -\log f(\upsilon)$ yields the maximum likelihood fuser. Several robust cost functions have been used in the robust statistics setting, such as Huber's robust cost function, Andrews' method, Vapnik's loss function, or the biweight approach. Here, we propose a more general M-estimate function that is generated from Huber's robust cost function.

$$\rho(e_i(t)) = \begin{cases} e_i^2(t)/2, & \text{for} \quad |e_i(t)| \le a \\ a|e_i(t)| - a^2/2, & \text{for} \quad a < |e_i(t)| \le b \\ ab - a^2/2, & \text{for} \quad b < |e_i(t)| \end{cases} \quad (16)$$

It can be seen that $\rho(\cdot)$ is an even real-valued function and it is quadratic when $e_i(t)$ is smaller than a, which is just the same as the maximum likelihood (ML) cost function and keeps the efficiency of the M-estimate; For larger values of $e_i(t)$ in the interval $[a,b]$, the function is linear and increase more slowly than ML; For residuals greater than *b*, the function is equal to a constant. Based on (16), the weighted matrix of innovations can be formulated as

$$D(e_i(t)) = \begin{cases} 1, & \text{for} \ |e_i(t)| \le a \\ a/|e_i(t)|, & \text{for} \ a < |e_i(t)| \le b \\ 0, & \text{for} \ |e_i(t)| > b \end{cases} \quad (17)$$

The three different intervals of $D(\cdot)$ are designed to deal with different kinds of residuals. In order to keep the accuracy and efficiency, when $|e_i(t)| \le a$, $D(\cdot)$ is set to be 1; when sampling from the moderate innovations, $D(\cdot)$ are decreased with the residuals while sampling from a heavy-tailed distribution or outliers, the weights are set to be zero.

Remark 2. When $\omega(t)$ and $\upsilon_i(t), i = 1, 2, 3, \cdots, N$ are independent white noises with zero mean and variance matrices Q and R_i, from *Remark 1* we can see that $D = I_n$. In this case, the proposed robust Kalman filter reduced to the conventional Kalman filter.

4. INTERNAL ELLIPSIOD APPROXIMATION BASED ESTIMATION FUSION ALGORITHM

Once the local estimation is obtained by the subsystems, we are facing the problem of how to fuse the estimation in a right way in the higher level. In the presence of outlier, the optimal KF-based approach may cause divergence. In avoid explicitly calculating the cross-covariance, a novel estimation fusion algorithm that calculates the largest volume ellipsoid within the intersection of the covariance ellipsoids using internal ellipsoid approximation approach, is proposed in this section.

The largest ellipsoid algorithm in (Benaskeur, 2002) solved the matrices orientation problem in the case of two sensors, and the largest ellipsoid within the intersection of two ellipsoids can be computed, unfortunately, it did not derive the computation of the center (it presents the resulting fusion estimate) of the largest ellipsoid correctly, and the estimation performance may degrade severely, as can be seen in the simulation examples in Section 5. Hence, an Internal Ellipsoid Approximation method, which can be formulated in the following theorem, is proposed.

Theorem 2. Given two ellipsoids $\varepsilon(x_{0_1}, P_1)$ and $\varepsilon(x_{0_2}, P_2)$ and define parameterized family of internal ellipsoids $\varepsilon(x_0^-, P^-)$ with

$$x_0^- = (\omega_1 P_1^{-1} + \omega_2 P_2^{-1})^{-1}(\omega_1 P_1^{-1}x_1 + \omega_2 P_2^{-1}x_2) \quad (18)$$
$$P^- = (1 - \omega_1 x_1^T P_1^{-1}x_1 - \omega_2 x_2^T P_2^{-1}x_2 + x_0^{-T}(P^-)^{-1}x_0^-)$$
$$(\omega_1 P_1^{-1} + \omega_2 P_2^{-1})^{-1} \quad (19)$$

The best internal ellipsoid $\varepsilon(\hat{x}_0^-, \hat{P}^-)$ in the class (18)-(19), namely, such that

$$\varepsilon(x_0^-, P^-) \subseteq \varepsilon(\hat{x}_0^-, \hat{P}^-) \subseteq \varepsilon(x_{0_1}, P_1) \cap \varepsilon(x_{0_2}, P_2)$$

(20)

for all $0 \le \omega_1, \omega_2 \le 1$, is specified by the parameters

$$\hat{\omega}_1 = \frac{1 - \min(1, \beta_2)}{1 - \min(1, \beta_1) \cdot \min(1, \beta_2)}$$
$$\hat{\omega}_2 = \frac{1 - \min(1, \beta_1)}{1 - \min(1, \beta_1) \cdot \min(1, \beta_2)} \quad (21)$$

where β_1 and β_2 are the parameters determined in (4)-(5).

Proof. The result is similar as in (Vazhentsev, 2000), which is omitted here. □

Remark. After the parameters β_1 and β_2 is determined by *Lemma 1*, the center and shape parameters of the best internal ellipsoid $\varepsilon(\hat{x}_0^-, \hat{P}^-)$ can be calculated by

(18)-(19), and (21). The consistency of the algorithm using IEA is granted by the Fig. 2 in (Benaskeur, 2002) graphically.

Corollary 1. Two special cases are

(1) If $\beta_1 \geq 1, \beta_2 < 1$, then $\hat{\omega}_1 = 1, \hat{\omega}_2 = 0$, we have $\varepsilon(\hat{x}_0^-, \hat{P}^-) = \varepsilon(x_{0_1}, P_1)$;

(2) If $\beta_1 < 1, \beta_2 \geq 1$, then $\hat{\omega}_1 = 0, \hat{\omega}_2 = 1$, we have $\varepsilon(\hat{x}_0^-, \hat{P}^-) = \varepsilon(x_{0_2}, P_2)$.

Proof. This result can be derived from *Lemma 1* readily.

5. SIMULATION EXAMPLES

To evaluate the robustness of proposed robust Kalman filter and effectiveness of the IEA approach, simulations are performed on target tracking system with constant velocity model. The objectives of the simulation examples are two-fold: (a) to verify the robustness of the proposed robust Kalman filter; and (b) to demonstrate the performance superiority of the IEA method. Consider a simple target dynamic model with two sensors, which is the same as the one used in (See Smith & Sameer, 2006; Bar-Shalom, 1986; Chang, Saha, & Bar-Shalom, 1997)

$$x(t+1) = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} T^2/2 \\ T \end{bmatrix} \omega(t)$$

$$y_i(t) = H_i x(t) + \upsilon_i(t), i\text{=}1, 2$$

where T=0.1s is the sampling period. The state $x(t) = [s(t) \ \ \dot{s}(t)]^T$, where $s(t)$ and $\dot{s}(t)$ are the position and velocity respectively of the target at time tT. $\upsilon_i(t)$ is with the contaminated Gaussian density function $F_i = (1-\alpha)N(0, \sigma_i^2) + \alpha N(0, k\sigma_i^2)$, where $\alpha = 0.1$, k=100, and σ_i^2 are 5 and 8 respectively; We add outliers into $\upsilon_i(t), i = 1, 2$ with the covariance of 100 during the sampling step from 100 to 105. $\upsilon_i(t)$ are independent of standard Gaussian white noise $\omega(t)$. Our goal is to find the estimation fusion based on the local robust Kalman filters suppressing the effect of outliers on the estimation performance. The Mean Square Error (MSE) versus the sampling periods is computed by

$$\text{MSE} = \frac{1}{M}\sum_{i=1}^{M}[\tilde{x}_k(i)^T \tilde{x}_k(i)]$$ where M is the number of runs and $\tilde{x}_k(i)$ represents the estimation error of the fused estimate in the ith run. Both results from single and multiple Monte Carlo runs are presented.

First, to verify the robustness of the proposed Kalman filter, the performance comparison over arbitrary one run taking the IEA as the fusion algorithm and using RKF as the local estimator is shown in Table 1. As can be seen from Table 1, the performance discrepancy with different ε and k is not very large, which demonstrates the robustness of the proposed Kalman filter. Also, the superiority of RKF can be seen from Table 2, where showing the 100 Monte Carlo runs' results in the case that the local estimation systems take either traditional Kalman filter (TKF) or the proposed robust Kalman filter (RKF) as the estimator, and the fusion center takes the fast covariance intersection (Niehsen, 2002) (FCI), the simple convex combination (SCC), the largest ellipsoid method in (Benaskeur, 2002) (TLE), and proposed robust estimation fusion based on internal ellipsoid approximation, respectively. From Table 2 it can be seen than if there is no outlier, the IEA has nearly same accuracy when the local estimator is the robust KF or not. Because the largest ellipsoid method cannot find the estimates correctly, the performance degrades obviously. On the other hand, the performance comparison between the case

Table 1. MSE Performance for different α and k

α	k	Sampling steps 10	50	100	150	200
0.02	100	0.0021	0.0088	0.0165	0.0206	0.0505
	500	0.0015	0.0220	0.0604	0.0934	0.1285
	1000	0.0037	0.0478	0.0586	0.0915	0.1057
0.05	100	0.0004	0.0102	0.0350	0.0583	0.1033
	500	0.0016	0.0252	0.0481	0.0628	0.0783
	1000	0.0010	0.0327	0.0692	0.1024	0.1831
0.10	100	0.0014	0.0108	0.0237	0.0837	0.1295
	500	0.0025	0.0524	0.2938	0.3669	0.4002
	1000	0.0120	0.0669	0.2163	0.4784	0.7951

Table 2. MSE performance comparison over 100 runs

Local estimators	Fusion center	Outlier -free	Outliers presented
Traditional KF	SCC	0.4450	14.2067
	FCI	0.4468	14.0858
	TLE	63.9791	71.6905
	IEA	0.4511	15.3431
Proposed RKF	SCC	0.4471	0.6079
	FCI	0.4442	0.6106
	TLE	107.9364	123.2528
	IEA	0.4760	0.7105

of outliers presented and case outlier free, one can easily see that the traditional KF estimation fuses degrade, while the RKF fuses changes little, which demonstrate the RKF is no sensitive to the outliers.

6. CONCLUSIONS

The problem of robust estimation fusion in decentralized architecture when the sensor noises are contaminated by outliers is considered in this paper. Attentions have been mainly focused on two aspects. On the one hand, in order to suppress or deduce the influence of outliers to the estimation performance, a novel simple robust Kalman filtering scheme with weighted matrices of innovation sequences was introduced for local estimation. It has been shown that the proposed RKF takes conventional Kalman filter as a special case when the noises are normal ones. On the other hand, an internal ellipsoid approximation method is proposed to fuse the local estimation in the fusion center. The explicit solution of the fusion estimation of the state and its covariance matrix was also given. A simulation example shows the robustness of the proposed robust Kalman filter and the effectiveness of IEA strategy, comparing to previous results, the proposed algorithm is more applicable and effective in the presence of outliers.

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