Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Strong practical stability and stabilization of uncertain discrete linear repetitive processes

P. Dabkowski * **K. Galkowski** *,[1] **E. Rogers** ** **A. Kummert** ***

* *Institute of Physics, Nikolaus Copernicus University in Torun, Poland, (e-mail: p.dabkowski@fizyka.umk.pl)*
** *School of Electronics and Computer Science, University of Southampton, Southampton SO17 1BJ, (e-mail: etar@ecs.soton.ac.uk)*
*** *Faculty of Electrical, Information and Media Engineering, University of Wuppertal, Rainer-Gruenter-Strasse 21, D-42119 Wuppertal, Germany, (e-mail: kummert@uni-wuppertal.de)*

Abstract: Repetitive processes are a distinct class of $2D$ systems of both theoretical and practical interest. The stability theory for these processes currently consists of two distinct concepts termed asymptotic stability and stability along the pass respectively where the former is a necessary condition for the latter. Recently applications have arisen where asymptotic stability is too weak and stability along the pass is too strong for meaningful progress to be made. Previously reported work has defined the concept of strong practical stability for such cases and produced Linear Matrix Inequality (LMI) based necessary and sufficient conditions for it to hold. These can then be used as a basis for the design of a stabilizing control law. In this paper the (more practically relevant) case when there is uncertainty associated with the process description which is assumed to be of the norm bounded form is considered.

Keywords: Uncertain repetitive processes, Strong practical stability

1. INTRODUCTION

The unique characteristic of a repetitive, or multipass Rogers et al. (2007), process is a series of sweeps, termed passes, through a set of dynamics defined over a fixed finite duration known as the pass length. On each pass an output, termed the pass profile, is produced which acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile. This, in turn, leads to the unique stabilization problem in that the output sequence of pass profiles generated can contain oscillations that increase in amplitude in the pass-to-pass direction.

Physical examples of these processes include long-wall coal cutting and metal rolling operations Smyth (1992); Rogers et al. (2007). Also in recent years applications have arisen where adopting a repetitive process setting for analysis has distinct advantages over alternatives. Examples of these so-called algorithmic applications include classes of iterative learning stabilization schemes Amann et al. (1998) and iterative algorithms for solving nonlinear dynamic optimal stabilization problems based on the maximum principle Roberts (2000). In this last case, for example, use of the repetitive process setting provides the basis for the development of highly reliable and efficient iterative solution algorithms and in the former it provides a stability theory which, unlike many alternatives, provides information concerning an absolutely critical problem in this application area, i.e. the trade-off between convergence and the learnt dynamics.

Attempts to stabilize these processes using standard (or 1D) systems theory/algorithms fail (except in a few very restrictive special cases) precisely because such an approach ignores their inherent 2D systems structure, i.e. information propagation occurs from pass-to-pass and along a given pass and also the initial conditions are reset before the start of each new pass. To remove these deficiencies, a rigorous stability theory has been developed Rogers et al. (2007) based on an abstract model of the dynamics in a Banach space setting which includes a very large number of processes with linear dynamics and a constant pass length as special cases. Also the results of applying this theory to a range of sub-classes, including the so-called discrete linear repetitive processes considered here, have been reported Rogers et al. (2007). This stability theory consists of the distinct concepts of asymptotic stability and stability along the pass respectively where the former is a necessary condition for the latter.

Recognizing the unique control problem, this stability theory is of the bounded input bounded output (BIBO) form, i.e. bounded inputs are required to produce bounded sequences of pass profiles (where boundedness is defined in terms of the norm on the underlying Banach space). Asymptotic stability guarantees this property over the finite and fixed pass length whereas stability along the pass is stronger in that it requires this property uniformly, i.e. for all possible values of the pass length (and hence it

[1] For the duration of the work reported here K. Galkowski was a Gerhard Mercator Guest Professor in the University of Wuppertal.

is not surprising that asymptotic stability is a necessary condition for stability along the pass).

If asymptotic stability holds for discrete linear repetitive processes then the sequence of pass profiles produced will converge in the pass-to-pass direction to a so-called limit profile which is described by a 1D discrete linear systems state-space model. This fact has clear implications for the design of stabilization schemes. Moreover, the condition for asymptotic stability is very easy to test whereas one of the extra for stability along the pass is much more involved. This raises the question of whether or not asymptotic stability alone would be sufficient for at least some practically relevant cases. The answer Gałkowski et al. (2000) for at least some applications is no but for these it may be acceptable to use so-called strong practical stability as an alternative to stability along the pass. Note here that in the optimal control application Roberts (2000) is all that can ever be achieved and see Rogers et al. (2007) for discussion centered round the iterative learning control application area where strong practical stability could be the most appropriate way forward.

The basis of strong practical stability was developed in Gałkowski et al. (2000) and in subsequent work Dabkowski et al. (2007) it was shown that necessary and sufficient conditions for this property could be formulated in LMI terms which immediately give algorithms for the design of a stabilizing control law. In this paper we extend all of these results to the (more practically relevant) case where the model matrices are not exactly known but belong to some convex set. In this work, the uncertainty is assumed to be of the so-called norm bounded uncertainty are investigated.

Throughout this paper, the null and identity matrices with the required dimensions are denoted by 0 and I respectively. Moreover, $M > 0$ (< 0) denotes a real symmetric positive (negative) definite matrix and $(*)$ is used to denote block entries in symmetric LMIs.

2. BACKGROUND

The state-space model of a discrete linear repetitive process Rogers et al. (2007) has the following form over $0 \le p \le \alpha - 1$, $k \ge 0$

$$\begin{aligned} x_{k+1}(p+1) &= Ax_{k+1}(p) + Bu_{k+1}(p) + B_0 y_k(p) \\ y_{k+1}(p) &= Cx_{k+1}(p) + Du_{k+1}(p) + D_0 y_k(p) \end{aligned} \tag{1}$$

where $\alpha < \infty$ is the pass length and on pass k $x_k(p) \in \mathbb{R}^n$ is the state vector, $y_k(p) \in \mathbb{R}^m$ is the pass profile vector, and $u_k(p) \in \mathbb{R}^r$ is the vector of control inputs. The boundary conditions (i.e. the pass state initial vector sequence and the initial pass profile) are

$$\begin{aligned} x_{k+1}(0) &= d_{k+1}, \quad k \ge 0 \\ y_0(p) &= f(p), \quad 0 \le p \le \alpha - 1 \end{aligned} \tag{2}$$

where the $n \times 1$ vector d_{k+1} has known constant entries and $f(p)$ is an $m \times 1$ vector whose entries are known functions of p. Note also that these are of the simplest possible form but some applications require the pass state initial vector on each pass to be a function of points along the previous pass, and it is possible that these alone can cause instability. These so-called dynamic boundary conditions are not considered in this work.

Applying the stability theory of Rogers et al. (2007) to (1) and (2) now gives the necessary and sufficient condition for asymptotic stability as $r(D_0) < 1$ where $r(\cdot)$ denotes the spectral radius of its matrix argument. At first sight, this result is somewhat surprising in that it is essentially independent of the plant state dynamics and, in particular, places no constraints the location of the eigenvalues of the matrix A which clearly influence the dynamics produced along any pass. This condition is a result of the finite pass length and its consequences are discussed next.

Suppose that asymptotic stability holds and the input sequence applied $\{u_{k+1}\}_k$ converges strongly as $k \to \infty$ (i.e. in the sense of the norm on the underlying function space) to u_∞. Then the strong limit $y_\infty := lim_{k\to\infty} y_k$ is termed the limit profile corresponding to this input sequence. Also the limit profile is given by

$$\begin{aligned} x_\infty(p+1) &= (A + B_0(I-D_0)^{-1}C)x_\infty(p) \\ &\quad + (B + B_0(I-D_0)^{-1}D)u_\infty(p) \\ y_\infty(p) &= (I-D_0)^{-1}Cx_\infty(p) \\ &\quad + (I-D_0)^{-1}Du_\infty(p) \\ x_\infty(0) &= d_\infty \end{aligned} \tag{3}$$

where d_∞ is the strong limit of the sequence $\{d_k\}_k$. In physical terms, this result states that under asymptotic stability the repetitive dynamics can, after a 'sufficiently large' number of passes have elapsed, be replaced by those of a 1D discrete linear system. This last fact has obvious implications in terms of the stabilization of these processes — see Rogers et al. (2007) for a more detailed discussion of this point.

As an example, consider the case when $A = -0.5$, $B = 1$, $B_0 = 0.5 + \beta$, $C = 1$, $D = 0$, $D_0 = 0$, where β is a real scalar. Asymptotic stability holds in this case with resulting limit profile

$$y_\infty(p+1) = \beta y_\infty(p) + u_\infty(p)$$

Hence if $|\beta| \ge 1$, the sequence of pass profiles converge (in the pass-to-pass direction (k)) to an unstable 1D discrete linear system. Note also that this occurs even though the state matrix A is stable in the 1D sense.

The problem here is the finite pass length over which duration even an unstable 1D discrete linear system can only produce a bounded output. If the limit profile is unstable (as a 1D discrete linear system) then clearly this is, in general, unacceptable. As noted previously in this paper, however, cases do exist where asymptotic stability is all that is required or can be achieved (in the optimal control algorithm Roberts (2000) the matrix corresponding to A in the discrete linear repetitive process state space model can never satisfy $r(A) < 1$).

Stability along the pass prevents this problem from arising by demanding the BIBO property uniformly with respect to the pass length and can be analyzed mathematically by letting $\alpha \to \infty$. This leads to several sets of necessary and sufficient conditions Rogers et al. (2007) for this property.

3. STRONG PRACTICAL STABILITY AND STABILIZATION

A repetitive process evolves over a semi-infinite strip in the positive quadrant of the 2D domain, i.e. over $0 \leq p \leq \alpha, k \geq 0$. Stability along the pass, however, treats the process as evolving over the complete positive quadrant, i.e. both p and k are of unbounded duration. For this reason, stability along the pass can be too strong in some cases of practical interest — see, for example, Smyth (1992); Rogers et al. (2007) for further discussion of this point and illustrative examples.

A similar situation arises for the class of 2D discrete linear systems, including those described by the extensively studied Roesser Roesser (1975) and Fornasini Marchesini Fornasini and Marchesini (1978) state-space models, and this has led to the concept of so-called practical stability first introduced in Agathoklis and Bruton (1983) (see also Xu et al. (1994) for other results on this property). To explain the motivation for this, it is instructive to briefly consider the 1D case. In particular, the standard definition of BIBO stability in the 1D case demands that the output sequence $y(i)$ of a BIBO stable system remains bounded in amplitude for all input signals that were bounded in amplitude, where the term 'bounded in amplitude' is interpreted in terms of the norm on the underlying function space. Note, however, that the input and output signals may be of unbounded duration, i.e. i may be unbounded, and in many cases i is a temporal variable.

The 1D definition of BIBO stability has been extended to the 2D (and nD, $n \geq 3$) case by considering input signals which are unbounded in both variables. The basic idea of practical stability in the sense of Agathoklis and Bruton (1983) for 2D systems is to consider BIBO stability when the input signals are of unbounded duration in at most one variable. Next we proceed in a similar manner to develop and characterize so-called strong practical stability for discrete linear repetitive processes described by (1) and (2).

Consider the case when $p = 0$. Then it is easy to see that $r(D_0) < 1$, i.e. asymptotic stability, is a necessary condition. Also consideration of the current pass dynamics alone, i.e. with the previous pass terms in (1) deleted, leads immediately to $r(A) < 1$ as another necessary condition. These two conditions alone can be regarded as a weak form of practical stability but, as the numerical example in the previous section illustrates, they cannot prevent the possibility of a limit profile which is unstable as a 1D linear system.

Suppose now that $r(D_0) < 1$ and $r(A) < 1$ hold. Then it follows from routine arguments that strong practical stability will be completely characterized by considering the cases of p finite and $k \to \infty$, and $p \to \infty$ and k finite respectively. Both these cases result in limit profiles described by 1D discrete linear systems — in the first case the limit profile is given by (3) and in the second it is the so-called vertical limit profile described by

$$\begin{aligned} y_{k+1}(\infty) &= \left(C(I-A)^{-1}B_0 + D_0\right) y_k(\infty) \\ &\quad + \left(C(I-A)^{-1}B + D\right) u_k(\infty) \\ x_{k+1}(\infty) &= (I-A)^{-1}B_0 y_k(\infty) \\ &\quad + (I-A)^{-1}Bu_k(\infty) \end{aligned} \tag{4}$$

Now we can introduce the following definition of strong practical stability.

Definition 1. A discrete linear repetitive process described by (1) and (2) is said to be strongly practically stable when

- it is asymptotically stable,
- $r(A) < 1$,
- the limit profile (3) is stable in the 1D sense, and
- the limit profile (4) is stable in the 1D sense.

The following result is a straightforward consequence of the above definition.

Theorem 2. A discrete linear repetitive process described by (1) and (2) is strongly practically stable if, and only if,

$$\begin{aligned} r(D_0) &< 1 \\ r(A) &< 1 \\ r(A + B_0(I-D_0)^{-1}C) &< 1 \\ r(C(I-A)^{-1}B_0 + D_0) &< 1 \end{aligned} \tag{5}$$

The essential difference with stability along the pass is that both limit profiles are stable, but in between growth can occur in the pass profile sequence but these must be damped out as the dynamics evolve in k and p. Note also that the third and the fourth conditions in (5) involve matrix sums, products and inversions. This makes them potentially very awkward for extension to control law design and robust stability when, for example, some or all of the matrices in the process state-space model are subject to additive perturbations.

The route to developing computationally efficient tests for the third condition in Theorem 2 (with a natural extension to control law design) is via 1D singular linear systems theory for the state-space model

$$\begin{aligned} Ex(h+1) &= \widehat{A}x(h) + \widehat{B}u(h) \\ y(h) &= \widehat{C}x(h) + \widehat{D}u(h) \end{aligned} \tag{6}$$

where E is a singular matrix. In particular, the condition $r(A+B_0(I_m-D_0)^{-1}C) < 1$ of Theorem 2 is easily seen to be equivalent to stability (admissibility) of the 1D singular linear system with state-space model

$$E_1 z(h+1) = \begin{bmatrix} A & B_0 \\ C & D_0 - I \end{bmatrix} z(h) + \begin{bmatrix} B \\ D \end{bmatrix} u(h) \tag{7}$$

where

$$E_1 = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \tag{8}$$

or

$$E_1 z(h+1) = A_1 z(h) + \Pi u(h) \tag{9}$$

Similarly, the condition $r(C(I-A)^{-1}B_0 + D_0) < 1$ of Theorem 2 is equivalent to stability (admissibility) of the 1D singular linear system

$$E_2 z(h+1) = \begin{bmatrix} A - I & B_0 \\ C & D_0 \end{bmatrix} z(h) + \begin{bmatrix} B \\ D \end{bmatrix} u(h) \tag{10}$$

where

$$E_2 = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} \quad (11)$$

or

$$E_2 z(h+1) = A_2 z(h) + \Pi u(h) \quad (12)$$

Also, as shown in Dabkowski et al. (2007), it is possible to develop LMI stability (admissibility) conditions for the models of (7) and (10) which are hence equivalent to the third and the fourth conditions of (5). Also these LMIs lead to control law design algorithms as discussed next.

The control law considered here is

$$u_{k+1}(p) = K_1 x_{k+1}(p) + K_2 y_k(p) \quad (13)$$

which is the sum of current pass state feedback ($x_{k+1}(p)$) and feedforward (in the k direction) from the previous pass pass profile. A simpler structure would result if current pass state feedback alone could be used but it is known that this is only possible in a few very restrictive special cases. Note also that the previous pass profile is a measured output and here we assume that it not significantly corrupted by noise etc. Moreover, the current pass state vector in this control law could be replaced by the current pass profile or estimated using an observer if not all entries are available for measurement.

LMI based stability tests and algorithms for designing K_1 and K_2 in (13) have been reported in Dabkowski et al. (2007) and are omitted here due to space limitations. Next, we extend these results to the robust case.

4. ROBUST STABILIZATION

In most practical cases, the process state-space model will not be known exactly. Instead, we have (at best) nominal values for the entries in these models and to proceed we must assume that the true entries lie in some uncertainty set around the nominal. If this set is convex then we can proceed to obtain control laws which will stabilize any state-space model where the entries of the matrices which define it lie in this set. In what follows, one commonly used approach to model the uncertainty is used, i.e. the so-called norm bounded uncertainty.

The state-space model of the uncertain discrete linear repetitive processes considered here is of the form

$$\begin{aligned} x_{k+1}(p+1) &= (A+\Delta A)x_{k+1}(p) + (B+\Delta B)u_{k+1}(p) \\ &+ (B_0+\Delta B_0)y_k(p) \\ y_{k+1}(p) &= (C+\Delta C)x_{k+1}(p) + (D+\Delta D)u_{k+1}(p) \\ &+ (D_0+\Delta D_0)y_k(p) \end{aligned} \quad (14)$$

where the matrices ΔA, ΔB, ΔB_0, ΔC, ΔD, ΔD_0 represent the admissible uncertainties and are assumed to satisfy

$$\begin{bmatrix} \Delta A & \Delta B_0 & \Delta B \\ \Delta C & \Delta D_0 & \Delta D \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \mathcal{F} \left[M_1 \;\; M_2 \;\; M_3 \right] \quad (15)$$

where H and M are some known constant matrices with compatible dimensions end $\mathcal{F}$ is an unknown matrix which satisfies

$$\mathcal{F}^T \mathcal{F} \leq I \quad (16)$$

We will also require the following well known result.

Lemma 3. Suppose that H and M are known real matrices of appropriate dimensions and the unknown matrix $\mathcal{F}$ satisfies $\mathcal{F}^T\mathcal{F} \leq I$. Then for any $\epsilon > 0$

$$H\mathcal{F}M + M^T\mathcal{F}^TH^T \preceq \epsilon HH^T + \frac{1}{\epsilon}M^TM \quad (17)$$

The following is the main result of this paper.

Theorem 4. Suppose that a control law of the form (13) is applied to a discrete linear repetitive process described by (14) with uncertainty satisfying (15)–(16). Then the controlled process is strongly practically stable if the following LMIs hold

$$\begin{bmatrix} W_2 - G_2 - G_2^T & * & * \\ D_0G_2 + DR_2 & -W_2 + \epsilon_1 H_2 H_2^T & * \\ M_2G_2 + M_3R_2 & 0 & -\epsilon_1 I_n \end{bmatrix} < 0 \quad (18)$$

$$\begin{bmatrix} W_1 - G_1 - G_1^T & * & * \\ AG_1 + BR_1 & -W_1 + \epsilon_2 H_1 H_1^T & * \\ M_1G_1 + M_3R_1 & 0 & -\epsilon_2 I_n \end{bmatrix} < 0 \quad (19)$$

$$\left[\begin{array}{cccccccc}
\begin{array}{c} -X_{11}^1 + AG_1 + BR_1 + (AG_1+BR_1)^T \\ +\epsilon_3 4H_1H_1^T \end{array} & * & * & * & * & * & * & * \\
CG_1 + DR_1 + (B_0G_2+BR_2)^T + \epsilon_3 4H_2H_1^T & \begin{array}{c} D_0G_2 - G_2 + DR_2 + (D_0G_2 - G_2 + DR_2)^T \\ +\epsilon_3 4H_2H_2^T \end{array} & * & * & * & * & * & * \\
-G_1 + (AG_1\beta_1 + BR_1\beta_1)^T & (CG_1\beta_1 + DR_1\beta_1)^T & X_{11}^1 - G_1\beta_1 - (G_1\beta_1)^T & * & * & * & * & * \\
(B_0G_2\beta_1 + BR_2\beta_1)^T & Y_{22}^1 - G_2 + (D_0G_2\beta_1 - G_2\beta_1 + DR_2\beta_1)^T & X_{21}^1 & X_{22}^1 - G_2\beta_1 - (G_2\beta_1)^T & * & * & * & * \\
M_1G_1 + M_3R_1 & 0 & 0 & 0 & -\epsilon_3 I_n & * & * & * \\
0 & M_2G_2 + M_3R_2 & 0 & 0 & 0 & -\epsilon_3 I_n & * & * \\
0 & 0 & M_1G_1\beta_1 + M_3R_1\beta_1 & 0 & 0 & 0 & -\epsilon_3 I_n & * \\
0 & 0 & 0 & M_2G_2\beta_1 + M_3R_2\beta_1 & 0 & 0 & 0 & -\epsilon_3 I_n
\end{array}\right] < 0 \quad (20)$$

$$
\begin{bmatrix}
\begin{matrix} AG_1 - G_1 + BR_1 + (AG_1 - G_1 + BR_1)^T \\ +4\epsilon_4 H_1 H_1^T \end{matrix} & * & * & * & * & * & * & * \\
CG_1 + DR_1 + (B_0G_2 + BR_2)^T + 4\epsilon_4 H_2 H_1^T & \begin{matrix} -X_{22}^2 + D_0G_2 + DR_2 + (D_0G_2 + DR_2)^T \\ +4\epsilon_4 H_2 H_2^T \end{matrix} & * & * & * & * & * & * \\
Y_{11}^2 - G_1 + (AG_1\beta_2 - G_1\beta_2 + BR_1\beta_2)^T & (CG_1\beta_2 + DR_1\beta_2)^T & X_{11}^2 - G_1\beta_2 - (G_1\beta_2)^T & * & * & * & * & * \\
(B_0G_2\beta_2 + BR_2\beta_2)^T & -G_2 + (D_0G_2\beta_2 + DR_2\beta_2)^T & X_{21}^2 & X_{22}^2 - G_2\beta_2 - (G_2\beta_2)^T & * & * & * & * \\
M_1G_1 + M_3R_1 & 0 & 0 & 0 & -\epsilon_4 I_n & * & * & * \\
0 & M_2G_2 + M_3R_2 & 0 & 0 & 0 & -\epsilon_4 I_n & * & * \\
0 & 0 & M_1G_1\beta_1 + M_3R_1\beta_1 & 0 & 0 & 0 & -\epsilon_4 I_n & * \\
0 & 0 & 0 & M_2G_2\beta_1 + M_3R_2\beta_1 & 0 & 0 & 0 & -\epsilon_4 I_n
\end{bmatrix} < 0 \qquad (21)
$$

for a given $\beta_1 > 1$, $\beta_2 > 1$ with the variables $W_1 > 0$, $W_2 > 0$, $X_{11}^1 = (X_{11}^1)^T$, $X_{22}^1 = (X_{22}^1)^T$, $X_{11}^2 = (X_{11}^2)^T$, $X_{22}^2 = (X_{22}^2)^T$, X_{21}^1, X_{21}^2, Y_{11}^2, Y_{22}^1, G_1, G_2, R_1, R_2, $\epsilon_1 > 0$, $\epsilon_2 > 0$, $\epsilon_3 > 0$, $\epsilon_4 > 0$.

If these LMIs hold, the control law matrices are given by

$$K_1 = R_1 G_1^{-1}, \quad K_2 = R_2 G_2^{-1} \qquad (22)$$

Proof. The proof relies on representing the requirements of (5) for the uncertain process of (14) with the control law of (13) applied in LMI form. This can be accomplished by the use of results for singular discrete linear systems given in Chaabane et al. (2007), Lemma 3 and routine (but complicated) application of the Schur's complement formula and congruence transforms. The details are omitted here due space limitations. (For the proof in the case with no uncertainty see Dabkowski et al. (2007).)

4.1 Numerical Example

Example 1. Consider the case when the pass length $\alpha = 50$ and

$$A = \begin{bmatrix} 1.82 & -1.66 & 1.56 \\ -1.97 & -1.99 & -1.06 \\ 1.29 & -2.43 & 1.87 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.673 & 0.0825 \\ 0.706 & -0.309 \\ -0.424 & -0.139 \end{bmatrix}, \quad B_0 = \begin{bmatrix} -0.975 \\ -1.23 \\ -1.25 \end{bmatrix}$$

$$C = [\,-0.553 \;\; -0.427 \;\; 1.12\,]$$

$$D = [\,-0.656 \;\; -0.0482\,], \quad D_0 = -1.18$$

with boundary conditions

$$x_{k+1}(0) = [\,10 \;\; 10 \;\; 10\,]^T$$
$$y_0(p) = 0, \quad 0 \le p \le 49$$

Also

$$H_1 = \begin{bmatrix} 0.053 & 0.0020 & 0.065 \\ 0.040 & 0.036 & 0.054 \\ 0.027 & 0.051 & 0.085 \end{bmatrix}$$

$$H_2 = [\,0.016 \;\; 0.0076 \;\; 0.011\,]$$

$$M_1 = \begin{bmatrix} 0.0062 & 0.018 & 0.042 \\ 0.030 & 0.025 & 0.039 \\ 0.040 & 0.036 & 0.027 \end{bmatrix}, M_2 = \begin{bmatrix} 0.050 \\ 0.039 \\ 0.22 \end{bmatrix}$$

$$M_3 = \begin{bmatrix} 0.038 & 0.016 \\ 0.0044 & 0.0024 \\ 0.038 & 0.0079 \end{bmatrix}$$

This process is asymptotically, and hence practically, unstable — as confirmed by the pass profile sequence shown in Fig. 1.

In this case, the LMIs of Theorem 4 LMIs are feasible and for $\beta_1 = \beta_1' = 5.0$, $\quad \beta_2 = \beta_2' = 19.0$

$$K_1 = \begin{bmatrix} 0.719 & -0.677 & 2.03 \\ -19.9 & 3.69 & -9.42 \end{bmatrix}, K_2 = \begin{bmatrix} -1.0 \\ -1.38 \end{bmatrix}$$

for $\beta_1 = \beta_1'' = 7.0$, $\quad \beta_2 = \beta_2'' = 17.0$

$$K_1 = \begin{bmatrix} 0.716 & -0.700 & 2.06 \\ -20.1 & 3.28 & -8.80 \end{bmatrix}, K_2 = \begin{bmatrix} -0.988 \\ -1.56 \end{bmatrix}$$

Sequences of pass profiles for these controlled processes are shown in Figures 2 and 3 respectively.

From these plots, it is clearly seen that strong practical stability guarantees the stable limit profile but, in contrast what would happen under stability along the pass, oscillations appear the pass profile sequence but are eventually damped out in both p and k. These responses also show that β_1 and β_2 can be used to influence these oscillations (note the difference in the values on the pass profile axis in these plots). How to exploit this feature to the maximum extent is the subject of current research. Finally, note that positive results from this will also have relevance in the design of control laws for 2D discrete linear systems described by the Roesser and Fornasini Marchesini state-space models given the strong links these have with the discrete linear repetitive processes considered here.

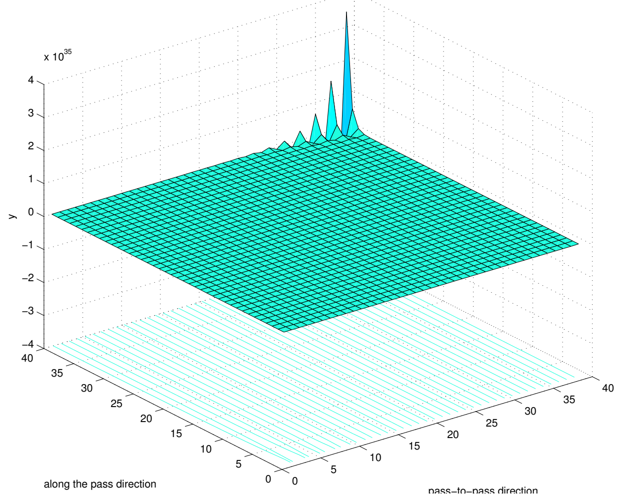

Fig. 1. Pass profile dynamics for the uncontrolled process.

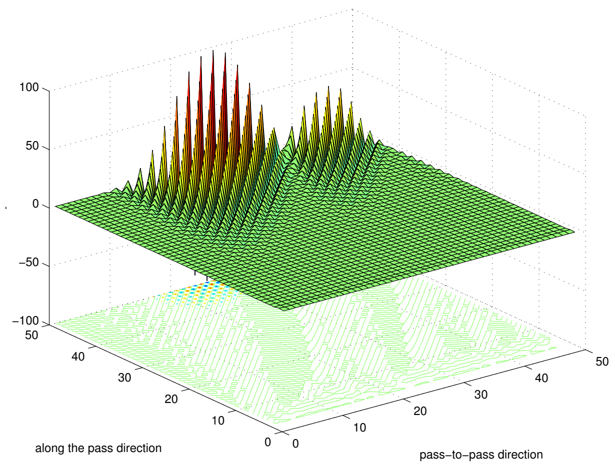

Fig. 2. Pass profile dynamics for the controlled process when $\beta_1 = 5.0$ and $\beta_2 = 19.0$.

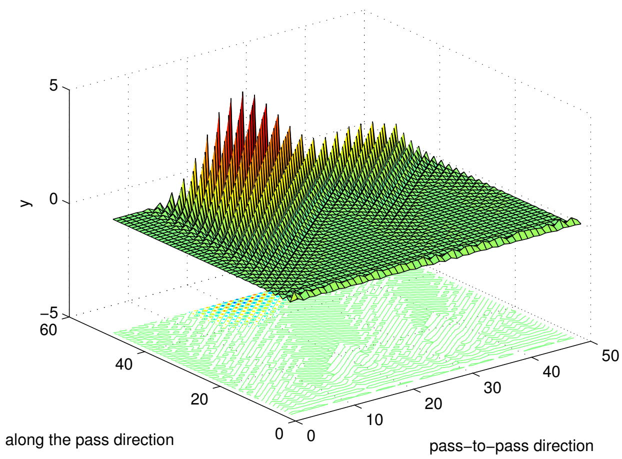

Fig. 3. Pass profile dynamics for the controlled process when $\beta_1 = 7.0$ and $\beta_2 = 17.0$.

5. CONCLUSIONS

This paper has produced new results on so-called strong practical stability of discrete linear repetitive processes. These extend previous work to allow for control law design when there is uncertainty associated with the process state-space model which here is assumed to be of the norm bounded type. The resulting design algorithms are LMI based. Future work should include the extension of these results to other models of uncertainty and rules for selecting the parameters β_1 and β_2 in the main new result to maximum advantage. In the longer term, the application to iterative learning control will be considered.

REFERENCES

P. Agathoklis and L. T. Bruton. Practical-BIBO stability of n-dimensional discrete systems. *IEE Proceedings. Part G. Electronic circuits and systems*, 130(6):236–242, 1983.

N. Amann, D. H. Owens, and E. Rogers. Predictive optimal iterative learning control. *International Journal of Control*, 69(2):203–226, 1998.

M. Chaabane, O. Bachelier, and D. Mehdi. Admissibility and state feedback admiss and stabilization of discrete singular systems: an LMI approach. Technical report 20070208DM, LAII-ESIP, Poitiers, France, 2007.

P. Dabkowski, K. Galkowski, E. Rogers, and A. Kummert. Strong practical stability and control of discrete linear repetitive processes. In *Proc. Int. Workshop on Multidimensional (nD) Systems, nDS 2007*, Aveiro, Portugal, June 27–29 2007.

E. Fornasini and G. Marchesini. Doubly indexed dynamical systems: state models and structural properties. *Mathematical Systems Theory*, 12:59–72, 1978.

K. Gałkowski, E. Rogers, J. Gramacki, A. Gramacki, and D. H. Owens. Strong practical stability for a class of 2D linear systems. In *Proc. Int. Symposium on Circuits and Systems (ISCAS)*, volume 1, pages 403–406, Geneva, Switzerland, 28-31 May 2000.

P. D. Roberts. Numerical investigation of a stability theorem arising from 2-dimensional analysis of an iterative optimal control algorithm. *Multidimensional Systems and Signal Processing*, 11(1-2):109–124, 2000.

R. P. Roesser. A discrete state-space model for linear image processing. *IEEE Transactions on Automatic Control*, 20(1):1–10, 1975.

E. Rogers, K. Gałkowski, and D. H. Owens. *Control Systems Theory and Applications for Linear Repetitive Processes*, volume 349 of *Lecture Notes in Control and Information Sciences*. Springer-Verlag, Berlin, Germany, 2007.

K. J. Smyth. *Computer Aided Analysis for Linear Repetitive Processes*. PhD thesis, University of Strathclyde, Glasgow, United Kingdom, 1992.

L. Xu, O. Saito, and K. Abe. The design of practically stable nD feedback systems. *Automatica*, 30(9):1389–1397, 1994.