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Using the Unscented Kalman Filter and a Non-linear Two-track Model for Vehicle State Estimation

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Abstract: In order to evaluate the driving stability of a motor vehicle, the accurate determination of the vehicle sideslip angle is of significant importance. With the help of the sensor signals in today's production vehicles, this state can only be determined with limited accuracy. We propose an algorithm for the determination and estimation of the vehicle state based on the Unscented Kalman Filter. In the described estimator a two-track model of the vehicle is used, which represents the road contact with the Pacejka's Magic Formula tyre model.

1. INTRODUCTION

For a reliable assessment of a vehicle's driving state and driving stability, the exact determination of the vehicle sideslip angle is of great importance. Using the sensors available in today's production vehicles, the sideslip angle can only be determined with limited accuracy. Sensors for lateral acceleration, yaw rate, steering wheel angle and wheel speeds can be regarded as standard equipment in most cases. Series applications of optical speed sensors providing direct measurement of longitudinal and lateral velocity or sideslip angle seem to be improbable for the near future. Sideslip angle determination based on a combination of inertial measurements and satellite navigation systems (Global Positioning System, GPS) is currently researched and might become available in future cars (Ryu, 2004).

In this context, this work is aimed at the determination of the vehicle's driving state. In literature, several works and approaches concerning vehicle state estimation can be found (e.g., Boßdorf-Zimmer *et al.*, 2006, Mao *et al.*, 2006, Pruckner, 2001, Zuurbier & Bremmer, 2002). In general, vehicle dynamics are represented by suitable models. In modelling and simulation of vehicle dynamics, the exact reproduction of the tyre behaviour is an essential criterion for model accuracy, since tyre characteristics significantly affect the horizontal forces acting upon the vehicle. Especially for high lateral accelerations and in transient vehicle states, the tyre behaviour shows various non-linearities.

All publications concerning vehicle state estimation known to the authors utilize highly simplified tyre models. In order to be able to employ a linear Kalman filter, a linear model must be used. In this case, the side-force behaviour of the tyre is reduced to a lateral stiffness. The extended Kalman filter, which can be applied to nonlinear systems, also requires simplified tyre models. The tyre forces must be differentiable with respect to all state variables. For this reason, approximations of the tyre characteristics based on exponential functions are used in some cases. The degressive dependency of lateral tyre force on wheel load and its impact on the vehicle's handling characteristics remain unconsidered in many publications.

For conventional variants of the Kalman filter, the plant model needs to be simplified to linear systems or at least to differentiable systems. For this reason, algorithms have been developed which compensate these constraints and are able to deal with non-differentiable systems. In this context the so-called Unscented Kalman Filter is an interesting approach, since it can be adopted to any non-linear system while the necessary computational effort is not significantly increased (Julier & Uhlmann, 1997).

The objective of this work is the use of a complex tyre-model in order to represent vehicle dynamics in a more accurate way. The widely used Pacejka tyre-model, the so-called Magic Formula (Bakker *et al.*, 1987), represents non-linear tyre characteristics quite well. However, it is not easily differentiable and therefore not used in any previous papers. The implementation of a vehicle state estimation based on the Unscented Kalman Filter allows the integration of this tyre-model.

2. VEHICLE MODELLING

For the design of a state estimator, it is necessary to establish a model of the corresponding system. For the representation of lateral dynamics with low lateral accelerations (approx. 3-4 m/s²), the linear single track model is often used, applying various simplifications compared to the real vehicle. The tyres of one axle are approximated to one tyre on the centre line of the vehicle, the centre of gravity is located on the road surface and the lateral forces are calculated as a linear function of tyre sideslip (cornering stiffness).

To arrive at a more exact modelling approach of lateral dynamics and validity for higher lateral accelerations, different more complex and non-linear models are used e.g., non-linear single track or two track models. In this work, an extension to a non-linear two-track model is made use of (see Fig. 2-1). For this purpose, a vehicle with four wheels is considered whose centre of gravity is located at a height h_{SP} above the road surface. There are no simplifications applied for small angles, and cornering stiffness depends on wheel vertical forces and slip angle in a non-linear way. The roll momentum caused by the lateral acceleration is distributed between front and rear axle based on the proportions in the real vehicle. Thus tyre lateral force characteristics are influenced, in case this dependency is represented by the tyre model, providing a possibility to tune the steering behaviour according to the reference vehicle.

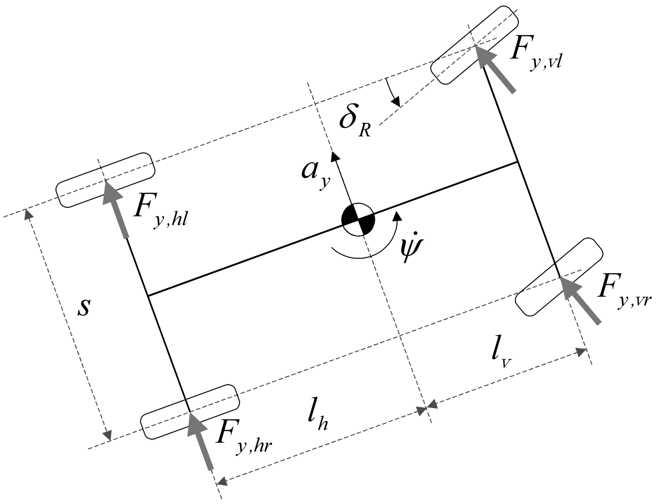

Fig. 2-1. Two-track vehicle model

The equations of the two-track model used are the following (Renner, 2006):

$$m\,a_y = F_{y,hr} + F_{y,hl} + F_{y,vr}\cos\delta_R + F_{y,vl}\cos\delta_R \ , \tag{1}$$

$$J_z\,\ddot{\psi} = F_{y,vl}l_v\cos\delta_R + F_{y,vl}\frac{s}{2}\sin\delta_R - F_{y,hl}l_h$$
$$\qquad - F_{y,hr}l_h + F_{y,vr}l_v\cos\delta_R - F_{y,vr}\frac{s}{2}\sin\delta_R \ . \tag{2}$$

In this, m describes the vehicle mass, a_y the lateral acceleration, $F_{y,vl}$, $F_{y,vr}$, $F_{y,hl}$, $F_{y,hr}$ the tyre lateral forces front left, front right, rear left and rear right respectively, δ_R the steering angle, J_z the moment of inertia around the yaw axis, $\ddot{\psi}$ the yaw rate, l_v, l_h the distance from the centre of gravity to front and rear axles respectively, and s the track width (see Fig. 2-1).

The tyre lateral forces result from the tyre vertical forces and the tyre model used. In the work of Zuurbier & Bremmer (2002) a differentiable tyre model, based on an simple differentiable exponential function, is adopted in a non-linear two-track model. The present paper uses a tyre model based on Pacejka´s Magic Formula (MF):

$$F_y(\alpha) = D\sin\{C\arctan[B\alpha - E\ [B\alpha - \arctan(B\alpha)]]\} \ . \tag{3}$$

The parameters B, C, D and E are estimated by optimization procedures based on tyre measurements (Bakker *et al.*, 1987). Moreover, α describes the slip angle, i.e. the angle between wheel circumferential direction and motion direction of the wheel contact point. The slip angles are:

$$\alpha_v = \delta_R - \arctan\left(\frac{l_v\,\dot{\psi}}{v} + \tan\beta\right) , \tag{4}$$

$$\alpha_h = \arctan\left(\frac{l_h\,\dot{\psi}}{v} - \tan\beta\right) \tag{5}$$

for both front and rear wheels (Renner, 2006) with the vehicle sideslip angle β and the velocity v of the center of gravity. Vertical tyre loads result from the stationary tyre load and the dynamic tyre load due to lateral and longitudinal acceleration. Tyre lateral forces depend on vertical tyre loads in a nonlinear way according to the drum test bench measurements taken to parameterise the tyre model.

Both non-linear tyre models and the linear tyre cornering stiffness are shown in Fig. 2-2. It becomes obvious that the three tyre models correspond well in the range of low slip angles. The tyre model based on an exponential function delivers good results up to the maximum of lateral force. But the degression of the lateral forces that do occur at higher slip angles can only be represented by the MF model, the exponential tyre model cannot reproduce this phenomenon. Furthermore, the degressive behaviour of lateral forces for increasing vertical loads is modelled. This offers the possibility of tuning the understeering properties using roll moment distribution.

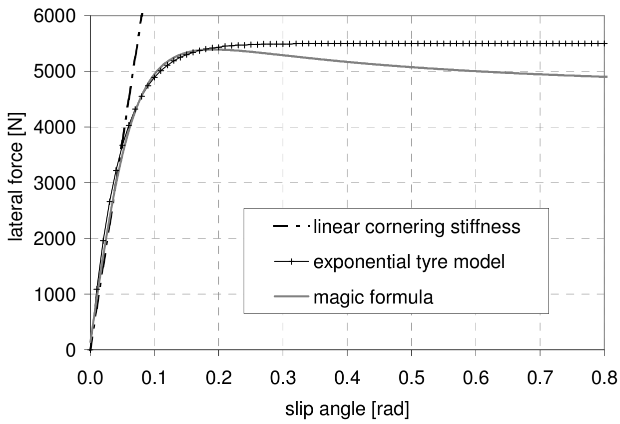

Fig. 2-2. Comparison of the tyre models

Finally, the non-linear two-track model is noted in state space formulation:

$$\dot{x} = f(x,u) \ , \tag{6}$$

$$y = h(x,u) \ , \tag{7}$$

where

$$x = [\beta\ \dot{\psi}\ a_y]^{\mathrm{T}} \ , \tag{8}$$

$$y = [a_y\ \dot{\psi}]^{\mathrm{T}} \ , \tag{9}$$

$$u = \delta_R \ . \tag{10}$$

The functions f and h are derived from (1), (2) and the relationship

$$\dot{\beta} = \dot{\psi} - \frac{a_y}{v} \quad . \tag{11}$$

In principle the inclusion of the lateral acceleration a_y in the state vector is not necessary. However, the numerical complexity for the practical implementation of the vehicle state estimation is decreased by this.

3. STATE ESTIMATION

The goal of a model based state estimation is to compute state variables which cannot to be measured directly. The state is estimated using a model of the system whereas the knowledge of the input value and the measurable output values is assumed (cf. Fig. 3-1). In lots of applications different variants of the Kalman Filter are employed.

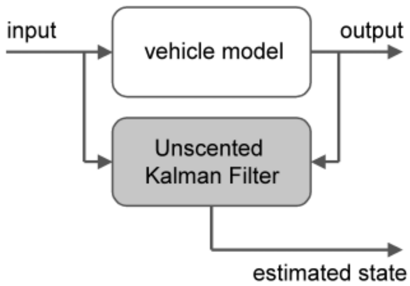

Fig. 3-1. Principle of the model-based state estimation

For the Kalman Filter based state estimation a model in state space is required. The estimation of the new system state is based on the actual state and the input values. For this estimated state the corresponding values of the measurable output are calculated and compared with the real values, as represented by the difference between the model and the real system. This difference is used for calculation of the necessary approximations or adjustments.

In order to be able to use the Kalman Filter based vehicle state estimator the vehicle model taken from (6) and (7) has to be discretised. For the transition of the state $\hat{x}_k^+$ at the time t_k to the state $\hat{x}_{k+1}^-$ at the time t_{k+1} we get:

$$\hat{x}_{k+1}^- = \hat{x}_k^+ + \int_{t_k}^{t_{k+1}} f(\hat{x}(\tau), u(\tau)) d\tau \quad , \tag{12}$$

where $\hat{x}(\tau)$ denotes the solution of the differential equation

$$\dot{\hat{x}}(t) = f(\hat{x}(t), u(t)) \tag{13}$$

in the time interval $[t_k \ , \ t_{k+1}]$ for the initial condition $\hat{x}(t_k) = \hat{x}_k^+$.

Equation (12) describes a typical initial value problem which could be solved during simulations using e.g. the Runge Kutta Method. For real-time implementation with short sampling times it is often possible to assume minor changes of $f(\hat{x}(t), u(t))$ in the time interval $[t_k \ , \ t_{k+1}]$. Hence in the considered time interval we can set

$$f(\hat{x}(\tau), u(\tau)) \approx f(\hat{x}(t_k), u(t_k)) \quad . \tag{14}$$

Inserting into (12) results in

$$\hat{x}_{k+1}^- \approx \hat{x}_k^+ + t_s \, f(\hat{x}(t_k), u(t_k)) \quad , \tag{15}$$

where a constant sampling time $t_s = t_{k+1} - t_k$ is assumed.

In the next step the measurable output values $y(t_k)$ need to be used to correct the estimated state $\hat{x}_k^-$ (estimated using (12) or (15)) as applicable. Both for the linear Kalman Filter and the Unscented Kalman Filter this is done by using the equation

$$\hat{x}_k^+ = \hat{x}_k^- + K_k(y(t_k) - t_s(\hat{x}_k^-, u(t_k))) \quad , \tag{16}$$

where K_k denotes the so-called Kalman Gain. In order to calculate the Kalman Gain we refer e. g. to Brown & Hwang, 1992, Grewal & Andrews, 1993, Reif *et al.*, 1999, Reif & Unbehauen, 1998.

Using the Unscented Kalman Filter, the non-linear functions f and h are not linearised. Instead, the probability distribution of the states being influenced by noise and the corresponding output values corrupted by noise, is approximated. An artificial ensemble of different weighted states having a given average and a given covariance is calculated. The states of this ensemble are not chosen randomly but by a given procedure (see e.g. Julier *et al.*, 2000). For every state of this ensemble $\hat{X}_k^+$ at the time t_k the corresponding state at the time t_{k+1} is calculated by (12) or (15). Thus the ensemble $\hat{X}_{k+1}^-$ of the states at the time t_{k+1} is obtained, and using (7), i. e.

$$\hat{y}_{k+1}^- = h(\hat{x}_{k+1}^-, u(t_{k+1})) \tag{17}$$

the corresponding ensemble $\hat{Y}_{k+1}^-$ of the measurable output values. Hence the actual estimation value and the covariance $P_{xx,k+1}^-$ of the (still uncorrected) estimated state, the covariance $P_{yy,k+1}^-$ of the corresponding output values and the cross covariance $P_{xy,k+1}^-$ between the estimated state and the corresponding output values could be calculated. The calculation of the Kalman Gain K_k is done according to least squares estimation. It yields (Catlin, 1989, Julier & Uhlmann, 2004):

$$K_k = P_{xy,k}^- (P_{yy,k}^-)^{-1} \quad . \tag{18}$$

4. RESULTS

As described above, the equations of the non-linear two-track model using tyre models according to the Magic Formula are used for the implementation of a state estimator based on an Unscented Kalman Filter. For the estimator design, the covariance of the measurement noise and the covariance of the system noise are determined empirically. Design parameters for the Unscented Kalman Filter are chosen according to Wan & van der Merve, 2000.

For adjusting the estimator design parameters and assessing the achieved estimation results, the estimator is integrated into a vehicle dynamics simulation environment. The basis for this simulation environment is formed by a MATLAB/Simulink full vehicle simulation model. Fundamentally, this model consists of the linear differential equations of motion of a 5 mass, 10 degree of freedom (DoF) system: translational and rotational motion of the body and vehicle respectively in three coordinate directions as well as translational motion of the 4 wheel masses. In addition to the linear time invariant (LTI) core of the model, non-linear elements such as a tyre model, steering gear, and kinematics and compliance subsystems are used in order to improve the accuracy of the model. Hence, the dynamics of the real vehicle are represented with a high level of accuracy.

This vehicle dynamics model substitutes the real vehicle in Fig. 3-1. The following values are fed into the vehicle state estimator by the vehicle dynamics model: steering angle, lateral acceleration, yaw rate, longitudinal acceleration and longitudinal velocity. The estimation results are validated by a comparison between the sideslip angle estimated and the sideslip angle calculated within the complex vehicle dynamics model.

To provide conditions similar as in a real vehicle, the signals given to the estimation are corrupted in a realistic way, representing signals available on a vehicle's CAN bus. To this end, the estimator inputs from the simuation environment are superimposed with noise and offset, quantised according to the CAN resolution of comparable signals in production vehicles and discretised. Therefore, the signals available for the estimator are of similar quality as the signals transmitted via CAN. An example of the lateral acceleration signal is shown in Fig. 4-1.

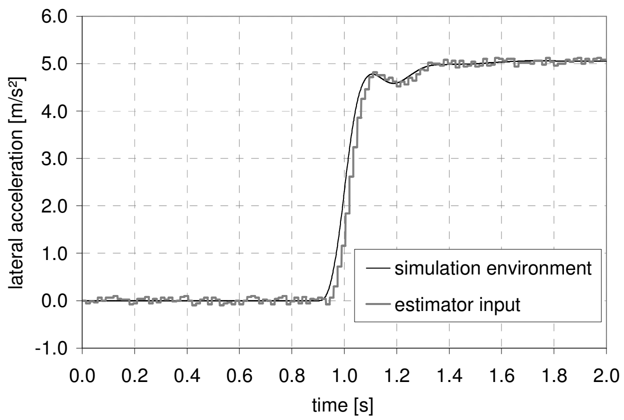

Fig. 4-1. Realistic corruption of the signals

The estimation quality becomes obvious when comparing the estimated sideslip angle and the sideslip angle of the vehicle dynamics model. Additionally, the results of the estimation are compared to the results of a Kalman Filter based on a linear single track model. Simulation results for the manoeuvres step steer input, double lane change and single sinus steering are shown in the following.

For the step steer input manoeuvre (40° steering angle, 65 km/h, Fig. 4-2), it can be seen that the limits of the linear single track model are exceeded. The sideslip angle is estimated inaccurately by the linear Kalman Filter, whereas the non-linear estimator observes the sideslip angle significantly better despite of a remaining offset. It can also be seen that both estimation results are affected by the transmission-caused signal corruption in a similar way.

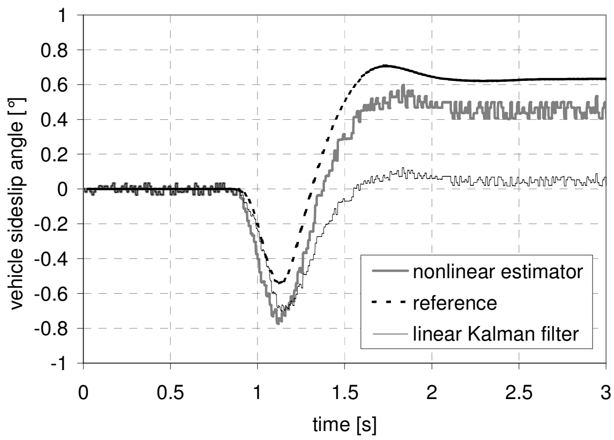

Fig. 4-2. Step steer input

Significant advantages of the non-linear estimation are demonstrated in highly dynamic manoeuvres with large steering angles and high sideslip angles. In Fig. 4-3, the results of a double lane change simulation are illustrated. The estimation values nearly reach the reference signal during the whole manoeuvre. Especially there are virtually no phase lags. On the other hand, the linear Kalman Filter is able to represent the driving behaviour only insufficiently. Therefore, the estimated values show significant differences concerning both amplitude and phase.

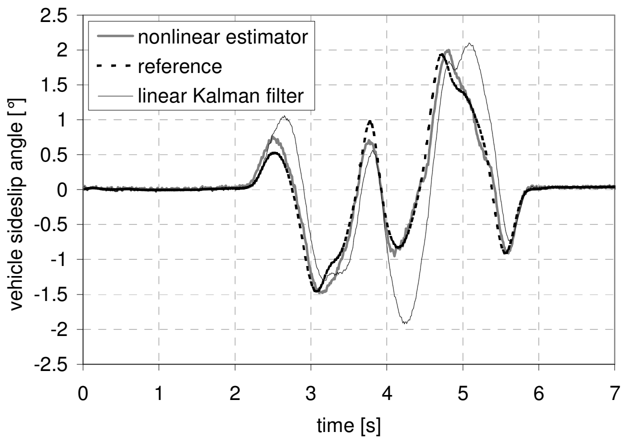

Fig. 4-3. Double lane change at 65 km/h

The situation represents similarly during the single sinus manoeuvre at 80 km/h vehicle velocity (see Fig. 4-4). Here a steering angle with an increasing amplitude is applied, 150° to the left and 225° to the right. Thereby high slip angles are reached and oversteering vehicle reaction is provoked. At the estimation result it becomes obvious that a linear tyre model is nowhere near enough to cover this manoeuvre. But it is possible to get a good estimation of the sideslip angle using the non-linear tyre model based on the Magic Formula.

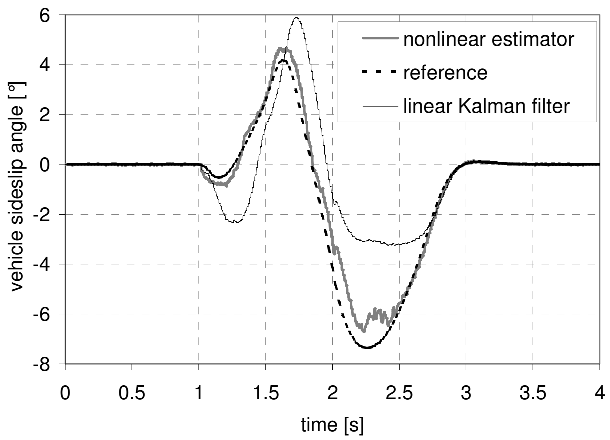

Fig. 4-4. Single sinus manoeuvre

The linear estimator exhibits higher sideslip angle values in most dynamic manoeuvres. This behaviour is due to the absence of a saturation in the side force behaviour, since the linear tyre produces unrealistically high lateral forces for large tyre sideslip angles, thus causing excessive yaw and vehicle sideslip reactions.

In spite of the adequate results of the non-linear estimator, especially compared to the linear Kalman Filter, there are some differences found between estimations based on the Unscented Kalman Filter and the simulated reference values during all considered situations. These differences are to be attributed only to a minor part to the quality of the input signals. The primary cause for these differences is the high degree of simplification of the vehicle model used in the estimator. In the two track model, body and wheels are represented by a rigid body, not allowing for any relative motion between these elements. Therefore, any influence on the driving behaviour caused by heave, pitch and roll motions of the body cannot be represented. Furthermore, kinematics and compliance of the suspension, exerting important influence on stationary and transient lateral dynamics of the vehicle, are not modelled due to the same reason. Regarding these limitations of the two track model, the exact reproduction of vehicle motion will be limited in spite of the more accurate tyre model.

As a secondary cause, some dynamic differences can be attributed to the transient tyre behaviour. The original modelling of Pacejka´s Magic Formula (Bakker *et al.*, 1987) is used within the non-linear estimation. Current versions of the Magic Formula tyre model also represent the dynamic behaviour (Pacejka & Besselink, 1997), for e.g. the Version MF 5.2 put into the vehicle dynamics model used above. In order to be able to implement such a tyre model in the estimator, the state vector has to be extended by a corresponding state variable of the tyre model. Since this applies to all four tyres independently, the state vector would have to be extended to at least a dimension of 7. The computation effort caused by this extension would be disproportionate relative to the improvement of the anticipated estimation results. Therefore only the stationary tyre model of the Magic Formula is used in this work.

Approaches to further improvements of the models are for instance an extension of the used two-track model (considering the tyre longitudinal forces) and an optimization of the vehicle parameters applied, as well as the design parameters of the Unscented Kalman Filter. Future work will also have to cover the issues of estimation robustness in different driving situations and the representation of low friction coefficients.

5. CONCLUSIONS

This paper shows an approach, how to implement a vehicle state estimator using a variant of the Kalman Filter, the so called Unscented Kalman Filter. This establishes the possibility to use non-linear elements, which are non-differentiable or whose derivative cannot be calculated with an acceptable computation effort. The described estimation is based on a two-track model of the vehicle, which simulates the tyre behaviour with the Magic Formula of Pacejka. The estimation designed that way, offers a significant improvement of the estimation accuracy, with a low increase of calculation complexity, compared to the standard Kalman Filter. These improvements, especially during high-dynamical manoeuvres with high sideslip angles, were demonstrated by means of a complex vehicle dynamics model.

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