Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Robust H_∞ Control for Neutral Uncertain Switched Nonlinear Systems using Multiple Lyapunov Functions ⋆

Li-li Li * **Georgi M. Dimirovski** ** **Jun Zhao** *,***

* *Key Laboratory of Integrated Automation of Process Industry, Ministry of Education, Northeastern University. Shenyang, P.R.China (E-mail: tolilili@gmail.com zhaojun@ise.neu.edu.cn)*
** *Department of Computer Engineering, Dogus University, Kadikoy, TR-34722, Istanbul, Turkey (E-mail: gdimirovski@dogus.edu.tr)*
*** *Department of Information Engineering, Research School of Information Sciences and Engineering, The Australian National University, Canberra ACT 0200, Australia*

Abstract: This paper focuses on the problem of robust H_∞ control for a class of switched nonlinear systems with neutral uncertainties via the multiple Lyapunov function approach. Uncertainties are allowed to appear in channels of state, control input and disturbance input. Conditions for the solvability of the robust H_∞ control problem and design of both switching law and controllers are presented. As an application, a hybrid state feedback strategy is proposed to solve the standard robust H_∞ control problem for nonlinear systems when no single continuous controller is effective.

Keywords: Switched systems; Robust H_∞ control; Multiple Lyapunov functions; Neutral uncertainties; Hybrid control.

1. INTRODUCTION

Due to theoretical significance and practical applications, the study of switched systems has attracted rapidly growing interest (Liberzon (2003); Persis et al. (2003); Cheng et al. (2005); Xie & Wang (2003); Zhao & Dimirovski (2004); Sun & Ge (2005)). Many systems encountered in practice exhibit switching between several subsystems depending on various environmental factors such as mechanical systems, the automotive industry, switching power converters and many other fields. A switched system can also be used to describe an overall system of a single process controlled by means of multi-controlller switching. Loosely speaking, a switched system consists of a family of continuous-time subsystems and a rule that specifies the switching among them. Regarding design of switched systems under some properly chosen switching law, the multiple Lyapunov function approach has been proven to be a powerful and effective tool regarding design of switched systems (Branicky (1998); El-Farra et al. (2005);).

On the other hand, the H_∞ control problem has been well understood and extensively explored for continuous and discrete systems. The remarkable achievements may be the algebraic Riccati inequalities for linear systems and Hamilton-Jacobi inequalities for nonlinear systems (Schaft (1996)). However, it has been rarely addressed for switched systems. This is mainly because more difficulties arise from the interaction between continuous variables and discrete switching signals. The H_∞ control problem was studied in Zhai et al. (2001) by using average dwell time approach incorporated with a piecewise Lyapunov function. Hespanha (2003) gave a method of computing the root-mean-square gains of switched linear systems. Other method such as LMI (Ji et al. (2006)) was also dedicated to the studies of the H_∞ control problem for switched linear systems. For the nonlinear case, however, results are relatively rare and mainly explored with special structures. The problem of the H_∞ control for switched nonlinear systems is addressed in Zhao & Hill (2004) and Zhao & Zhao (2006) via the multiple Lyapunov function approach.

Since uncertainties are unavoidable in practice, robust control is of great importance and has been extensively studied in the control field. However, switched systems with neutral uncertainties have not been investigated so far. This paper considers the problem of robust H_∞ control for a class of switched nonlinear systems with neutral uncertainties. On the basis of the multiple Lyapunov function technique, a sufficient condition for the switched nonlinear systems to be asymptotically stable with H_∞-norm bound is derived for all admissible uncertainties. Then, for a non-switched nonlinear system with neutral uncertainties, when a single continuous feedback control law can not solve the standard robust H_∞ control problem, the problem is solved by controller switching among finite candidate controllers. Finally, an example illustrates the effectiveness of the proposed approach. Compared with the existing results, this paper considers neutral uncertainties since practical parameter perturbations are often nonlinearly state and nonlinearly state derivative depen-

⋆ This work was supported by Dogus University Fund for Science and the NSF of P. R. China under the grant No. 60574013.

dent. Additionally, it also allows uncertainties to appear in channels of state, control input and disturbance input.

2. PROBLEM FORMULATION

Consider switched nonlinear systems described by the state-space model of the form

$$\begin{aligned}\dot{x}+\Delta j_\sigma(\dot{x},t)&=f_\sigma(x)+\Delta f_\sigma(x,t)+(g_\sigma(x)+\Delta g_\sigma(x,t))u_\sigma\\&\quad+(p_\sigma(x)+\Delta p_\sigma(x,t))\,\omega_\sigma,\\ z&=h_\sigma(x)+k_\sigma(x)u_\sigma,\end{aligned}\tag{1}$$

where $\sigma(t):\Re^+\to M=\{1,2,\dots,m\}$ is the right continuous piecewise constant switching signal to be designed, $x\in\Re^n$ is the state vector, $u_i\in\Re^{m_i}$ and $\omega_i\in\Re^{p_i}$ which belong to $L_2[0,\infty)$ denote the control input and disturbance input of the i-th subsystem respectively, $z\in\Re^{q_i}$ is the regulated output, $f_i(x)$, $g_i(x)$, $p_i(x)$, $h_i(x)$ and $k_i(x)$ are known smooth nonlinear function matrices of appropriate dimensions with $f_i(0)=0$ and $h_i(0)=0$, $\Delta j_i(\dot{x},t)$, $\Delta f_i(x,t)$, $\Delta g_i(x,t)$ and $\Delta p_i(x,t)$ denote unknown smooth nonlinear function matrices, $i\in M$. Additionally, we assume all uncertainties satisfy the following assumptions.

Assumption 1. The uncertain functions Δj_i, Δf_i, Δg_i, and Δp_i are gain bounded smooth functions described by

$$\begin{aligned}\Delta j_i(\dot{x},t)&=e_{j_i}\delta_{j_i}(\dot{x},t), && \|\delta_{j_i}\|\le\|W_{j_i}\dot{x}\|,\\ \Delta f_i(x,t)&=e_{f_i}\delta_{f_i}(x,t), && \|\delta_{f_i}\|\le\|W_{f_i}(x)\|,\\ \Delta g_i(x,t)&=e_{g_i}\delta_{g_i}(x,t), && \|\delta_{g_i}\|\le\|W_{g_i}(x)\|,\\ \Delta p_i(x,t)&=e_{p_i}\delta_{p_i}(x,t), && \|\delta_{p_i}\|\le\|W_{p_i}\|\end{aligned}$$

with known constant matrices e_{j_i}, e_{f_i}, e_{g_i}, e_{p_i} and unknown function vectors δ_{j_i}, δ_{f_i}, δ_{g_i}, δ_{p_i} satisfying $\delta_{j_i}(0,t)=0$ and $\delta_{f_i}(0,t)=0$. W_{j_i},W_{f_i},W_{g_i} are known smooth function matrices, W_{p_i} are given weighting matrices, $i\in M$.

For convenience, we adopt the following notations (Branicky (1998)) for switched system (1). Let

$$\Sigma=\{x_0;(i_0,t_0),(i_1,t_1),\cdots,(i_n,t_n),\cdots,|i_k\in M,k\in N\}$$

denote a switching sequence with the initial state x_0 and the initial time t_0, where (i_k,t_k) means that the i_k-th subsystem is active for $t_k\le t<t_{k+1}$.

Now, the robust H_∞ control problem for switched system (1) can be formulated as follows:
Given a constant $\gamma>0$, design a continuous state feedback controller $u_i(x)$ for each subsystem and a switching law $i=\sigma(t)$ such that
(a) The closed-loop system is asymptotically stable when $\omega_i=0$.
(b) System (1) has finite robust L_2-gain γ from ω_i to z for all admissible uncertainties, i.e., there holds

$$\int_0^T z^T(t)z(t)\,\mathrm{d}t\le\gamma^2\int_0^T\omega_i^T(t)\omega_i(t)\,\mathrm{d}t+\beta(x_0)$$

for all $T>0$ and all admissible uncertainties, where $\beta(\cdot)$ is some real-valued function.

Throughout this paper, $\Re^n$ denotes the n-dimensional Euclidean space, and for a matrix P, $P>0$ denotes that P is positive definite, the superscript "T" stands for matrix transpose, I is the identity matrix, $\|\cdot\|$ represents either the Euclidean vector norm or the induced matrix 2-norm, and $\bar{\sigma}(\cdot)$ denotes the largest singular value of a matrix.

3. MAIN RESULTS

In this section, we shall present a condition for the robust H_∞ control problem to be solvable, and design continuous controllers for subsystems and a switching law.

First, we consider the robust H_∞ control problem of the switched system

$$\begin{aligned}\dot{x}+\Delta j_\sigma(\dot{x},t)&=f_\sigma(x)+\Delta f_\sigma(x,t)+(p_\sigma(x)+\Delta p_\sigma(x,t))\omega_\sigma,\\ z&=h_\sigma(x).\end{aligned}\tag{2}$$

Theorem 1. Let a constant $\gamma>0$ be given. Suppose that
(1) $(f_i+\Delta f_i,\ h_i)$ is detectable.
(2) There exist nonnegative functions $\beta_{ij}(x)$ $(i,j\in M)$, positive constants λ_{j_i}, λ_{f_i}, λ_{p_i}, and radially unbounded, positive definite smooth functions $V_i(x)$, $V_i(x(0))=0$ $(i\in M)$ such that the following partial differential inequalities

$$\begin{aligned}&\frac{\partial V_i}{\partial x}f_i+\gamma_i^2C_i^TC_i+\gamma_i^2\left(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x}B_i+C_i^TD_i\right)R_i^{-1}\\&\cdot\left(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x}B_i+C_i^TD_i\right)^T+\sum_{j=1}^m\beta_{ij}(V_i-V_j)\le 0,\ i\in M\end{aligned}\tag{3}$$

hold, where

$$\gamma_i^2=\frac{\gamma^2}{1+\bar{\sigma}(W_{p_i})/\lambda_{p_i}^2},\ B_i=[\,p_i\ \ \lambda_{j_i}e_{j_i}\ \ \lambda_{f_i}e_{f_i}\ \ \lambda_{p_i}e_{p_i}\,],$$

$$C_i=\begin{bmatrix}(1/\gamma_i)h_i\\(1/\lambda_{j_i})W_{j_i}f_i\\(1/\lambda_{f_i})W_{f_i}\\0\end{bmatrix},\ D_i=\begin{bmatrix}0\\(1/\lambda_{j_i})W_{j_i}B_i\\0\\0\end{bmatrix},\ R_i=I-D_i^TD_i.$$

Then, the robust H_∞ control problem for (2) is solved under some switching law.

Proof. Obviously, for any $x\in R^n\backslash\{0\}$, there exists an $i\in M$ such that $V_i(x)-V_j(x)\ge 0$, $\forall j\in M$. Then, the switching law is taken as

$$\sigma(t)=\min_i\{i:i=\arg\max_{j\in M}V_j(x)\}.\tag{4}$$

Associated with the switching law (4) and nonnegative functions $\beta_{ij}(x)$, for any fixed $x\in\Re^n$, it follows that $\sum_{j=1}^m\beta_{ij}\,(V_i-V_j)\ge 0$ for some $i\in M$ and $\forall j\in M$. It can be easily obtained from (3) that

$$\begin{aligned}&\frac{\partial V_i}{\partial x}f_i+\gamma_i^2C_i^TC_i+\gamma_i^2\left(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x}B_i+C_i^TD_i\right)R_i^{-1}\\&\cdot\left(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x}B_i+C_i^TD_i\right)^T\le 0,\qquad i\in M.\end{aligned}\tag{5}$$

Consider neutral uncertainty $\Delta j_i(\dot{x},t)$ as an exogenous disturbance and define a new extended disturbance input including it. To this end, let

$$d_i^T=\left[\,\omega_i^T\ \ -(1/\lambda_{j_i})\delta_{j_i}^T\ \ (1/\lambda_{f_i})\delta_{f_i}^T\ \ (1/\lambda_{p_i})\omega_i^T\delta_{p_i}^T\,\right].$$

In view of

$$\begin{aligned}d_i^Td_i&=\|\omega_i\|^2+\frac{1}{\lambda_{j_i}^2}\delta_{j_i}^T\delta_{j_i}+\frac{1}{\lambda_{f_i}^2}\delta_{f_i}^T\delta_{f_i}+\frac{\bar{\sigma}(W_{p_i})}{\lambda_{p_i}^2}\|\omega_i\|^2\\&=\left(1+\frac{\bar{\sigma}(W_{p_i})}{\lambda_{p_i}^2}\right)\|\omega_i\|^2+\frac{1}{\lambda_{j_i}^2}\delta_{j_i}^T\delta_{j_i}+\frac{1}{\lambda_{f_i}^2}\delta_{f_i}^T\delta_{f_i},\end{aligned}$$

we obtain that

$$-\gamma^2\|\omega_i\|^2=-\gamma_i^2d_i^Td_i+\frac{\gamma_i^2}{\lambda_{j_i}^2}\delta_{j_i}^T\delta_{j_i}+\frac{\gamma_i^2}{\lambda_{f_i}^2}\delta_{f_i}^T\delta_{f_i}.\tag{6}$$

Considering Assumption 1, there holds

$$\begin{aligned}&\dot V_i(x(t))+\|z\|^2-\gamma^2\|\omega_i\|^2\\&=\frac{\partial V_i}{\partial x}(f_i+\Delta f_i+p_i\omega_i+\Delta p_i\omega_i-\Delta j_i)+\|z\|^2-\gamma^2\|\omega_i\|^2\\&=\frac{\partial V_i}{\partial x}(f_i+B_id_i)+h_i^Th_i-\gamma_i^2d_i^Td_i+\frac{\gamma_i^2}{\lambda_{j_i}^2}\delta_{j_i}^T\delta_{j_i}+\frac{\gamma_i^2}{\lambda_{f_i}^2}\delta_{f_i}^T\delta_{f_i}.\quad(7)\end{aligned}$$

Furthermore,

$$\begin{aligned}\frac{\gamma_i^2}{\lambda_{j_i}^2}\delta_{j_i}^T\delta_{j_i}\le&\frac{\gamma_i^2}{\lambda_{j_i}^2}(f_i+\Delta f_i+p_i\omega_i+\Delta p_i\omega_i-\Delta j_i)^TW_{j_i}^T\\&\cdot W_{j_i}(f_i+\Delta f_i+p_i\omega_i+\Delta p_i\omega_i-\Delta j_i)\\=&\frac{\gamma_i^2}{\lambda_{j_i}^2}f_i^TW_{j_i}^TW_{j_i}f_i+\frac{\gamma_i^2}{\lambda_{j_i}^2}d_i^TB_i^TW_{j_i}^TW_{j_i}B_id_i\\&+\frac{2\gamma_i^2}{\lambda_{j_i}^2}f_i^TW_{j_i}^TW_{j_i}B_id_i.\end{aligned}\qquad(8)$$

Substituting (8) into (7) and considering (5), then, by completing the squares, we have

$$\begin{aligned}&\dot V_i(x(t))+\|z\|^2-\gamma^2\|\omega_i\|^2\\&=\frac{\partial V_i}{\partial x}(f_i+B_id_i)+h_i^Th_i-\gamma_i^2d_i^Td_i+\frac{\gamma_i^2}{\lambda_{j_i}^2}f_i^TW_{j_i}^TW_{j_i}f_i\\&\quad+\frac{\gamma_i^2}{\lambda_{j_i}^2}d_i^TB_i^TW_{j_i}^TW_{j_i}B_id_i+\frac{2\gamma_i^2}{\lambda_{j_i}^2}f_i^TW_{j_i}^TW_{j_i}B_id_i+\frac{\gamma_i^2}{\lambda_{f_i}^2}\delta_{f_i}^T\delta_{f_i}\\&=\frac{\partial V_i}{\partial x}(f_i+B_id_i)+\gamma_i^2C_i^TC_i-\gamma_i^2d_i^TR_id_i+2\gamma_i^2C_i^TD_id_i\\&=\frac{\partial V_i}{\partial x}f_i+\gamma_i^2C_i^TC_i\\&\quad-\gamma_i^2\left\|R_i^{\frac12}d_i-R_i^{-\frac12}\left(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x}B_i+C_i^TD_i\right)^T\right\|^2\\&\quad+\gamma_i^2\left(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x}B_i+C_i^TD_i\right)R_i^{-1}\left(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x}B_i+C_i^TD_i\right)^T\\&\le\frac{\partial V_i}{\partial x}f_i+\gamma_i^2C_i^TC_i\\&\quad+\gamma_i^2\left(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x}B_i+C_i^TD_i\right)R_i^{-1}\left(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x}B_i+C_i^TD_i\right)^T\\&\le 0.\end{aligned}$$

Therefore, we can obtain that

$$\dot V_i(x(t))+\|z\|^2-\gamma^2\|\omega_i\|^2\le 0.\qquad(9)$$

Now, we introduce

$$J_T=\int_0^T\left(z^Tz-\gamma^2\omega_i^T\omega_i\right)\,\mathrm{d}t.$$

According to (9) and the switching sequence Σ, suppose $t_0=0$, $x(t_0)=x(0)$, when $T\in[t_k,t_{k+1})$, for any admissible uncertainties, we have

$$\begin{aligned}J_T=&\sum_{j=0}^{k-1}\int_{t_j}^{t_{j+1}}\left(z^Tz-\gamma^2\omega_{i_j}^T\omega_{i_j}+\dot V_{i_j}(x(t))\right)\,\mathrm{d}t\\&-\sum_{j=0}^{k-1}\left(V_{i_j}(x(t_{j+1}))-V_{i_j}(x(t_j))\right)\\&+\int_{t_k}^{T}\left(z^Tz-\gamma^2\omega_{i_k}^T\omega_{i_k}+\dot V_{i_k}(x(t))\right)\,\mathrm{d}t\\&-(V_{i_k}(x(t_T))-V_{i_k}(x(t_k)))\\\le&-\sum_{j=0}^{k-1}\left(V_{i_j}(x(t_{j+1}))-V_{i_j}(x(t_j))\right)\\&-(V_{i_k}(x(t_T))-V_{i_k}(x(t_k)))\\=&V_{i_0}(x(0))-V_{i_k}(x(t_T))\\&+\sum_{j=0}^{k-1}\left(V_{i_{j+1}}(x(t_{j+1}))-V_{i_j}(x(t_{j+1}))\right)\end{aligned}\qquad(10)$$

Since $V_{\sigma(t_{k-1})}(x(t_k))=V_{\sigma(t_k)}(x(t_k))$, (10) leads to

$$\begin{aligned}J_T\le&V_{i_0}(x(0))-V_{i_k}(x(t_T))\\&+\sum_{j=0}^{k-1}\left(V_{i_{j+1}}(x(t_{j+1}))-V_{i_j}(x(t_{j+1}))\right)\\\le&V_{i_0}(x(0))-V_{i_k}(x(t_T))\\\le&V_{i_0}(x(0))\end{aligned}$$

Let $\beta(x(0))=\max_{i_0\in M}\{V_{i_0}(x(0))\}$. Therefore, we conclude that

$$\int_0^Tz^T(t)z(t)\,\mathrm{d}t\le\gamma^2\int_0^T\omega_i^T(t)\omega_i(t)\,\mathrm{d}t+\beta(x(0))$$

holds for all admissible uncertainties and disturbance input ω_i, which means switched system (2) has finite L_2-gain.

When $\omega_i=0$, it follows from (9) that $\dot V_i(x(t))\le\|z\|^2+\dot V_i(x(t))\le 0$. The detectability of $(f_i+\Delta f_i,h_i)$ gives asymptotical stability of the switched system (2) by LaSalle's invariance principle. This completes the proof.

Next, we consider the robust H_∞ control problem of the switched system (1). We shall derive such a state feedback control law that the closed-loop system has robust L_2-gain performance.

Theorem 2. Let a constant $\gamma>0$ be given. Suppose that
(1) $(f_i+\Delta f_i,\ h_i)$ is detectable.
(2) There exist nonnegative functions $\beta_{ij}(x)$ $(i,j\in M)$, positive constants λ_{j_i}, λ_{f_i}, λ_{p_i}, λ_{g_i} and radially unbounded, positive definite smooth functions $V_i(x)$, $V_i(x(0))=0$ $(i\in M)$ such that the following partial differential inequalities

$$\begin{aligned}&\frac{\partial V_i}{\partial x}f_i+\gamma_i^2\tilde C_i^T\tilde C_i+\gamma_i^2\left(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x}\tilde B_i+\tilde C_i^T\tilde D_{1i}\right)\tilde R_i^{-1}\\&\cdot\left(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x}\tilde B_i+\tilde C_i^T\tilde D_{1i}\right)^T-\left(\frac{\partial V_i}{\partial x}\tilde E_i+\tilde C_i^TS_i\tilde D_{2i}\right)\tilde S_i^{-1}\\&\cdot\left(\frac{\partial V_i}{\partial x}\tilde E_i+\tilde C_i^TS_i\tilde D_{2i}\right)^T+\sum_{j=1}^m\beta_{ij}(V_i-V_j)\le 0,\ i\in M.\end{aligned}\qquad(11)$$

hold, where $\tilde B_i=[p_i\ \ \lambda_{j_i}e_{j_i}\ \ \lambda_{f_i}e_{f_i}-\lambda_{g_i}e_{g_i}\ \ \lambda_{g_i}e_{g_i}\ \ \lambda_{p_i}e_{p_i}]$,

$$\tilde C_i=\begin{bmatrix}(1/\gamma_i)h_i\\(1/\lambda_{j_i})W_{j_i}f_i\\(1/\lambda_{f_i})W_{f_i}\\(1/\lambda_{f_i})W_{f_i}\\0\end{bmatrix},\ \tilde D_{1i}=\begin{bmatrix}0\\(1/\lambda_{j_i})W_{j_i}\tilde B_i\\0\\0\\0\end{bmatrix},\ \tilde D_{2i}=\begin{bmatrix}(1/\gamma_i)k_i\\0\\0\\(1/\lambda_{g_i})W_{g_i}\\0\end{bmatrix},$$

$$\tilde{R}_i = I - \tilde{D}_{1i}^T \tilde{D}_{1i},\ S_i = \gamma_i^2 \left(\tilde{D}_{1i} \tilde{R}_i^{-1} \tilde{D}_{1i}^T + I\right),\ \tilde{S}_i = \tilde{D}_{2i}^T S_i \tilde{D}_{2i},$$

$$\tilde{E}_i = \frac{1}{2}\left(g_i + \tilde{B}_i \tilde{R}_i^{-1} \tilde{D}_{1i}^T \tilde{D}_{2i}\right),\ \gamma_i^2 = \frac{\gamma^2}{1+\bar{\sigma}(W_{p_i})/\lambda_{p_i}^2}.$$

Then, the hybrid state feedback controllers

$$u_i = u_i(x) = -\hat{S}_i^{-1}\left(\frac{\partial V_i}{\partial x}\hat{B}_i + \tilde{C}_i^T S_i \tilde{D}_{2i}\right)^T \tag{12}$$

solve robust H_∞ control problem under switching law (4).

Proof. The closed-loop system of (1) with state feedback $u_i(x)$ is given by

$$\dot{x} + \Delta j_i(\dot{x}, t) = f_{k_i}(x) + \Delta f_{k_i}(x,t) + (p_i(x) + \Delta p_i(x,t))\omega_i,$$
$$z = h_{k_i}(x), \tag{13}$$

where $f_{k_i}(x) = f_i(x) + g_i(x)u_i(x)$, $\Delta f_{k_i}(x,t) = \Delta f_i(x,t) + \Delta g_i(x,t)u_i(x)$, $h_{k_i}(x) = h_i(x) + k_i(x)u_i(x)$, and function Δf_{k_i} are defined by

$$\Delta f_{k_i}(x,t) = e_{k_i}\delta_{k_i}, \quad \|\delta_{k_i}\| \le \|W_{k_i}(x)\|, \quad i \in M$$

with $e_{k_i} = [e_{f_i} - \mu_i e_{g_i}\ \ \mu_i e_{g_i}]$, $\delta_{k_i}^T = \left[\delta_{f_i}^T\ \ \delta_{f_i}^T + (1/\mu_i)u_i^T \delta_{g_i}^T\right]$, and $W_{k_i}^T = \left[W_{f_i}^T\ \ W_{f_i}^T + (1/\mu_i)u_i^T W_{g_i}^T\right]$ where $\mu_i\ (i \in M)$ are positive constants. Hence, from Theorem 1, the robust H_∞ control problem of (13) is solved under switching law (4), if there exist nonnegative functions $\beta_{ij}(x)\ (i,j \in M)$, positive constants λ_{j_i}, λ_{f_i}, λ_{p_i} such that the following partial differential inequalities

$$\frac{\partial V_i}{\partial x} f_{k_i} + \gamma_i^2 C_{k_i}^T C_{k_i} + \gamma_i^2 \left(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x} B_{k_i} + C_{k_i}^T D_{k_i}\right) R_{k_i}^{-1}$$
$$\cdot \left(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x} B_{k_i} + C_{k_i}^T D_{k_i}\right)^T + \sum_{j=1}^m \beta_{ij}(V_i - V_j) \le 0,\ i \in M. \tag{14}$$

have radially unbounded, positive definite solutions $V_i(x)$, $V_i(x(0)) = 0\ (i \in M)$, where

$$R_{k_i} = I - D_{k_i}^T D_{k_i},\ B_{k_i} = [p_i\ \ \lambda_{j_i} e_{j_i}\ \ \lambda_{f_i} e_{k_i}\ \ \lambda_{p_i} e_{p_i}],$$

$$C_{k_i} = \begin{bmatrix} (1/\gamma_i) h_{k_i} \\ (1/\lambda_{j_i}) W_{j_i} f_i \\ (1/\lambda_{f_i}) W_{k_i} \\ 0 \end{bmatrix},\ D_{k_i} = \begin{bmatrix} 0 \\ (1/\lambda_{j_i}) W_{j_i} B_{k_i} \\ 0 \\ 0 \end{bmatrix}.$$

Let $\lambda_{g_i} = \mu_i \lambda_{f_i}$. Then, it can be shown that (14) is equivalent to (11). In fact, it follows from the switching law (4) and (12) that

$$\frac{\partial V_i}{\partial x} f_{k_i} + \gamma_i^2 C_{k_i}^T C_{k_i} + \gamma_i^2 \left(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x} B_{k_i} + C_{k_i}^T D_{k_i}\right) R_{k_i}^{-1}$$
$$\cdot \left(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x} B_{k_i} + C_{k_i}^T D_{k_i}\right)^T + \sum_{j=1}^m \beta_{ij}(V_i - V_j)$$
$$= \frac{\partial V_i}{\partial x} f_i + \frac{\partial V_i}{\partial x} g_i u_i + \gamma_i^2 \tilde{C}_i^T \tilde{C}_i + 2\gamma_i^2 \tilde{C}_i^T \tilde{D}_{2i} u_i + \gamma_i^2 u_i \tilde{D}_{2i}^T \tilde{D}_{2i} u_i$$
$$+ \gamma_i^2 \left(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x}\tilde{B}_i + \tilde{C}_i^T \tilde{D}_{1i}\right) \tilde{R}_i^{-1} \left(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x}\tilde{B}_i + \tilde{C}_i^T \tilde{D}_{1i}\right)^T$$
$$+ 2\gamma_i^2 \left(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x}\tilde{B}_i + \tilde{C}_i^T \tilde{D}_{1i}\right) \tilde{R}_i^{-1} \tilde{D}_{1i}^T \tilde{D}_{2i} u_i$$
$$+ \gamma_i^2 u_i^T \tilde{D}_{2i}^T \tilde{D}_{1i} \tilde{R}_i^{-1} \tilde{D}_{1i}^T \tilde{D}_{2i} u_i + \sum_{j=1}^m \beta_{ij}(V_i - V_j)$$
$$= \frac{\partial V_i}{\partial x} f_i + \gamma_i^2 \tilde{C}_i^T \tilde{C}_i + u_i^T \tilde{S}_i u_i + 2\left(\frac{\partial V_i}{\partial x}\tilde{E}_i + \tilde{C}_i^T S_i \tilde{D}_{2i}\right) u_i$$
$$+ \gamma_i^2 \left(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x}\tilde{B}_i + \tilde{C}_i^T \tilde{D}_{1i}\right) \tilde{R}_i^{-1} \left(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x}\tilde{B}_i + \tilde{C}_i^T \tilde{D}_{1i}\right)^T$$
$$+ \sum_{j=1}^m \beta_{ij}(V_i - V_j)$$
$$= \frac{\partial V_i}{\partial x} f_i + \gamma_i^2 \tilde{C}_i^T \tilde{C}_i + \gamma_i^2 \left(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x}\tilde{B}_i + \tilde{C}_i^T \tilde{D}_{1i}\right) \tilde{R}_i^{-1}$$
$$\cdot \left(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x}\tilde{B}_i + \tilde{C}_i^T \tilde{D}_{1i}\right)^T - \left(\frac{\partial V_i}{\partial x}\tilde{E}_i + \tilde{C}_i^T S_i \tilde{D}_{2i}\right) \tilde{S}_i^{-1}$$
$$\cdot \left(\frac{\partial V_i}{\partial x}\tilde{E}_i + \tilde{C}_i^T S_i \tilde{D}_{2i}\right)^T + \sum_{j=1}^m \beta_{ij}(V_i - V_j).$$

Finally, using the same arguments as in the proof of Theorem 1, the desired result follows.

Remark 1. When $M = \{1\}$, the switched system (1) degenerates into a regular nonlinear system and the robust H_∞ control problem becomes the standard robust H_∞ control problem for nonlinear systems. Additionally, if $f(x) = Ax$, $g(x) = B_2$, $p(x) = B_1$, $h(x) = Cx$, and $k(x) = D$, this result is equivalent to the condition given by Shen et al. (1996).

Remark 2. For the switched linear system

$$[I + E_{j_i}\Sigma_{j_i}(t)F_{j_i}]\dot{x} = [A_i + E_{a_i}\Sigma_{a_i}(t)F_{a_i}]x$$
$$+ [B_i + E_{b_i}\Sigma_{b_i}(t)F_{b_i}]u_i$$
$$+ [H_i + E_{h_i}\Sigma_{h_i}(t)F_{h_i}]\omega_i,$$
$$z = C_i x + D_i u_i, \tag{15}$$

with state feedback $u_i = K_i x$, where uncertain matrices satisfy $\Sigma_\epsilon^T(t)\Sigma_\epsilon(t) \le I$, $\epsilon \in \{j_i, a_i, b_i, h_i, i \in M\}$. Let $\delta_{j_i} = \Sigma_{j_i}(t)F_{j_i}\dot{x}$, $\delta_{f_i} = \Sigma_{a_i}(t)F_{a_i}x$, $\delta_{g_i} = \Sigma_{b_i}(t)F_{b_i}$, $\delta_{p_i} = \Sigma_{h_i}(t)F_{h_i}$, then it is clear that $\delta_\epsilon, \epsilon \in \{j_i, a_i, b_i, h_i, i \in M\}$ satisfy Assumption 1 with $W_{j_i} = F_{j_i}$, $W_{f_i} = F_{a_i}$, $W_{g_i} = F_{b_i}$, $W_{p_i} = F_{h_i}$. (11) turns out to be matrix inequalities

$$P_i A_i + A_i^T P_i + \gamma_i^2 \tilde{C}_i^T \tilde{C}_i + \gamma_i^2 \left(\frac{1}{\gamma_i^2} P_i \tilde{B}_i + \tilde{C}_i^T \tilde{D}_{1i}\right) \tilde{R}_i^{-1}$$
$$\cdot \left(\frac{1}{\gamma_i^2} P_i \tilde{B}_i + \tilde{C}_i^T \tilde{D}_{1i}\right)^T - \left(2P_i \tilde{E}_i + \tilde{C}_i^T S_i \tilde{D}_{2i}\right) \tilde{S}_i^{-1}$$
$$\cdot \left(2P_i \tilde{E}_i + \tilde{C}_i^T S_i \tilde{D}_{2i}\right)^T + \sum_{j=1}^m \beta_{ij}(P_i - P_j) < 0,\ i \in M. \tag{16}$$

where $\tilde{B}_i = [H_i\ \ \lambda_{j_i} E_{j_i}\ \ \lambda_{f_i} E_{a_i} - \lambda_{g_i} E_{b_i}\ \ \lambda_{g_i} E_{b_i}\ \ \lambda_{p_i} E_{h_i}]$,

$$\tilde{C}_i = \begin{bmatrix} (1/\gamma_i) C_i \\ (1/\lambda_{j_i}) F_{j_i} A_i \\ (1/\lambda_{f_i}) F_{a_i} \\ (1/\lambda_{f_i}) F_{a_i} \\ 0 \end{bmatrix},\ \tilde{D}_{1i} = \begin{bmatrix} 0 \\ (1/\lambda_{j_i}) F_{j_i} \tilde{B}_i \\ 0 \\ 0 \\ 0 \end{bmatrix},\ \tilde{D}_{2i} = \begin{bmatrix} (1/\gamma_i) D_i \\ 0 \\ 0 \\ (1/\lambda_{g_i}) F_{b_i} \\ 0 \end{bmatrix},$$

$$\tilde{R}_i = I - \tilde{D}_{1i}^T \tilde{D}_{1i},\ S_i = \gamma_i^2 \left(\tilde{D}_{1i} \tilde{R}_i^{-1} \tilde{D}_{1i}^T + I\right),\ \tilde{S}_i = \tilde{D}_{2i}^T S_i \tilde{D}_{2i},$$

$$\tilde{E}_i = \frac{1}{2}\left(B_i + \tilde{B}_i \tilde{R}_i^{-1} \tilde{D}_{1i}^T \tilde{D}_{2i}\right),\ \gamma_i^2 = \frac{\gamma^2}{1+\bar{\sigma}(F_{h_i})/\lambda_{p_i}^2}.$$

Then, the hybrid state feedback controllers are

$$u_i = K_i x = -\tilde{S}_i^{-1}\left(2P_i\tilde{E}_i+\tilde{C}_i^T S_i\tilde{D}_{2i}\right)^T x.$$

Remark 3. For the switched system (15), suppose that $D_i^T[C_i\ D_i] = [0\ I]$. If uncertain function $E_{j_i}\Sigma_{j_i}(t)F_{j_i}=0$, and $E_{h_i}\Sigma_{h_i}(t)F_{h_i}=0$, we can choose $E_{a_i}=E_{b_i}=E_i$, $E_{j_i}=0$, $F_{j_i}=0$, $E_{h_i}=0$, and $F_{h_i}=0$, then (16) becomes

$$\begin{aligned}&P_iA_i+A_i^TP_i+C_i^TC_i+\frac{2\gamma_i^2}{\lambda_{f_i}^2}F_{a_i}^TF_{a_i}+\frac{1}{\gamma_i^2}P_iH_iH_i^TP_i\\&+\frac{\lambda_{f_i}^2}{\gamma_i^2}P_iE_iE_i^TP_i-\left(P_iB_i+\frac{\gamma_i^2}{\lambda_{f_i}\lambda_{g_i}}F_{a_i}^TF_{b_i}\right)\left(I+\frac{\gamma_i^2}{\lambda_{g_i}^2}F_{b_i}^TF_{b_i}\right)^{-1}\\&\cdot\left(P_iB_i+\frac{\gamma_i^2}{\lambda_{f_i}\lambda_{g_i}}F_{a_i}^TF_{b_i}\right)^T+\sum_{j=1}^m\beta_{ij}(P_i-P_j)<0,\quad i\in M.\end{aligned}$$

In fact, the same result for switched linear system has been shown by Ji et al. (2006).

Next, we consider how to apply the obtained results to non-switched nonlinear systems by controller switching. For a nonlinear system, a continuous robust H_∞ controller may not exist or may be sometimes too complex to implement. Thus, in some control problems, control actions are decided by switching between finite candidate controllers. Subsequently, we try to use hybrid state feedback strategy to solve the robust H_∞ control problem for uncertain nonlinear systems.

Consider the following nonlinear system

$$\begin{aligned}\dot{x}+\Delta j(\dot{x},t)&=f(x)+\Delta f(x,t)+(g(x)+\Delta g(x,t))u\\&\quad+(p(x)+\Delta p(x,t))\,\omega,\\z&=h(x)+k(x)u,\end{aligned}\tag{17}$$

where $x\in\Re^n$ is the state vector, $u\in\Re^m$ and $\omega\in\Re^p$ denote the control input and disturbance input respectively, $z\in\Re^q$ is the regulated output, $f(x)$, $g(x)$, $p(x)$, $h(x)$ and $k(x)$ are known smooth nonlinear vector functions of appropriate dimensions with $f(0)=0$ and $h(0)=0$, $\Delta j(\dot{x},t)$, $\Delta f(x,t)$, $\Delta g(x,t)$ and $\Delta p(x,t)$ denote unknown smooth nonlinear vector functions. Additionally, we assume all uncertainties satisfy the following assumptions.

Assumption 2. The uncertain functions Δj, Δf, Δg and Δp are gain bounded smooth functions described by

$$\begin{aligned}\Delta j(\dot{x},t)&=e_j\delta_j(\dot{x},t), &\|\delta_j\|\le\|W_j\dot{x}\|,\\\Delta f(x,t)&=e_f\delta_f(x,t), &\|\delta_f\|\le\|W_f(x)\|,\\\Delta g(x,t)&=e_g\delta_g(x,t), &\|\delta_g\|\le\|W_g(x)\|,\\\Delta p(x,t)&=e_p\delta_p(x,t), &\|\delta_p\|\le\|W_p\|\end{aligned}$$

with known constant matrices e_j, e_f, e_g, e_p and unknown function vectors δ_j, δ_f, δ_g, δ_p satisfying $\delta_j(0,t)=0$ and $\delta_f(0,t)=0$. W_j, W_f, W_g are known smooth function vectors, W_p is given weighting matrix.

For system (17), suppose that there exists the following class of finite candidate state feedback controllers

$$u_i=u_i(x)=-\tilde{S}^{-1}\left(\frac{\partial V_i}{\partial x}\tilde{E}+\tilde{C}^TS\tilde{D}_2\right)^T,\tag{18}$$

where $V_i(x)$ will be specified later, the control law u is generated by switching among them.

Theorem 3. Let a constant $\gamma>0$ be given. Suppose that
(1) $(f+\Delta f,\ h)$ is detectable.
(2) There exist nonnegative functions $\beta_{ij}(x)$ $(i,j\in M)$, positive constants λ_j, λ_f, λ_p, λ_g and radially unbounded, positive definite smooth functions $V_i(x)$, $V_i(x(0))=0$ $(i\in M)$ such that the following partial differential inequalities

$$\begin{aligned}&\frac{\partial V_i}{\partial x}f+\gamma_1^2\tilde{C}^T\tilde{C}+\gamma_1^2\left(\frac{1}{2\gamma_1^2}\frac{\partial V_i}{\partial x}\tilde{B}+\tilde{C}^T\tilde{D}_1\right)\tilde{R}^{-1}\\&\cdot\left(\frac{1}{2\gamma_1^2}\frac{\partial V_i}{\partial x}\tilde{E}+\tilde{C}^T\tilde{D}_1\right)^T-\left(\frac{\partial V_i}{\partial x}\tilde{E}+\tilde{C}^TS\tilde{D}_2\right)\tilde{S}^{-1}\\&\cdot\left(\frac{\partial V_i}{\partial x}\tilde{E}+\tilde{C}^TS\tilde{D}_2\right)^T+\sum_{j=1}^m\beta_{ij}(V_i-V_j)\le0,\ i\in M.\end{aligned}\tag{19}$$

hold, where $\tilde{B}=[p\ \ \lambda_je_j\ \ \lambda_fe_f-\lambda_ge_g\ \ \lambda_ge_g\ \ \lambda_pe_p]$,

$$\tilde{C}=\begin{bmatrix}(1/\gamma_1)h\\(1/\lambda_j)W_jf\\(1/\lambda_f)W_f\\(1/\lambda_f)W_f\\0\end{bmatrix},\ \tilde{D}_1=\begin{bmatrix}0\\(1/\lambda_j)W_j\tilde{B}\\0\\0\\0\end{bmatrix},\ \tilde{D}_2=\begin{bmatrix}(1/\gamma_1)k\\0\\0\\(1/\lambda_g)W_g\\0\end{bmatrix},$$

$$\tilde{R}=I-\tilde{D}_1^T\tilde{D}_1,\ S=\gamma_1^2\left(\tilde{D}_1\tilde{R}^{-1}\tilde{D}_1^T+I\right),\ \tilde{S}=\tilde{D}_2^TS\tilde{D}_2,$$

$$\tilde{E}=\frac{1}{2}\left(g+\tilde{B}\tilde{R}^{-1}\tilde{D}_1^T\tilde{D}_2\right),\ \gamma_1^2=\frac{\gamma^2}{1+\bar{\sigma}(W_p)/\lambda_p^2}.$$

Then, the hybrid controllers (18) with the switching law (4) solve the robust H_∞ control problem for (17).

Proof. Substituting the designed controllers (18) into the system (17) results in a switched nonlinear system. Then, applying Theorem 2 yields the result.

4. EXAMPLE

In this section, we give an example to demonstrate the effectiveness of the proposed design method. Consider the following switched nonlinear system

$$\begin{aligned}\dot{x}+\Delta j_i(\dot{x},t)&=f_i(x)+\Delta f_i(x,t)+(g_i(x)+\Delta g_i(x,t))u_i\\&\quad+(p_i(x)+\Delta p_i(x,t))\omega_i,\\z&=h_i(x)+k_i(x)u_i,\quad i=1,2,\end{aligned}\tag{20}$$

where

$$\begin{aligned}&f_1(x)=-2x^3,\ g_1(x)=x^2,\ p_1(x)=-1,\ h_1(x)=x^3,\ k_1(x)=1,\\&f_2(x)=-2x,\ g_2(x)=2x,\ p_2(x)=1,\ h_2(x)=-x,\ k_2(x)=1,\\&\Delta j_1(\dot{x},t)=a_1\dot{x}\sin t,\ e_{j_1}=1,\ \delta_{j_1}(\dot{x},t)=a_1\dot{x}\sin t,\ W_{j_1}=1,\\&\Delta j_2(\dot{x},t)=a_2\dot{x}\cos t,\ e_{j_2}=1,\ \delta_{j_2}(\dot{x},t)=a_2\dot{x}\cos t,\ W_{j_2}=1,\\&\Delta f_1(x,t)=b_1x\cos t, e_{f_1}=1, \delta_{f_1}(x,t)=b_1x\cos t, W_{f_1}=x,\\&\Delta f_2(x,t)=b_2x\sin t, e_{f_2}=1, \delta_{f_2}(x,t)=b_2x\sin t, W_{f_2}=x,\\&\Delta g_1(x,t)=c_1e^{-t}\cos x, e_{g_1}=1, \delta_{g_1}(x,t)=c_1e^{-t}\cos x, W_{g_1}=1,\\&\Delta g_2(x,t)=c_2e^{-t}\sin x, e_{g_2}=1, \delta_{g_2}(x,t)=c_2e^{-t}\sin x, W_{g_2}=1,\\&\Delta p_1(x,t)=d_1e^{-t},\ e_{p_1}=1,\ \delta_{p_1}(x,t)=d_1e^{-t},\ W_{p_1}=1,\\&\Delta p_2(x,t)=d_2e^{-t},\ e_{p_2}=1,\ \delta_{p_2}(x,t)=d_2e^{-t},\ W_{p_2}=1,\end{aligned}$$

and a_i, b_i, c_i, d_i are unknown constants belonging to $[0,1]$.

Let $\gamma^2 = 2$, $\lambda_{j_i} = \lambda_{f_i} = \lambda_{g_i} = \lambda_{p_i} = 1$, $\beta_1(x) = \frac{1}{2}x^2$, $\beta_2(x) = 2x^2+1$, then following Theorem 2, we have $\gamma_1 = \gamma_2 = 1$,

$\tilde{B}_1 = [-1\ 1\ 0\ 1\ 1]$, $\tilde{B}_2 = [1\ 1\ 0\ 1\ 1]$, $\tilde{C}_1^T = \left[x^3\ -2x^3\ x\ x\ 0\right]$,

$\tilde{C}_2^T = [-x\ -2x\ x\ x\ 0]$, $\tilde{D}_{11}^T = \left[0\ \tilde{B}_1^T\ 0\ 0\ 0\right]$,

$\tilde{D}_{12}^T = \left[0\ \tilde{B}_2^T\ 0\ 0\ 0\right]$, $\tilde{D}_{21}^T = [1\ 0\ 0\ 1\ 0]$, $\tilde{D}_{22}^T = [1\ 0\ 0\ 1\ 0]$,

$\tilde{R}_i = I - \tilde{D}_{1i}^T \tilde{D}_{1i}$, $S_i = \tilde{D}_{1i} \tilde{R}_i^{-1} \tilde{D}_{1i}^T + I$, $\tilde{S}_i = \tilde{D}_{2i}^T S_i \tilde{D}_{2i}$, $\tilde{E}_i = \frac{1}{2}\left(g_i + \tilde{B}_i \tilde{R}_i^{-1} \tilde{D}_{1i}^T \tilde{D}_{2i}\right)$, $i = 1, 2$.

We choose

$$V_1(x) = 2x^2, \quad V_2(x) = x^4, \quad x \in R^n.$$

Both of them are globally positive definite and $V_1(0) = V_2(0) = 0$. Then

$$\begin{aligned}
&\frac{\partial V_1}{\partial x} f_1 + \gamma_1^2 \tilde{C}_1^T \tilde{C}_1 + \gamma_1^2 \left(\frac{1}{2\gamma_1^2} \frac{\partial V_1}{\partial x} \tilde{B}_1 + \tilde{C}_1^T \tilde{D}_{11}\right) \tilde{R}_1^{-1} \\
&\quad \cdot \left(\frac{1}{2\gamma_1^2} \frac{\partial V_1}{\partial x} \tilde{B}_1 + \tilde{C}_1^T \tilde{D}_{11}\right)^T - \left(\frac{\partial V_1}{\partial x} \tilde{E}_1 + \tilde{C}_1^T S_1 \tilde{D}_{21}\right) \tilde{S}_1^{-1} \\
&\quad \cdot \left(\frac{\partial V_1}{\partial x} \tilde{E}_1 + \tilde{C}_1^T S_1 \tilde{D}_{21}\right)^T + \beta_1 (V_1 - V_2) \\
&= -\frac{5}{6}x^6 - \frac{5}{6}x^4 - \frac{23}{6}x^2 \\
&\leq 0
\end{aligned}$$

and

$$\begin{aligned}
&\frac{\partial V_2}{\partial x} f_2 + \gamma_2^2 \tilde{C}_2^T \tilde{C}_2 + \gamma_2^2 \left(\frac{1}{2\gamma_2^2} \frac{\partial V_2}{\partial x} \tilde{B}_2 + \tilde{C}_2^T \tilde{D}_{12}\right) \tilde{R}_2^{-1} \\
&\quad \cdot \left(\frac{1}{2\gamma_2^2} \frac{\partial V_2}{\partial x} \tilde{B}_2 + \tilde{C}_2^T \tilde{D}_{12}\right)^T - \left(\frac{\partial V_2}{\partial x} \tilde{E}_2 + \tilde{C}_2^T S_2 \tilde{D}_{22}\right) \tilde{S}_2^{-1} \\
&\quad \cdot \left(\frac{\partial V_2}{\partial x} \tilde{E}_2 + \tilde{C}_2^T S_2 \tilde{D}_{22}\right)^T + \beta_2 (V_2 - V_1) \\
&= -8x^8 - \frac{10}{3}x^6 - \frac{1}{3}x^4 - \frac{1}{3}x^2 \\
&\leq 0
\end{aligned}$$

The switching law

$$\sigma(t) = \begin{cases} 1 & \text{if} \quad -\sqrt{2} \leq x \leq \sqrt{2}, \\ 2 & \text{otherwise.} \end{cases}$$

and the hybrid controllers

$$u_1 = -\tilde{S}_1^{-1}\left(\tilde{E}_1^T \frac{\partial^T V_1}{\partial x} + \tilde{D}_{21}^T S_1 \tilde{C}_1\right) = -\frac{3}{2}x^3 - \frac{1}{2}x,$$

$$u_2 = -\tilde{S}_2^{-1}\left(\tilde{E}_2^T \frac{\partial^T V_2}{\partial x} + \tilde{D}_{22}^T S_2 \tilde{C}_2\right) = -2x^4.$$

solve the robust H_∞ control problem.

5. CONCLUSION

This paper has discussed the robust H_∞ control problem for a class of uncertain switched nonlinear systems. Uncertainties are considered to be nonlinearly dependent on state and state derivative and allowed to appear in channels of state, control input and disturbance input. A sufficient condition has been derived by designing a switching law and hybrid state feedback controllers via the multiple Lyapunov function approach. Moreover, a hybrid state feedback strategy is proposed to solve the robust H_∞ control problem for uncertain nonlinear systems.

REFERENCES

A.van der Schaft. *L_2-gain and passivity techniques in nonlinear control*. Springer-Verlag, 1996.

C.D. Persis, R.D. Santis, and A.S. Morse. Switched nonlinear systems with state-dependent dwell-time. *Systems and Control Letters*, 50(4):291–302, 2003.

D. Liberzon. *Switching in systems and control*, Birkhauser, Boston, 2003.

D.Z. Cheng, L. Guo, Y.D. Lin, and Y. Wang. Stabilization of switched linear systems. *IEEE Transactions on Automatic Control*, 50(5):661–666, 2005.

G.M. Xie, L. Wang. Controllability and stabilizability of switched linear systems. *Systems and Control Letters*, 48(2):135–155, 2003.

G.S. Zhai, B. Hu, K. Yasuda, and A.N. Michel. Disturbance attenuation properties of time-controlled switched systems. *Journal of the Franklin Institute*, 338(7):765–779, 2001.

J.P. Hespanha. Root-mean-square gains of switched linear systems. *IEEE Transactions on Automatic Control*, 48(11):2040-2045, 2003.

J. Zhao, D.J. Hill. Hybrid H_∞ control based on multiple Lyapunov functions. *Proceedings of the 6th IFAC Symposium on Nonlinear Control Systems*, 2:567–572, 2004.

J. Zhao, G.M. Dimirovski. Quadratic stability of a class of switched nonlinear systems. *IEEE Transactions on Automatic Control*, 49(4):574–578, 2004.

M.S. Branicky. Multiple Lyapunov functions and other analysis tools for switched and hybrid systems. *IEEE Transactions on Automatic Control*, 43(4):475–482, 1998.

N.H. El-Farra, P. Mhaskar, and P.D. Christofides. Output feedback control of switched nonlinear systems using multiple Lyapunov functions. *Systems and Control Letters*, 54(12):1163–1182, 2005.

S.Z. Zhao, J. Zhao. Robust H_∞ control for uncertain switched nonlinear systems. *Control Theory and Applications*, 23(4):606–610, 2006.(In chinese)

T.L. Shen, H.Q. Zang, and K. Tamura. Riccati equation approach to robust L_2-gain synthesis for a class of uncertain nonlinear systems. *International Journal of control*, 64(6):1177–1188, 1996.

Z.D. Sun, S.S. Ge. Analysis and synthesis of switched linear control systems. *Automatica*, 41(2):181–195, 2005.

Z.J. Ji, X.X. Guo, L. Wang, and G.M. Xie. Robust H_∞ control and stabilization of uncertain switched linear systems:a multiple Lyapunov functions approach. *Journal of Dynamic Systems, Measurement and Control, Transactions of the ASME*, 128(3):696–700, 2006.