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A Dual Mode MPC Scheme for Nonlinear Processes

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Abstract: This paper presents a new dual mode nonlinear model predictive controller (NMPC) that is based on the combination of the finite horizon NMPC with the infinite horizon predictive controller (IHMPC). The resulting nonlinear controller is shown to be stable when the IHMPC is globally stabilizing. The main advantage of the proposed controller in comparison to the IHMPC is a better performance as the model nonlinearities are taken into account in the computation of the control law. The advantage of the proposed controller compared to the existing dual mode NMPC is that constraints are also considered in the linear controller that is supposed to control the system when the state enters the terminal set. The performance of the proposed controller is compared to the stable IHMPC through simulation of an industrial styrene polymerization reactor.

1. INTRODUCTION

Nowadays, the objective of any control system focuses on the manipulation of the control inputs in such a way that it can satisfy a variety of operating criteria (economic, safety, environmental or product quality) that may change depending on the characteristics of the process (Camacho and Bordons, 1999). Model Predictive Control (MPC) was developed in the 70's when the conventional PID controllers showed to be inadequate to comply with this sort of demand in the controller performance (Henson, 1998). MPC demonstrated to be a powerful tool to face such control challenges.

The acronym MPC stands for a class of control algorithms related to the control of the future behavior of the plant through the explicit use of a model of the process. At each sampling step, MPC computes in open loop a sequence of control actions that optimizes the future behavior of the plant. The first control action is injected into the plant and at the next sampling step the procedure is repeated. In general, MPC designates the class of controllers in which a linear model is used to predict the dynamic behavior, a linear or quadratic cost function is considered and linear constraints are considered in the states and inputs of the system. Analogously, NMPC refers to those model predictive controllers, which are based on non-linear models and/or consider a non-quadratic cost function and non linear constraints (Qin and Badgwell, 2000; Allgöwer et al., 2004).

The use of NMPC tends to increase as many chemical processes are essentially non-linear, which makes inadequate the application of linear models to describe the process dynamics (Findeisen and Allgöwer, 2002). In the usual approach of NMPC, the closed loop stability of the system is not assured, and for each specific system, a heuristic tuning should be pursued and there is no guarantee that this approach will be successful (Camacho and Bordons, 2004). The usual practice of NMPC is to use finite output and control horizons, which do not necessarily produce closed loop stability. Theoretically, we could consider infinite control and output horizons, but this would lead to an infinite dimension optimization problem. Usually, it is not possible to compute the optimal control sequence with the infinite horizon for a nonlinear system. There are several different ways to obtain closed loop stability with the NMPC with finite horizon (Allgöwer et al., 2004). The common approach to guarantee stability is to make use of the Lyapunov method, by adding equality and inequality constraint as well as penalty weights to the optimization problem (Allgöwer, et al., 2004, Camacho and Bordons, 2004).

Some key ingredients of a stabilizing MPC are the terminal set and the terminal cost. The terminal state corresponds to the predicted state at the end of the prediction horizon. In the usual stabilizing approach, we force the terminal state to reach a terminal set that contains the terminal state at steady state. Associated to the terminal state, there is a terminal cost that appears as a new term in the control cost. The MPCs with guaranteed stability can be classified as follows: MPC with equality terminal constraint (Mayne and Michalska, 1990); MPC with terminal cost (Bitmead, 1990, Rawlings and Muske, 1993, Alamir and Bornard, 1995); MPC with terminal inequality constraint (Michalska and Main, 1993) and MPC with terminal cost and terminal constraint (Chen and Allgöwer, 1998).

A method to obtain a stable NMPC was proposed by Mayne and Michalska (1993). The proposed strategy, which is denominated dual mode control, is based on the inclusion of an inequality constraint that forces the terminal state to lie in a terminal region Ω at the end of the of the prediction horizon p. When the state is outside the terminal region, the control action is provided by the conventional NMPC with the above constraint. Once, the state is in Ω, the controller is switched to a linear controller previously defined and that is assumed positive invariant in Ω. One limitation of this approach is that the linear controller that is activated when the system state enters the terminal set Ω does not consider explicitly the constraints that are included in the NMPC. Thus, the assumptions behind this controller are not completely true when, for instance, it is expected that one or more inputs will be saturated at the terminal steady state. Usually, this case may happen when there is, in the control structure, an optimization layer that defines the optimal terminal steady state.

In this work, it is proposed a NMPC with dual mode in which the controller that is activated when the predicted state at the end of the output horizon enters the terminal set, is the extended infinite horizon MPC (Odloak, 2004). This controller can be designed to be stabilizing inside Ω, even when a constraint as the input saturation becomes active. The basic idea is to modify the control cost function of the NMPC by adding the cost of the IHMPC in such a way that the global cost becomes a Lyapunov function for the closed loop system.

In the next section of this work, it is presented the NMPC with dual mode that is proposed here. Next it is discussed the conditions, which can guarantee the stability of the proposed dual mode NMPC. Then, it is simulated the application of the proposed controller to a polymerization reactor. Finally, in the last section it is presented the conclusions of this work.

2. THE DUAL MODE NMPC

The dual NMPC proposed in this work is composed of a NMPC with finite control and prediction horizons and an extended IHMPC that is based on a linear model with finite control horizon and infinite output horizon. This controller differs from the one proposed by Mayne and Michalska (1993) in the sense that, here, it is assumed that the IHMPC is globally stabilizing but has a poor performance when the operating point is far from the normal operating point. So, the dual mode approach is adopted in order to improve the performance of the closed loop system while maintaining stability.

Suppose that, the nonlinear deterministic system is represented by the following discrete time model

$$x(k+1) = f\left(x(k),u(k)\right) \qquad y(k) = C\,x(k) \tag{1}$$

where $x \in \Re^{nx}$ is the state vector, $u \in \Re^{nu}$ is the input vector, k is the present sampling instant and $y \in \Re^{ny}$ is the output vector. Suppose also, that at normal operating conditions $x \in \aleph \subset \Re^{nx}$ and $u \in \mathfrak{A} \subset \Re^{nu}$.

In the dual mode controller proposed here (Figure 1), the NMPC, which uses the model defined in (1), has a finite prediction horizon equal to p and a finite control horizon, which is also equal to p. So, it is assumed that the NMPC part of the controller actuates from time k until time k+p-1. The linear IHMPC has a control horizon equal to 1 and an infinite output prediction horizon. Thus, it defines the control action applied at time k+p and considers the output predictions from time k+p+1 to ∞. Then, the optimization problem that defines the dual mode controller considers an infinite output horizon and computes a sequence of control actions from time k until time k+p.

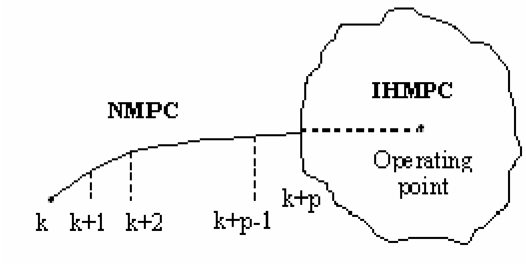

Fig. 1. The proposed dual mode NMPC

The infinite horizon MPC is based on the following time invariant linear model:

$$x(k+1) = Ax(k) + Bu(k) \qquad y(k) = Cx(k) \tag{2}$$

where A and B are matrices of appropriate dimensions. Suppose also that A has all its eigenvalues strictly inside the unit circle.

The cost function of the IHMPC is defined as follows (Odloak, 2004):

$$V_1 = \sum_{j=0}^{\infty}\left[e(k+j)-\delta_k\right]^T Q\left[e(k+j)-\delta_k\right] + \sum_{j=0}^{m-1}\Delta u(k+j)^T R\Delta u(k+j) + \delta_k^T S\delta_k \tag{3}$$

where $e(k+j) = y(k+j) - y^{SP}$ is the error of the output prediction taking into account the effect of the future control actions, $\Delta u(k) = u(k) - u(k-1)$, y^{SP} is the reference for the output and $\delta_k \in \Re^{ny}$ is a vector of slack variables. Matrices $Q \in \Re^{ny\times ny}$, $R \in \Re^{nu\times nu}$ and $S \in \Re^{ny\times ny}$ are assumed to be positive definite. The vector of slacks is included in the control cost to prevent the cost to become unbounded due to a possible offset in the controlled output. With this purpose, each slack corresponds to the steady state error in a controlled output. The weight matrix S should be selected such that the controller will prefer to force the slacks to zero or at least minimize them depending on the available degrees of freedom of the system (Odloak, 2004).

In the context of the dual mode MPC, the cost defined in (3) will correspond to the cost related to time instants k+p to ∞

and the control horizon is equal to one, which corresponds to a single control move applied to the system at time $k+p$. So, it is convenient to write this cost as follows:

$$V_1\left(x(k+p)\right)= \sum_{j=p+1}^{\infty}\left[e(k+j)-\delta_k\right]^T Q\left[e(k+j)-\delta_k\right] + \Delta u(k+p)^T R\Delta u(k+p)+\delta_k^T S\delta_k \quad (4)$$

At any future time instant, the state can be computed by applying recursively the model defined in (2):

$$x(k+p+j)=A^j x(k+p)+\sum_{i=1}^{j}A^{j-i}Bu(k+p) \quad (5)$$

Substituting (5) into the IHMPC cost represented in (4), the following expression is obtained:

$$V_1\left(x(k+p)\right)= \sum_{j=1}^{\infty}\left[\left(CA^j x(k+p)+\sum_{i=1}^{j}CA^{j-i}Bu(k+p)\right)-y^{SP}-\delta_k\right]^T Q\left[\left(CA^j x(k+p)+\sum_{i=1}^{j}CA^{j-i}Bu(k+p)\right)-y^{SP}-\delta_k\right] + \Delta u(k+p)^T R\Delta u(k+p)+\delta_k^T S\delta_k \quad (6)$$

In order to the above cost to be bounded, it can be proved that the following constraint has to be satisfied:

$$\left(\sum_{i=1}^{\infty}CA^i B\right)u(k+p)-y^{SP}-\delta_k=0 \quad (7)$$

Now, as the linear system is stable, one can define

$$K=\sum_{i=1}^{\infty}CA^i B$$

and (7) becomes

$$K\,u(k+p)-y^{SP}-\delta_k=0 \quad (8)$$

Substituting the stability condition defined in the above equation into the IHMPC cost represented in (6), the cost can be written as follows:

$$V_1\left(x(k+p)\right)= x(k+p)^T\bar{Q}x(k+p)+x(k+p)^T\bar{G}u(k+p) + u(k+p)^T\bar{H}u(k+p)+\Delta u(k+p)^T R\Delta u(k+p) + \left(Ku(k+p)-y^{SP}\right)^T S\left(Ku(k+p)-y^{SP}\right) \quad (9)$$

where:

$$\bar{Q}=\sum_{j=1}^{\infty}\left(CA^j\right)^T QCA^j$$

$$\bar{G}=2\sum_{j=1}^{\infty}\left(CA^j\right)^T Q\left(\sum_{i=1}^{j}CA^{j-i}B-K\right)$$

$$\bar{H}=\sum_{j=1}^{\infty}\left(\sum_{i=1}^{j}CA^{j-i}B-K\right)^T Q\left(\sum_{i=1}^{j}CA^{j-i}B-K\right)$$

and $\bar{Q}$ is obtained as the solution to the following Lyapunov equation:

$$A^T\bar{Q}A-\bar{Q}=-A^T C^T QCA$$

The cost function of the conventional NMPC is expressed as follows:

$$V_2\left(x(k)\right)=\sum_{j=1}^{p}\left(e(k+j)-\delta_k\right)^T Q\left(e(k+j)-\delta_k\right) + \sum_{j=0}^{p-1}\Delta u(k+j)^T R\Delta u(k+j) \quad (10)$$

where the slack is also included in the term related to NMPC to preserve the continuity between the nonlinear and linear parts of the controller.

In the controller proposed here, it is assumed that $m = p$ and the cost function for the dual mode controller is defined as follows:

$$V\left(x(k)\right)=V_2\left(x(k)\right)+V_1\left(x(k+p)\right) \quad (11)$$

Then, substituting (9) and (10) in Eq. (11), one obtain:

$$V\left(x(k)\right)= \sum_{j=1}^{p}\left(Cx(k+j)+Ku(k+p)\right)^T Q\left(Cx(k+j)+Ku(k+p)\right) + \sum_{j=0}^{p}\Delta u(k+j)^T R\Delta u(k+j)+x(k+p)^T\bar{Q}x(k+p) + x(k+p)^T\bar{G}u(k+p)+u(k+p)^T\bar{H}u(k+p) + \left(Ku(k+p)-y^{SP}\right)^T S\left(Ku(k+p)-y^{SP}\right) \quad (12)$$

Equation (12) defines the cost of the dual mode NMPC, which should be minimized taking into the usual constraints in the inputs of the system:

$$\begin{aligned} &u_{\min}\le u(k+j)\le u_{\max}\ , \quad j=0,1,\ldots,p \\ &-\Delta u_{\max}\le \Delta u(k+j)\le \Delta u_{\max}\end{aligned} \quad (13)$$

3. STABILITY OF THE DUAL MODE NMPC

The proposed dual mode NMPC will be stable if the following conditions are satisfied:

a). The set $\aleph$ is contained in the set Ω, or the normal operating set is contained in the terminal set. This means that the IHMPC is stable for all the normal operating conditions of the system. However, the stability of this controller may result in a poor performance of the closed loop system in such a way that it cannot be used in practice.

The infinite horizon MPC considered here is obtained from the solution to the optimization problem where V_1 is minimized subject to constraints (13). If the model defined in (2) represents the plant exactly, then, it can be shown that cost V_1 will be bounded and strictly decreasing as long as weight S is sufficiently large. Thus, the controller is stabilizing for the linear system.

b). For any $x(k+p)\in\aleph$ and $u(k+p)\in\Omega$ there is a control action $\tilde{u}(k+p+1)$ such that following inequality is feasible

$$\begin{aligned} V_1\left(x(k+p),u(k+p)\right) &\geq l\left(x(k+p),u(k+p)\right) \\ &+V_1\left(f\left(x(k+p),u(k+p)\right),\tilde{u}(k+p+1)\right) \end{aligned} \quad (14)$$

where

$$l\left(x(k+p),u(k+p)\right)=$$
$$\left(Cx\left(k+p\right)+Ku\left(k+p\right)\right)^T Q\left(Cx\left(k+p\right)+Ku\left(k+p\right)\right)$$

and $\tilde{u}(k+p+1)$ satisfies (13).

If conditions (a) and (b) are satisfied, then the optimization problem that minimizes $V\left(x(k)\right)$ subject to constraints (13) is always feasible for any $x(k)\in\aleph$. Also, $V\left(x(k)\right)$ will be bounded and decreasing, and so the dual mode NMPC will be converging.

4. EXAMPLE: INDUSTRIAL POLYMERIZATION REACTOR

In order to compare the performance of the proposed dual mode NMPC to the globally stable linear IHMPC, the controllers were tested by simulation in a free radical initiated bulk and solution of styrene polymerization in a 3000 liter jacketed CSTR (see Figure 2).

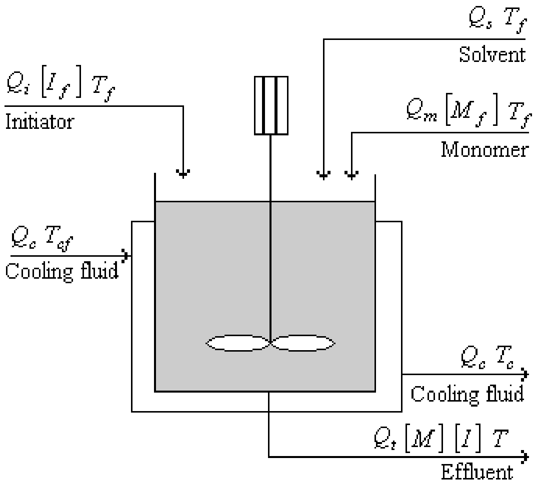

Fig. 2. Styrene polymerization reactor.

The model of the polymerization process is described by the following equations (Sotomayor *et al*., 2007):

$$\frac{d[I]}{dt}=\frac{\left(Q_i\left[I_f\right]-Q_t\left[I\right]\right)}{V}-k_d\left[I\right] \quad (15)$$

$$\frac{d[M]}{dt}=\frac{\left(Q_m\left[M_f\right]-Q_t\left[M\right]\right)}{V}-k_p\left[M\right]\left[P\right] \quad (16)$$

$$\begin{aligned}\frac{dT}{dt}&=\frac{Q_t\left(T_f-T\right)}{V}+\frac{\left(-\Delta H_r\right)}{\rho Cp}\times k_p\left[M\right]\left[P\right] \\ &-\frac{hA}{\rho CpV}\times\left(T-T_c\right)\end{aligned} \quad (17)$$

$$\frac{dT_c}{dt}=\frac{Q_c\left(T_{cf}-T_c\right)}{V_c}+\frac{hA}{\rho_c Cp_c V_c}\left(T-T_c\right) \quad (18)$$

$$\left[P\right]=\left[\frac{2fk_d\left[I\right]}{k_t}\right]^{1/2}$$

$$k_i=A_i\exp\frac{-E_i}{T},\quad i=d,p,t$$

$$Q_t=Q_i+Q_s+Q_m$$

$$Q_s=1.5Q_m-Q_i$$

where:

A	:	area of heat transfer
Cp	:	specific heat of the fluid in the reactor
Cp_c	:	specific heat of the fluid in the cooling jacket
f	:	initiator efficiency
h	:	global heat transfer coefficient
$[I]$	:	concentration of initiator in the reactor
$[I_f]$	:	concentration of initiator in feed stream
k_i	:	kinetic constants of the polymerization reaction
$[M]$	:	concentration of monomer in the reactor
$[M_f]$	:	concentration of monomer in the feed stream
Q_i	:	flowrate of initiator
Q_c	:	flowrate of cooling fluid to the jacket
Q_m	:	flowrate of monomer
Q_s	:	flowrate of solvent
Q_t	:	flowrate of the outlet stream of the reactor
t	:	time
T	:	temperature in the reactor
T_c	:	temperature of the cooling fluid
T_{cf}	:	inlet temperature of the cooling fluid
T_f	:	temperature of the feed
V	:	volume of the reactor
V_c	:	volume of the cooling jacket
$-\Delta H_r$	:	heat of the polymerization reaction
ρ	:	density of the reacting fluid
ρ_c	:	density of the cooling fluid

The numerical values of the model parameters and usual operating conditions of the reactor can be found in Sotomayor *et al*. (2007). The control objective in the reactor is to maintain the reaction temperature (T) at the desired value by manipulating the flow of initiator (Q_i) and the flow of fluid to the cooling jacket (Q_c).

The tuning parameters of the dual mode NMPC are the following:

$$p = 2\,,\ Q = 10\,,\ R = 1\times 10^{-4}\begin{bmatrix}4 & 0\\ 0 & 2\end{bmatrix},\ S = 2000$$

$$u_{\max} = \begin{bmatrix}600 & 600\end{bmatrix}^T,\ u_{\min} = \begin{bmatrix}0 & 0\end{bmatrix}^T,\ \Delta u_{\max} = \begin{bmatrix}40 & 50\end{bmatrix}^T$$

Figure 3(a) shows the responses of the NMPC when the set point of the reactor temperature suffers a change from 353.56 K to 356.56 K. It is also shown the responses of the IHMPC with the same tuning parameters as the dual mode NMPC except the control horizon m, which is made equal to one. As the system has two manipulated inputs and only one controlled output, the trajectories of the inputs and the resulting steady states are quite different. Figure 3(b) shows the cost V for the two controllers. It is clear that the performance of the proposed nonlinear controller is substantially better than the performance of the linear controller.

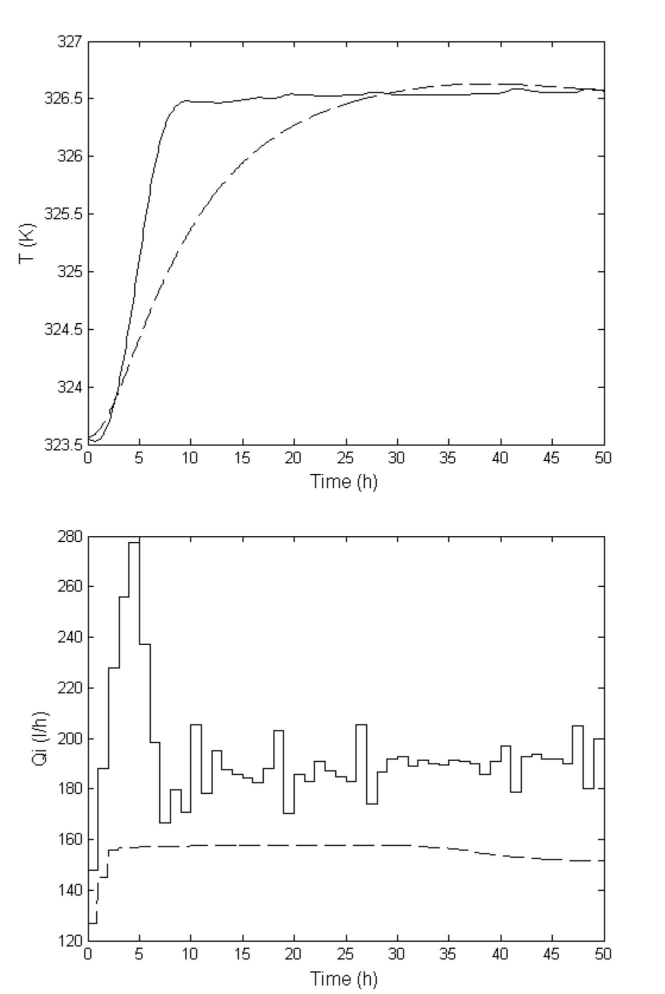

Fig. 3(a). Closed loop responses for an increase of 3 K in the set point of the reactor temperature: (——) dual mode NMPC, (— —) IHMPC.

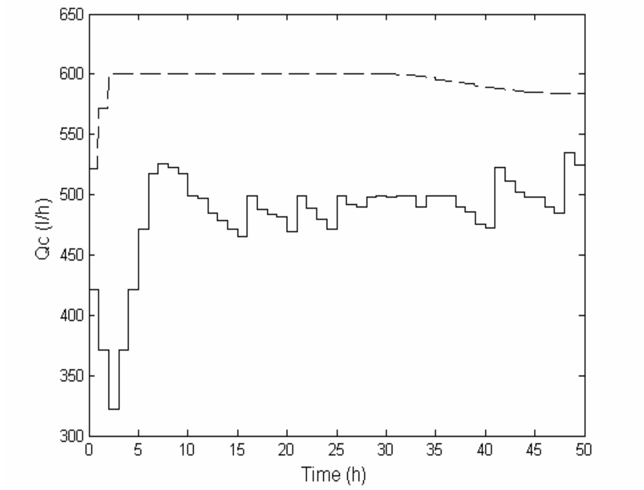

Fig. 3(a)(Cont.). Closed loop responses for an increase of 3 K in the set point of the reactor temperature: (——) dual mode NMPC, (— —) IHMPC.

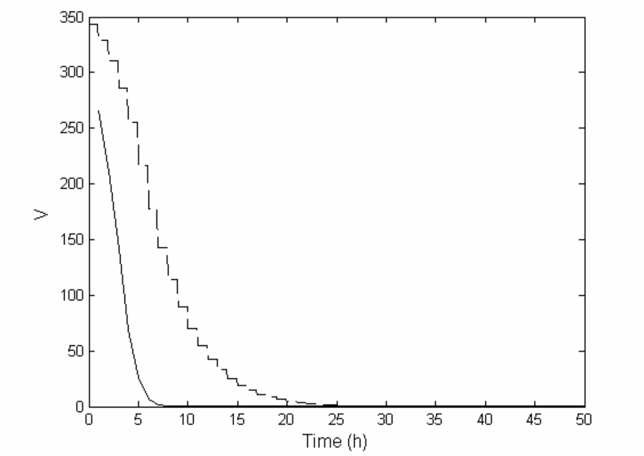

Fig. 3(b). Cost function profile: (——) dual mode NMPC, (— —) IHMPC.

Figure 4 shows the responses of the closed loop system with these controllers for a step change of 5 K in the temperature of the feed stream, while the set point of the reactor temperature is kept at 353.56 K . Again, the performance of the proposed nonlinear controller is much better than the performance of IHMPC. Observing the response of flow of cooling fluid (Q_c), it can be noted that this input is kept saturated at is maximum value for time instants after 12 h. This means that, in this case, we cannot apply the usual approach of the dual mode NMPC, which assumes that inside the terminal set we use a linear unconstrained controller.

5. CONCLUSIONS

In this work it is proposed a dual mode NMPC that combines the conventional finite horizon NMPC with the infinite horizon linear MPC. It is assumed that the system is working in an operating region where IHMPC is globally stabilizing and the controller is developed according to the ideas of Odloak (2004).

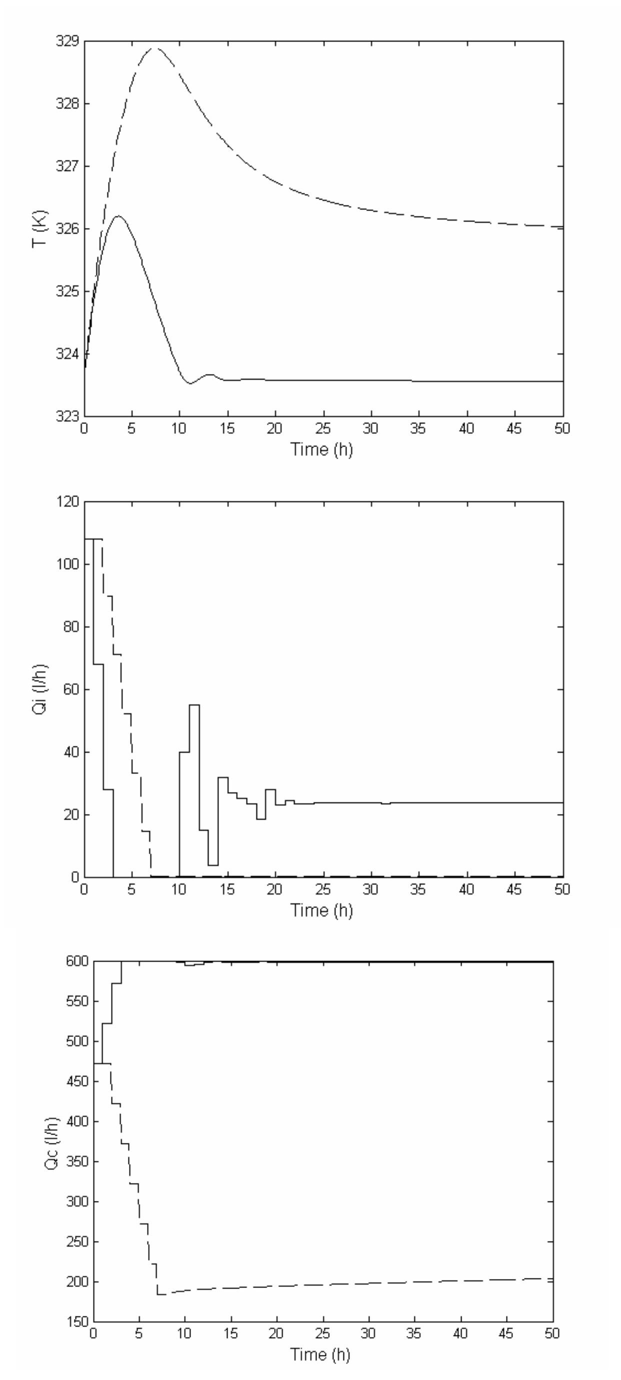

Fig. 4. Closed loop responses for an increase of 5 K in the feed temperature: (——) dual mode NMPC, (— —) IHMPC.

It is shown that the cost function of the conventional NMPC can be extended to include the cost of the IHMPC and the resulting infinite horizon cost can be made bounded without the inclusion of hard constraints. In the proposed controller, the nonlinear part of the controller has finite output and input horizons, while the linear part of the controller has infinite output horizon and control horizon equal to one. The basic difference between the proposed approach and the existing ones is that constraints can also be imposed in the linear controller that is activated when the state enters the terminal set, which enforces a more realist scenario to the control problem. The proposed approach was tested by simulation in a polymerization reactor for temperature tracking and temperature regulation.

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