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Sampled-data Networked Control Systems with Random Time Delay

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Abstract: The stability and performance of a networked control system (NCS) strongly depends on the transmission delay. However, the randomness of the transmission delay is an intrinsic property in the network communication, e.g. Ethernet. Aiming at NCS with random transmission delay, a novel control approach is proposed. The transmission delays from sensor-to-controller (SC) and controller-to-actuator (CA) are modelled by two independent Markovian processes. A controller, which is able to monitor the SC delay and synchronously switches according to it, is considered. The resulting closed-loop system is a Markovian jump linear system with randomly piecewise continuous delay. The exponential mean square stability for the given model is established by using a Lyapunov-Krasovskii functional. The performance benefits of the proposed approach are demonstrated in a numerical example.

1. INTRODUCTION

In views of affordability, widespread usage and well developed infrastructure, communication networks have increasing potential in industrial applications. One of the most apparent example comes with the networked control system (NCS). A NCS is a feedback control system using a shared network for the communication between spatially distributed sensors, actuators and physical plants. The NCS has advantages such as it allows flexible control structures, reduced wiring and easy maintenance. Typical examples are unmanned aerial vehicles, e.g. Seiler [2001], Ethernet-based car control network, e.g Daoud et al. [2006], teleoperation, e.g Hirche [2005].

The use of communication network comes, however, at the price of non-ideal signal transmission: the sampled data sent through the networks experience variable time delays and suffer transmission losses (or packet dropouts), see Hespanha et al. [2007], Baillieul and Antsaklis [2007]. Particularly, the delay is well known as a source of instability and deteriorates the control performance, see Gu et al. [2003]. So far, various approaches have been proposed in the literature to cope with time delay, see Ray and Galevi [1988], Zhang et al. [2001], Lin et al. [2003], Fridman et al. [2004]. Ray and Galevi [1988] introduces the augmented state vector method for constant delay. A hybrid system analysis approach is applied to NCS for known delay by Zhang et al. [2001] and for uncertain delay by Lin et al. [2003]. Time-varying delay and robust control are dealt by Fridman et al. [2004]. More approaches with deterministic delay can be found in the work of Kharitonov [1999], Richard [2003], Gu and Niculescu [2003] and references therein.

Systems with random time delay are studied by Nilsson [1998], Xiao et al. [2000], Yang et al. [2006], Zhang et al. [2005]. Nilsson [1998] models the delay as a Markovian process and the effect of random delay is treated as an LQG problem. However, the network-induced delay has to be less than one sampling interval. Therefore, this approach may be unsuitable for systems with longer time delay. A stochastic hybrid system approach involving bounded random delay and switching feedback control laws is considered by Xiao et al. [2000]. The approach results in a bilinear matrix inequality (BMI). An iteration algorithm is formulated for targeting the BMI difficulties. Yang et al. [2006] proposes a H_∞ control problem for Bernoulli random binary delay and derives an LMI problem for stochastic exponential stability. Zhang et al. [2005] considers a Markovian jump linear system (MJLS) approach for NCS. Based on the Lyapunov method, an iterative linear matrix inequality (LMI) for mode-dependent controller preserving stochastic stability is established.

Although the network-induced delay has been frequently discussed for NCS, only the sensor-to-controller (SC) delay has been taken into account in most of the previous works. The controller-to-actuator (CA) delay remains less explored. The stabilization results of NCS for SC and CA delays are first considered by Witrant et al. [2003], where a delay compensation predictive control approach is proposed for time-varying input delay systems. The random SC and CA delays are considered by Yang et al. [2006] and Zhang et al. [2005]. However, the SC and CA delays by Yang et al. [2006] have to be less than one sampling interval. Zhang et al. [2005] augments the state vector by delayed signal and results in complicated and higher dimensional systems.

This paper considers random transmission time delay. The SC and CA delays are modelled by two independent Markovian processes: r_t^{sc} and r_t^{ca}. The sampled-data system approach is applied and a switching output-feedback controller is proposed. The resulting time delay contains a random part related to the transmission delay and a linear time-varying part bounded by the sampling interval. The switching controller monitors the SC transmission

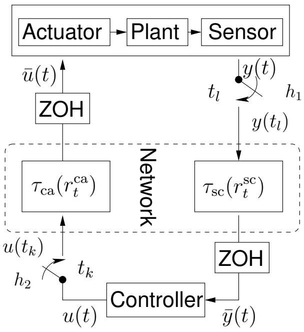

Fig. 1. Illustration of NCS over communication network, the transmission delay from sensor-to-controller $\tau_{\mathrm{sc}}(r_t^{\mathrm{sc}})$ and from controller-to-actuator $\tau_{\mathrm{ca}}(r_t^{\mathrm{ca}})$.

delay and synchronously switches with it. As a result, an MJLS with randomly piecewise continuous delay is formulated. The condition for exponential mean square stability is derived by the delay-dependent Lyapunov-Krasovskii approach. The controller design is presented in terms of linear matrix inequalities (LMI's). In the simulation the performance benefit of the proposed switching controller is demonstrated.

The reminder of the paper is organized as follows: In section II the sampled-data MJLS is introduced. The system contains two mode-dependent delays and a mode-dependent switching output-feedback controller. In section III the exponential mean square stability condition and controller design are shown. Finally, a numerical example is given to illustrate the proposed method.

Notation. Throughout the paper we let $\lambda_{\min}(M)$ and $\lambda_{\max}(M)$ denote the maximal and the minimal eigenvalue of matrix M. M^T and $||M||$ denote the transpose and induced Euclidean norm of matrix (or vector) M. M^+ denotes the pseudo-inverse of matrix M. The symbol $*$ denotes the transpose of the blocks outside the main diagonal block in symmetric matrices. $\mathbb{E}$ stands for mathematical expectation and $\mathbf{P}$ for probability. Let $\{r_t, t \geq 0\}$ denote a Markovian process governing the mode switching in the finite set $\mathcal{S} := \{1, \ldots, N\}$ having the generator $\mathcal{A} = (\alpha_{i,j})$, $i, j \in \mathcal{S}$, $\alpha_{i,j} > 0$, $i \neq j$, $\alpha_{i,i} = -\sum_{i\neq j} \alpha_{i,j}$. Then the mode transition probability can be defined as

$$\mathbf{P}_{i,j}(r_{t+\delta} = j | r_t = i) = e^{\mathcal{A}\delta}.$$

2. PROBLEM DEFINITION

2.1 NCS Model

Consider an LTI system as a plant:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B\bar{u}(t), \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $y \in \mathbb{R}^q$ is the measurement output and $\bar{u} \in \mathbb{R}^m$ is the control input; A, B and C are constant matrices with appropriate dimensions. The plant is interconnected with the controller over a communication network, see Fig. 1.

We now consider the network transmission delay $\tau_{\mathrm{sc}}(r_t^{\mathrm{sc}})$ and $\tau_{\mathrm{ca}}(r_t^{\mathrm{ca}})$ as Markovian delays. The mode switching is governed by the Markovian processes $r_t^{\mathrm{sc}} \in \mathcal{S}_{\mathrm{sc}}$ and $r_t^{\mathrm{ca}} \in \mathcal{S}_{\mathrm{ca}}$ which are independent and taking values in the finite set $\mathcal{S}_{\mathrm{sc}} := \{1, \ldots, N_{\mathrm{sc}}\}$ and $\mathcal{S}_{\mathrm{ca}} := \{1, \ldots, N_{\mathrm{ca}}\}$. The switching rates from mode i to j of both delays are defined by $\alpha_{i,j}^{\mathrm{sc}}$ and $\alpha_{i,j}^{\mathrm{ca}}$. According to (1) and Fig. 1, the piecewise constant measurement from SC at sampled time t_l is expressed by

$$\begin{aligned} \bar{y}(t) &= y(t - \tau_1(t, r_t^{\mathrm{sc}})) = Cx(t - \tau_1(t, r_t^{\mathrm{sc}})), \\ \tau_1(t, r_t^{\mathrm{sc}}) &= t - t_l + \tau_{\mathrm{sc}}(r_t^{\mathrm{sc}}), \quad t_l \leq t < t_{l+1}. \end{aligned} \tag{2}$$

The transmission delay $\tau_1(t, r_t^{\mathrm{sc}})$ in the SC channel can be known by the controller using the time-stamping technique. Hence, we consider an output-feedback controller which switches synchronously with the delay $\tau_{\mathrm{sc}}(r_t^{\mathrm{sc}})$. The switching output-feedback controller has the form

$$\begin{aligned} \dot{x}_c(t) &= A_c(r_t^{\mathrm{sc}})x_c(t) + B_c(r_t^{\mathrm{sc}})\bar{y}(t), \\ u(t) &= C_c(r_t^{\mathrm{sc}})x_c(t), \end{aligned} \tag{3}$$

where $x_c \in \mathbb{R}^n$ and $x_c = 0$ for $t \leq 0$. The piecewise constant control output in the CA side at sampled time t_k is expressed by

$$\begin{aligned} \bar{u}(t) &= u(t - \tau_{\mathrm{ca}}(r_t^{\mathrm{ca}})) = C_c x_c(t - \tau_2(t, r_t^{\mathrm{ca}})), \\ \tau_2(t, r_t^{\mathrm{ca}}) &= t - t_k + \tau_{\mathrm{ca}}(r_t^{\mathrm{ca}}), \quad t_k \leq t < t_{k+1}. \end{aligned} \tag{4}$$

Define $z^T = [x^T \ x_c^T]$. The closed-loop system in Fig. 1 is obtained as follows

$$\begin{aligned} \dot{z}(t) = &\bar{A}_0(r_t^{\mathrm{sc}})z(t) + \bar{A}_1(r_t^{\mathrm{sc}})z(t - \tau_1(t, r_t^{\mathrm{sc}})) \\ &+ \bar{A}_2(r_t^{\mathrm{sc}})z(t - \tau_2(t, r_t^{\mathrm{ca}})), \end{aligned} \tag{5}$$

where

$$\bar{A}_0(r_t^{\mathrm{sc}}) = \begin{bmatrix} A & 0 \\ 0 & A_c(r_t^{\mathrm{sc}}) \end{bmatrix}, \ \bar{A}_1(r_t^{\mathrm{sc}}) = \begin{bmatrix} 0 & 0 \\ B_c(r_t^{\mathrm{sc}})C & 0 \end{bmatrix},$$

$$\bar{A}_2(r_t^{\mathrm{sc}}) = \begin{bmatrix} 0 & BC_c(r_t^{\mathrm{sc}}) \\ 0 & 0 \end{bmatrix}.$$

System (5) is an MJLS with two mode-dependent time-varying delays $\tau_1(t, r_t^{\mathrm{sc}})$ and $\tau_2(t, r_t^{\mathrm{ca}})$.

2.2 Time Delay Model

The switching of transmission delays may result in the disorder of the sampled sequence. In this paper, we exclude the disordering in the sampled sequence, i.e. we assume that

A1: $\mathbf{P}(|\tau_1(t_{l+1}, r_t^{\mathrm{sc}}) - \tau_1(t_l, r_t^{\mathrm{sc}})| \geq h_1) = 0,$.
A2: $\mathbf{P}(|\tau_2(t_{k+1}, r_t^{\mathrm{ca}}) - \tau_2(t_k, r_t^{\mathrm{ca}})| \geq h_2) = 0.$

Assumptions A1 and A2 restrict the switching of any two consecutive delays is less than one sampling interval. These assumptions are not unreasonable as the current transmission delay in the real communication network is usually correlated to the previous delay.

The delays $\tau_1(t, r_t^{\mathrm{sc}})$ and $\tau_2(t, r_t^{\mathrm{ca}})$ contain a randomly piecewise constant part $\tau_{\mathrm{sc}}(r_t^{\mathrm{sc}})$ (or $\tau_{\mathrm{ca}}(r_t^{\mathrm{ca}})$ for τ_2) related to the network transmission delay and a time-varying part $t - t_l$ (or $t - t_k$) related to the inter-sampling effect as shown in Fig. 2 (b). The time-varying part is bounded by sampling interval h_i and has the derivative $\dot{\tau}_i = 1, \ i = 1, 2$. The switching probability between two consecutive transmission delays are

$$\begin{aligned} \mathbf{P}_{i,j}(r_{t_{l+1}}^{\mathrm{sc}} = j | r_{t_l}^{\mathrm{sc}} = i) &= e^{\mathcal{A}_{\mathrm{sc}} h_1}, \\ \mathbf{P}_{i,j}(r_{t_{k+1}}^{\mathrm{sc}} = j | r_{t_k}^{\mathrm{ca}} = i) &= e^{\mathcal{A}_{\mathrm{ca}} h_2}, \end{aligned}$$

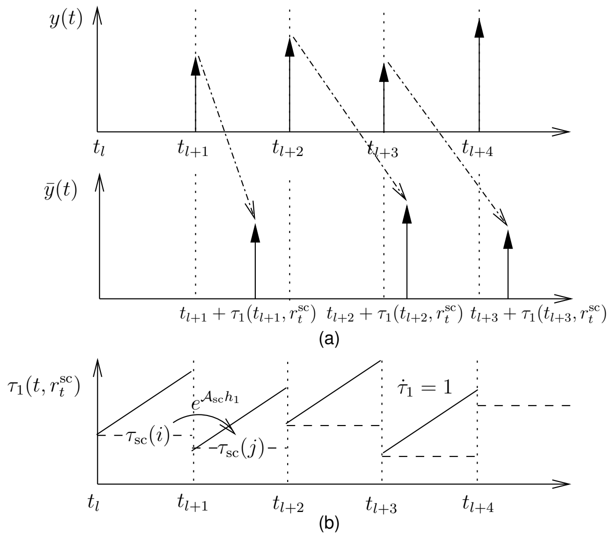

Fig. 2. The sampled output measurement $y(t)$, $\bar{y}(t)$ (a) and the evolution of time delay $\tau_1(t, r_t^{sc})$ for certain sample path of $\tau_{sc}(r_t^{sc})$ (b).

where $\mathcal{A}_{sc} = (\alpha_{i,j}^{sc})$, $\mathcal{A}_{ca} = (\alpha_{i,j}^{ca})$ are the transition generators of Markovian processes r_t^{sc} and r_t^{ca}. The upper and lower bounds of delays are defined as

$$\begin{aligned} \bar{\tau}_1 &= h_1 + \max_{i\in\mathcal{S}_{sc}}\{\tau_{sc}(i)\}, \quad \underline{\tau}_1 = \min_{i\in\mathcal{S}_{sc}}\{\tau_{sc}(i)\}, \\ \bar{\tau}_2 &= h_2 + \max_{i\in\mathcal{S}_{ca}}\{\tau_{ca}(i)\}, \quad \underline{\tau}_2 = \min_{i\in\mathcal{S}_{ca}}\{\tau_{ca}(i)\}. \end{aligned} \tag{6}$$

Before the main result is introduced, the following definition and lemma have to be given.

Definition 1. System (5) is said to be is exponential mean square stable if for any initial condition $z_0(r_0^{sc}, r_0^{ca})$, there exist positive constants b, and ρ such that for all $t \geq 0$

$$\mathbb{E}\{||z(t)||^2 | z_0(r_0^{sc}, r_0^{ca})\} \leq b||z_0(r_0^{sc}, r_0^{ca})||^2 e^{-\rho t}.$$

Lemma 1. (Boukas and Liu [2002]). Let X and Y be real constant matrices with appropriate dimensions. Then

$$X^TY + Y^TX \leq \varepsilon X^TX + \frac{1}{\varepsilon}Y^TY$$

holds for any $\varepsilon > 0$.

3. MAIN RESULT

In this section, a delay-dependent stability condition as well as output-feedback controller design for NCS with random input delay are presented. The approach is derived by using the Lyapunov-Krasovskii approach and descriptor transformation. Henceforth, we let $\tau_1(r_t^{sc})$, $\tau_2(r_t^{ca})$ denote $\tau_1(t, r_t^{sc})$ and $\tau_2(t, r_t^{ca})$ if no ambiguity occurs. Consider the integral

$$z(t) - z(t-\tau_1(r_t)) = \int_{t-\tau_1(r_t)}^{t} \dot{z}(s)ds \tag{7}$$

and take (7) into (5). The closed-loop system becomes

$$\begin{aligned} \dot{z}(t) &= \big(\bar{A}_0(r_t^{sc}) + \bar{A}_1(r_t^{sc}) + \bar{A}_2(r_t^{sc})\big)z(t) \\ &- \bar{A}_1(r_t^{sc})\int_{t-\tau_1(r_t^{sc})}^{t}\dot{z}(s)ds \\ &- \bar{A}_2(r_t^{sc})\int_{t-\tau_2(r_t^{ca})}^{t}\dot{z}(s)ds. \end{aligned}$$

Let $\xi^T(t) = [z^T(t)\ \dot{z}^T(t)]$, the closed-loop system has the descriptor form

$$\begin{aligned} E\dot{\xi}(t) &= \hat{A}(r_t^{sc})\xi(t) - \hat{A}_1(r_t^{sc})\int_{t-\tau_1(r_t^{sc})}^{t}\xi(s)ds \\ &- \hat{A}_2(r_t^{sc})\int_{t-\tau_2(r_t^{ca})}^{t}\xi(s)ds, \end{aligned}$$

where

$$\hat{A}_0(r_t^{sc}) = \begin{bmatrix} 0 & I \\ \bar{A}_0(r_t^{sc}) + \bar{A}_1(r_t^{sc}) + \bar{A}_2(r_t^{sc}) & -I \end{bmatrix}, \ E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix},$$

$$\hat{A}_1(r_t^{sc}) = \begin{bmatrix} 0 & 0 \\ 0 & \bar{A}_1(r_t^{sc}) \end{bmatrix}, \ \hat{A}_2(r_t^{sc}) = \begin{bmatrix} 0 & 0 \\ 0 & \bar{A}_2(r_t^{sc}) \end{bmatrix}.$$

Theorem 1. If there exist matrices $G(l,k)$, $H(l,k)$, $W(l,k)$, symmetric positive definite matrices $X_1(l,k)$, Q_1, Q_2 and scalars $n_1(l,k) > 0$, $n_2(l,k) > 0$, $l \in \mathcal{S}_{sc}$, $k \in \mathcal{S}_{ca}$ such that the following LMI holds

$$\begin{bmatrix} \Psi_1(l,k) & * & * \\ \hat{\tau}_1(l)\Psi_2(l,k) & -\hat{\tau}_1(l)Q_1 & * \\ \hat{\tau}_2(k)\Psi_3(l,k) & 0 & -\hat{\tau}_2(k)Q_2 \end{bmatrix} < 0, \tag{8}$$

where

$$\begin{aligned} \hat{\tau}_1(l) &= \tau_1(l) + \frac{1}{2}\bar{\alpha}^{sc}(\bar{\tau}_1^2 - \underline{\tau}_1^2), \\ \hat{\tau}_2(k) &= \tau_2(k) + \frac{1}{2}\bar{\alpha}^{ca}(\bar{\tau}_2^2 - \underline{\tau}_2^2), \\ \bar{\alpha}^{sc} &= \max\{|\alpha_{ii}^{sc}|, i \in \mathcal{S}_{sc}\}, \\ \bar{\alpha}^{ca} &= \max\{|\alpha_{ii}^{ca}|, i \in \mathcal{S}_{ca}\}, \end{aligned}$$

$$\begin{aligned} \Psi_1(l,k) &= \psi_1(l,k) + \psi_1^T(l,k) + \sum_{h=1}^{N_{sc}} \alpha_{l,h}^{sc} EX^T(h,k) \\ &+ \sum_{j=1}^{N_{ca}} \alpha_{k,j}^{ca} EX^T(l,j) + \tau_1(l)Q_1 + \tau_2(k)Q_2, \\ X(l,k) &= \begin{bmatrix} X_1(l,k) & 0 \\ -n_1(l,k)X_1(l,k) & n_2(l,k)X_1(l,k) \end{bmatrix}, \\ X_1(l,k) &= \begin{bmatrix} X_{11}(l,k) & 0 \\ 0 & X_{12}(l,k) \end{bmatrix}, \\ \Psi_2(l,k) &= \begin{bmatrix} 0 & 0 \\ -n_1(l,k)\psi_2(l,k) & n_2(l,k)\psi_2(l,k) \end{bmatrix}, \\ \Psi_3(l,k) &= \begin{bmatrix} 0 & 0 \\ -n_1(l,k)\psi_3(l,k) & n_2(l,k)\psi_3(l,k) \end{bmatrix}, \\ \psi_2(l,k) &= \begin{bmatrix} 0 & 0 \\ H(l,k) & 0 \end{bmatrix}, \quad \psi_3(l,k) = \begin{bmatrix} 0 & W(l,k) \\ 0 & 0 \end{bmatrix}, \end{aligned}$$

then the system (5) is exponential mean square stable under the controller (3) of the form

$$\begin{aligned} A_c(l) &= G(l,k)X_{12}^{-1}(l,k), \\ B_c(l) &= H(l,k)X_{11}^{-1}(l,k)C^{+}, \\ C_c(l) &= B^{+}W(l,k)X_{12}^{-1}(l,k). \end{aligned} \tag{9}$$

Proof: The state $\{\xi(t), r_t^{sc}, r_t^{ca}, t \geq 0\}$ depends on the history $\xi(t+\theta)$, $\theta \in [-2\tau_1(r_t^{sc}) - 2\tau_2(r_t^{ca}), 0]$, which implies however $\{\xi(t), r_t^{sc}, r_t^{ca}, t \geq 0\}$ is not a Markovian process. We modify our problem into a new Markovian process by defining a new process $\{\Xi(t), r_t^{sc}, r_t^{ca}\ t \geq 0\}$ taking values as the following

$$\Xi(t) = \xi(s+t), \ s \in \big[t - 2\tau_1(r_t^{sc}) - 2\tau_2(r_t^{ca}), t\big].$$

Define a set of positive definite matrices

$$P(r_t^{sc}, r_t^{ca}) = X^{-1}(r_t^{sc}, r_t^{ca})$$

and consider a Lyapunov candidate as (10) in the next page.

Suppose $r_t^{sc} = l \in \mathcal{S}_{sc}$, $r_t^{ca} = k \in \mathcal{S}_{ca}$ and let $\mathcal{L}(\cdot)$ be the infinitesimal generator of $\{\Xi(t), r_t^{sc}, r_t^{ca}\}$; then

$$\begin{aligned}
&\mathcal{L}V_1(\Xi(t), r_t^{sc}, r_t^{ca}) \\
&= \xi^T(t)\Bigg[\hat{A}^T(r_t^{sc})P(r_t^{sc}, r_t^{ca}) + P^T(r_t^{sc}, r_t^{ca})\hat{A}(r_t^{sc}) \\
&+ \sum_{h=1}^{N_{sc}} \alpha_{l,h}^{sc} EP(h, r_t^{ca}) + \sum_{j=1}^{N_{ca}} \alpha_{k,j}^{ca} EP(r_t^{sc}, j)\Bigg]\xi(t) \\
&- 2\xi^T(t)P^T(r_t^{sc}, r_t^{ca})\hat{A}_1(r_t^{sc})\int_{t-\tau_1(r_t^{sc})}^{t} \xi(s)ds \\
&- 2\xi^T(t)P^T(r_t^{sc}, r_t^{ca})\hat{A}_2(r_t^{sc})\int_{t-\tau_2(r_t^{ca})}^{t} \xi(s)ds.
\end{aligned}$$

According to Lemma A.1, it results in

$$\begin{aligned}
&\mathcal{L}V_1(\Xi(t), r_t^{sc}, r_t^{ca}) \\
&\leq \xi^T(t)\Bigg[\hat{A}^T(r_t^{sc})P(r_t^{sc}, r_t^{ca}) + P^T(r_t^{sc}, r_t^{ca})\hat{A}(r_t^{sc}) \\
&+ \sum_{h=1}^{N_{sc}} \alpha_{l,h}^{sc} P(h, r_t^{ca}) + \sum_{j=1}^{N_{ca}} \alpha_{k,j}^{ca} P(r_t^{sc}, j) \\
&+ \tau_1(r_t^{sc})P^T(r_t^{sc}, r_t^{ca})Q_1P(r_t^{sc}, r_t^{ca}) \\
&+ \tau_2(r_t^{ca})P^T(r_t^{sc}, r_t^{ca})Q_2P(r_t^{sc}, r_t^{ca})\Bigg]\xi(t) \\
&+ \int_{t-\tau_1(r_t^{sc})}^{t} \xi^T(s)\hat{A}_1^T(r_t^{sc})Q_1^{-1}\hat{A}_1(r_t^{sc})\xi(s)ds \\
&+ \int_{t-\tau_2(r_t^{ca})}^{t} \xi^T(s)\hat{A}_2^T(r_t^{sc})Q_2^{-1}\hat{A}_2(r_t^{sc})\xi(s)ds.
\end{aligned} \tag{11}$$

Similarly,

$$\begin{aligned}
&\mathcal{L}V_2(\Xi(t), r_t^{sc}, r_t^{ca}) \\
&\leq \tau_1(r_t^{sc})\xi^T(t)\hat{A}_1^T(r_t^{sc})Q_1^{-1}\hat{A}_1(r_t^{sc})\xi(t) \\
&+ \tau_2(r_t^{ca})\xi^T(t)\hat{A}_2^T(r_t^{sc})Q_2^{-1}\hat{A}_2(r_t^{sc})\xi(t) \\
&- \int_{t-\tau_1(r_t^{sc})}^{t} \xi^T(s)\hat{A}_1^T(r_t^{sc})Q_1^{-1}\hat{A}_1(r_t^{sc})\xi(s)ds \\
&- \int_{t-\tau_2(r_t^{ca})}^{t} \xi^T(s)\hat{A}_2^T(r_t^{sc})Q_2^{-1}\hat{A}_2(r_t^{sc})\xi(s)ds \\
&+ \bar{\alpha}^{sc}\int_{-\bar{\tau}_1}^{\underline{\tau}_1}\int_{t+\theta}^{t} \xi^T(s)\hat{A}_1^T(r_t^{sc})Q_1^{-1}\hat{A}_1(r_t^{sc})\xi(s)dsd\theta \\
&+ \bar{\alpha}^{ca}\int_{-\bar{\tau}_2}^{-\underline{\tau}_2}\int_{t+\theta}^{t} \xi^T(s)\hat{A}_2^T(r_t^{sc})Q_2^{-1}\hat{A}_2(r_t^{sc})\xi(s)dsd\theta.
\end{aligned} \tag{12}$$

$$\begin{aligned}
&\mathcal{L}V_3(\Xi(t), r_t^{sc}, r_t^{ca}) \\
&= \frac{1}{2}\bar{\alpha}^{sc}(\bar{\tau}_1^2 - \underline{\tau}_1^2)\xi^T(t)\hat{A}_1^T(r_t^{sc})]Q_1^{-1}\hat{A}_1(r_t^{sc})\xi(t) \\
&+ \frac{1}{2}\bar{\alpha}^{ca}(\bar{\tau}_2^2 - \underline{\tau}_2^2)\xi^T(t)\hat{A}_2^T(r_t^{sc})]Q_2^{-1}\hat{A}_2(r_t^{sc})\xi(t) \\
&- \bar{\alpha}^{sc}\int_{-\bar{\tau}_1}^{-\underline{\tau}_1}\int_{t+\theta}^{t} \xi^T(s)\hat{A}_1^T(r_t^{sc})Q_1^{-1}\hat{A}_1(r_t^{sc})\xi(s)dsd\theta \\
&- \bar{\alpha}^{ca}\int_{-\bar{\tau}_2}^{-\underline{\tau}_2}\int_{t+\theta}^{t} \xi^T(s)\hat{A}_2^T(r_t^{sc})Q_2^{-1}\hat{A}_2(r_t^{sc})\xi(s)dsd\theta.
\end{aligned} \tag{13}$$

Combine (11)-(13) and set $\hat{\tau}_1(r_t^{sc}) = \tau_1(r_t^{sc}) + \frac{\bar{\alpha}^{sc}}{2}(\bar{\tau}_1^2 - \underline{\tau}_1^2)$, $\hat{\tau}_2(r_t^{ca}) = \tau_2(r_t^{ca}) + \frac{\bar{\alpha}^{ca}}{2}(\bar{\tau}_2^2 - \underline{\tau}_2^2)$, it results in

$$\begin{aligned}
&\mathcal{L}V(\Xi(t), r_t^{sc}, r_t^{ca}) \\
&\leq \xi^T(t)\Bigg[\hat{A}^T(r_t^{sc})P(r_t^{sc}, r_t^{ca}) + P^T(r_t^{sc}, r_t^{ca})\hat{A}^T(r_t^{sc}) \\
&+ \sum_{h=1}^{N_{sc}} \alpha_{l,h}^{sc} EP(h, r_t^{ca}) + \sum_{j=1}^{N_{ca}} \alpha_{k,j}^{ca} EP(r_t^{sc}, j) \\
&+ \tau_1(r_t^{sc})P^T(r_t^{sc}, r_t^{ca})Q_1P(r_t^{sc}, r_t^{ca}) \\
&+ \tau_2(r_t^{ca})P^T(r_t^{sc}, r_t^{ca})Q_2P(r_t^{sc}, r_t^{ca})\Bigg]\xi(t) \\
&+ \hat{\tau}_1(r_t^{sc})\xi^T(t)\hat{A}_1^T(r_t^{sc})Q_1^{-1}\hat{A}_1(r_t^{sc})\xi(t) \\
&+ \hat{\tau}_2(r_t^{ca})\xi^T(t)\hat{A}_2^T(r_t^{sc})Q_2^{-1}\hat{A}_2(r_t^{sc})\xi(t) \\
&= \xi^T(t)\Theta(r_t^{sc}, r_t^{ca})\xi(t),
\end{aligned} \tag{14}$$

Pre- and post-multiply $\Theta(r_t^{sc}, r_t^{ca})$ by $X^T(r_t^{sc}, r_t^{ca})$ and $X(r_t^{sc}, r_t^{ca})$, it gives

$$\begin{aligned}
0 &> \hat{A}(r_t^{sc})X(r_t^{sc}, r_t^{ca}) + X^T(r_t^{sc}, r_t^{ca})\hat{A}^T(r_t^{sc}) \\
&+ \sum_{h=1}^{N_{sc}} \alpha_{l,h}^{sc} EX^T(h, r_t^{ca}) + \sum_{j=1}^{N_{ca}} \alpha_{k,j}^{ca} EX^T(r_t^{sc}, j) \\
&+ \hat{\tau}_1(r_t^{sc})X^T(r_t^{sc}, r_t^{ca})\hat{A}_1^T(r_t^{sc})Q_1^{-1}\hat{A}_1(r_t^{sc})X(r_t^{sc}, r_t^{ca}) \\
&+ \hat{\tau}_2(r_t^{ca})X^T(r_t^{sc}, r_t^{ca})\hat{A}_2^T(r_t^{sc})Q_2^{-1}\hat{A}_2(r_t^{sc})X(r_t^{sc}, r_t^{ca}).
\end{aligned} \tag{15}$$

Take

$$\begin{aligned}
G(r_t^{sc}, r_t^{ca}) &= A_c(r_t^{sc})X_{12}(r_t^{sc}, r_t^{ca}), \\
H(r_t^{sc}, r_t^{ca}) &= B_C(r_t^{sc})CX_{11}(r_t^{sc}, r_t^{ca}), \\
W(r_t^{sc}, r_t^{ca}) &= BC_c(r_t^{sc}, r_t^{ca})X_{12}(r_t^{sc}, r_t^{ca})
\end{aligned}$$

and apply Schur complement to (15) it results in (8). By simple matrix manipulation, the output-feedback controller is derived in (9).

Since $\max_{\theta\in[-2\tau,0]}\{||\xi(t+\theta)||\} \leq \varphi||\xi(t)||$ for some $\varphi > 0$ by Mahmoud and Al-Muthairi [1984], it has

$$\begin{aligned}
V(\Xi(t), r_t^{sc}, r_t^{ca}) \leq &\Bigg[\lambda_{\max}(EP(r_t^{sc}, r_t^{ca})) + \varphi_1\lambda_{\max}(R_1(r_t^{sc})) \\
&+ \varphi_2\lambda_{\max}(R_2(r_t^{sc}))\Bigg]||\xi(t)||^2 \leq \Lambda_{\max}(r_t^{sc}, r_t^{ca})||\xi(t)||^2,
\end{aligned}$$

where

$$\begin{aligned}
R_1(r_t^{sc}) &= \hat{A}_1^T(r_t^{sc})Q_1^{-1}\hat{A}_1(r_t^{sc}),\ \varphi_1 = \frac{\bar{\tau}_1^2}{2} + \frac{\bar{\alpha}^{sc}}{6}(\bar{\tau}_1^3 - \underline{\tau}_1^3), \\
R_2(r_t^{sc}) &= \hat{A}_2^T(r_t^{sc})Q_2^{-1}\hat{A}_2(r_t^{sc}),\ \varphi_2 = \frac{\bar{\tau}_2^2}{2} + \frac{\bar{\alpha}^{ca}}{6}(\bar{\tau}_2^3 - \underline{\tau}_2^3), \\
\Lambda_{\max}(r_t^{sc}, r_t^{ca}) &= \lambda_{\max}(EP(r_t^{sc}, r_t^{ca})) + \varphi_1\lambda_{\max}(R_1(r_t^{sc})) \\
&+ \varphi_2\lambda_{\max}(R_2(r_t^{sc})).
\end{aligned}$$

Combining with (14), it becomes

$$\begin{aligned}
&\frac{\mathcal{L}V(\Xi(t), r_t^{sc}, r_t^{ca})}{V(\Xi(t), r_t^{sc}, r_t^{ca})} \\
&\leq -\min_{r_t^{sc}\in\mathcal{S}_{sc}, r_t^{ca}\in\mathcal{S}_{ca}}\left\{\frac{\lambda_{\min}(-\Theta(r_t^{sc}, r_t^{ca}))}{\Lambda_{\max}(r_t^{sc}, r_t^{ca})}\right\} \triangleq -\rho_0
\end{aligned}$$

and yields in

$$\mathbb{E}\mathcal{L}V(\Xi(t), r_t^{sc}, r_t^{ca}) \leq -\rho_0\mathbb{E}V(\Xi(t), r_t^{sc}, r_t^{ca}). \tag{16}$$

Applying Dynkin's formula into (16), we have

$$V(\Xi(t), r_t^{\rm sc}, r_t^{\rm ca}) = V_1(\Xi(t), r_t^{\rm sc}, r_t^{\rm ca}) + V_2(\Xi(t), r_t^{\rm sc}, r_t^{\rm ca}) + V_3(\Xi(t), r_t^{\rm sc}, r_t^{\rm ca}), \quad (10)$$

where

$$\begin{aligned}
V_1(\Xi(t), r_t) &= \xi^T(t) E P(r_t^{\rm sc}, r_t^{\rm ca}) \xi(t),\\
V_2(\Xi(t), r_t) &= \int_{-\tau_1(r_t^{\rm sc})}^{0} \int_{t+\theta}^{t} \xi^T(s) \hat{A}_1^T(r_t^{\rm sc}) Q_1^{-1} \hat{A}_1(r_t^{\rm sc}) \xi(s) ds d\theta + \int_{-\tau_2(r_t^{\rm ca})}^{0} \int_{t+\theta}^{t} \xi^T(s) \hat{A}_2^T(r_t^{\rm sc}) Q_2^{-1} \hat{A}_2(r_t^{\rm sc}) \xi(s) ds d\theta,\\
V_3(\Xi(t), r_t) &= \bar{\alpha}^{\rm sc} \int_{-\bar{\tau}_1}^{-\underline{\tau}_1} \int_{t+\theta}^{t} \xi^T(s) \hat{A}_1^T(r_t^{\rm sc}) Q_1^{-1} \hat{A}_1(r_t^{\rm sc}) \xi(s)(s-t-\theta) ds d\theta\\
&\quad + \bar{\alpha}^{\rm ca} \int_{-\bar{\tau}_2}^{-\underline{\tau}_2} \int_{t+\theta}^{t} \xi^T(s) \hat{A}_2^T(r_t^{\rm sc}) Q_2^{-1} \hat{A}_2(r_t^{\rm sc}) \xi(s)(s-t-\theta) ds d\theta.
\end{aligned}$$

$$\begin{aligned}
&\mathbb{E}V(\Xi(t), r_t^{\rm sc}, r_t^{\rm ca}) - \mathbb{E}V(\Xi(0), r_0^{\rm sc}, r_0^{\rm ca})\\
&= \mathbb{E}\left[\int_0^t \mathcal{L}V(\Xi(s), r_s^{\rm sc}, r_s^{\rm ca}) ds\right]\\
&\le -\rho_0 \int_0^t \mathbb{E}\mathcal{L}V(\Xi(s), r_s^{\rm sc}, r_s^{\rm ca}) ds.
\end{aligned} \quad (17)$$

Using the Gronwall-Bellman lemma, (17) results in

$$\mathbb{E}V(\Xi(t), r_t^{\rm sc}, r_t^{\rm ca}) \le e^{-\rho_0 t} \mathbb{E}V(\Xi(0), r_0^{\rm sc}, r_0^{\rm ca}).$$

Since

$$V(\Xi(t), r_t^{\rm sc}, r_t^{\rm ca}) \ge \Big[\lambda_{\min}(EP(r_t^{\rm sc}, r_t^{\rm ca})) + \varphi_1 \lambda_{\max}(R_1(r_t^{\rm sc})) + \varphi_2 \lambda_{\max}(R_2(r_t^{\rm sc})\Big] ||\xi(t)||^2 = \Lambda_{\min}(r_t^{\rm sc}, r_t^{\rm ca}) ||\xi(t)||^2,$$

it it becomes

$$\mathbb{E}||\xi(t)||^2 \le \frac{e^{-\rho_0 t} \mathbb{E}V(\Xi(0), r_0^{\rm sc}, r_0^{\rm ca})}{\min_{r_t^{\rm sc} \in \mathcal{S}_{\rm sc}, r_t^{\rm ca} \in \mathcal{S}_{\rm ca}} \{\Lambda_{\min}(r_t^{\rm sc}, r_t^{\rm ca})\}}. \quad (18)$$

Equation (18) implies exponential mean square stability and completes the proof. ■

Remark 1. The delays $\tau_1(t, r_t^{\rm sc})$ and $\tau_2(t, r_t^{\rm ca})$ contain the transmission delays and the time-varying component bounded by the corresponding sampling intervals, see (2) and (4). Accordingly, the transmission delay as well as the sampling rate are conjointly treated by a single stability condition in Theorem 1. The solution of Theorem 1 indicates the trade-off between the sampling intervals h_1, h_2 and transmission delays $\tau_{\rm sc}(r_t^{\rm sc})$, $\tau_{\rm ca}(r_t^{\rm ca})$ whereby the exponential mean square stability is guaranteed.

Remark 2. In case of the constant transmission delay, i.e. $\tau_{\rm sc}(r_t^{\rm sc}) = \tau_{\rm sc}$ and $\tau_{\rm ca}(r_t^{\rm ca}) = \tau_{\rm ca}$, Theorem 1 reduces to the delay-dependent stability and controller design condition for systems with input delay.

Remark 3. The switching output-feedback controller is obtained in an LMI by the diagonal requirement of $X_1(r_t^{\rm sc}, r_t^{\rm ca})$. This setting comes with some conservatism. Theorem 1 can be applied to certain restricted unstable plants, e.g. two-mode input delay MJLS with one unstable subsystem. A similar example can be found in Boukas and Liu [2002]. However, in the example the delay is constant and appears only in the state. A more feasible LMI problem setting for general unstable plants remains still open and belongs to future research.

4. NUMERICAL EXAMPLE

To illustrate the efficacy of the proposed approach, a numerical example with switching output-feedback controller design is given in this section.

Example 1. Consider a LTI plant:

$$\dot{x} = -1.3x + u,$$
$$y = x,$$

which is interconnected with an output-feedback controller of the form (3) through a communication network. Assume the plant and the controller have the same sampling intervals $h_1 = h_2 = 15$ ms, the network has the set of transmission delays: $\tau_{\rm sc}(r_t^{\rm sc}) = \{15, 25\}$ ms in the SC channel and $\tau_{\rm ca}(r_t^{\rm ca}) = \{5, 15\}$ ms in the CA channel with mode transition generator

$$\mathcal{A}_{\rm sc} = \begin{bmatrix} -3 & 3 \\ 2 & -2 \end{bmatrix}, \quad \mathcal{A}_{\rm ca} = \begin{bmatrix} -3 & 3 \\ 1 & -1 \end{bmatrix}.$$

The resulting delays, i.e. the sum of transmission delays and sampling intervals, are $\tau_1(r_t^{\rm sc}) = \{30, 40\}$ ms and $\tau_2(r_t^{\rm ca}) = \{20, 30\}$ ms. Solving Theorem 1 by YALMIP toolbox in Matlab, the feasible output-feedback controllers associated with SC delays are

$$A_c(1) = -9.6121, \ B_c(1) = -3.9316, \ C_c(1) = -0.0852$$

for $\tau_{\rm sc}(1) = 15$ ms and

$$Ac(2) = -5.3656, \ Bc(2) = -2.8489, \ Cc(2) = -0.4981$$

for $\tau_{\rm sc}(2) = 25$ ms.

The closed-loop system is simulated with 100 random initial distribution probabilities of SC and CA delays using the `dde23` solver from MATLAB. One of the 100 sample paths for the SC and CA delays is shown in Fig. 3 (a). The initial condition of the closed-loop system is given by $x_0 = 1$, $-\bar{\tau}_1 - \bar{\tau}_2 < t < 0$. The output-feedback controller is synchronously switched with transmission delay $\tau_{\rm sc}(l)$, $l = 1, 2$. The mean state trajectory of switching output-feedback controller as well as the standard design with buffering at the controller side, i.e. holding the SC delay constant by $\tau_{\rm sc}(2)$, are presented in Fig. 3 (b). The mean state trajectory of switching output-feedback controller converges exponentially to a ball bounded by the radius $||x|| = 0.01$ after $t_{0.01} = 4.14$ s and $t_{0.01} = 4.75$ s for the standard design with buffering. Clearly, the switching output-feedback controller has superior performance over standard design approach. It can be seen that the proposed approach has the performance benefit even when the Markovian delay has only two modes, i.e. the total delay difference is 10 ms. If the SC delay has $N_{\rm sc} > 2$ modes, the delay difference is higher. The benefit is expected to be more obvious.

Open problems that will be addressed in the future research includes: i) the implementation of delay-dependent switched controller over real communication network using

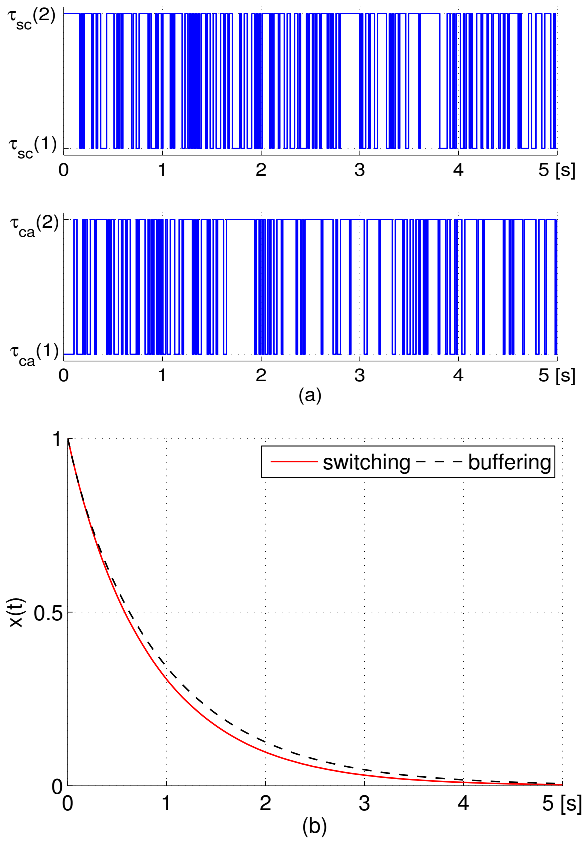

Fig. 3. The sample path of SC and CA delays (a) and the mean state trajectory of switching controller (solid); for comparison, controller with buffering SC delay (dashed).

time-stamping technique. This requires precise synchronization between sensors and controller; ii) the packet dropout results from sampled sequence disorder and network transmission.

5. CONCLUSION

Motivated by the random transmission delay in networked control systems (NCS) this paper concerns a novel control approach towards Markovian jump linear systems with random input delays and gives a sufficient stability condition and controller design method. Exponential mean square stability is guaranteed for independently random transmission delays from sensor-to-controller (SC) and controller-to-actuator (CA) using a Lyapunov-Krasovskii functional. A delay-dependent output-feedback controller is proposed and the performance benefit over the standard buffering approach is demonstrated. The example shows the proposed control approach is very promising for future NCS applications.

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