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LMI based stabilization of a class of switching LPV systems

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Abstract: The stabilization problem of a class of discrete-time LPV systems is considered. The plant switches among different operating conditions and, as long as the process operates in a fixed mode, the physical parameters are varying inside a known compact set. The possibility of noisy parameter measurements is taken into account, the measure and the control matrices corresponding to each mode are assumed to be known and time-invariant. The problem solvability conditions are stated in terms of feasibility of a set of LMIs, and the closed-loop stability is proved assuming a sufficiently long permanence of each mode.

1. INTRODUCTION

This paper considers the stabilization problem of a discrete-time LPV plant with the following mode-switch dynamics. A limited number of different operating conditions are possible and, as long as the process operates under a fixed mode, its dynamical matrix depends on parameters which are varying inside a known compact set. From time to time a sudden change of the operating condition occurs and the physical parameters describing the process dynamics suddenly switch to another compact set relative to the new situation. The bounds of the compact sets containing the physical parameters and the minimum permanence interval of each mode are usually available. The physical parameters are not known "a priori" but are assumed to be observed in real time. In general, the actuator and sensor equipments used in all the operating conditions are known and time-invariant as long as the plant operates in the same mode. For situations of this kind see e.g. Ippoliti et al. (2005), Jetto and Orsini (2006) and references therein. A similar class of systems has been also considered in Apkarian et al. (1995), Blanchini and Miani (2003).

The large amount of results for LPV systems can be classified making reference to the class of systems, to the type of Lyapunov function, to the kind of control algorithm. The gain-scheduling approach proposed in Becker and Packard (1994),Packard (1994),Apkarian and Gahinet (1995),Apkarian et al. (1995), Scorletti and Ghaoui (1998), gives computationally simple methods, but a parameter-independent Lyapunov function is used, so that conservative results can be obtained. More general parameter dependent Lyapunov functions have been exploited in Feron et al. (1996), Gahinet et al. (1996),Scherer (1996),Yu and Sideris (1997),Apkarian and R.J.Adams (1998),Wang and Balakrishnan (2002), Souza et al. (2003),Souza and Trofino (2004), to derive $\mathcal{H}_2$ and $\mathcal{H}_\infty$ gain scheduling controllers. Switching control of LPV systems using multiple-parameter dependent Lyapunov functions has been considered in Lu and Wu (2004). Model predictive controllers have been proposed in Casavola et al. (2002), Park and Jeong (2004). Stabilizability problems for LPV systems with switching and/or polytopic uncertainties have been considered in Blanchini et al. (2007), Lee (2007). The most part of the above articles assume an affine dependence of the system matrices on the physical parameters Apkarian et al. (1995), Feron et al. (1996), Gahinet et al. (1996), Casavola et al. (2002), Blanchini and Miani (2003), Souza et al. (2003), Souza and Trofino (2004), Park and Jeong (2004) or a LFT structure, Packard (1994), Apkarian and Gahinet (1995), Apkarian and R.J.Adams (1998), Scorletti and Ghaoui (1998).

Given the precedent literature, this paper has the three following salient features: i) a family of parameter-scheduled, observer based controllers is designed. Each controller robustly stabilizes a fixed mode using constant observer and regulator gains obtained by a set of LMIs. This greatly reduces the computational burden of the control algorithm; ii) the only assumption on the parameter dependence is the uniform boundedness; iii) the possibility of noisy parameter measurements is taken into account.

The overall control algorithm is given by a switching law driven by a supervisor whose task is to choose the appropriate controller according to the parameter measures. The problem solvability conditions are stated in terms of feasibility of a set of LMIs, and the closed-loop stability is proved assuming that each mode is kept for a sufficiently long time interval.

The paper is organized in the following way. Some basic notations and the problem statement are provided in Section 2, the synthesis procedure of the controller family is reported in Sections 3 and 4, the extension to noisy parameter measurements is considered in Section 5. The stability conditions are stated in Section 6. A numerical example and concluding remarks end the paper.

2. NOTATION AND PROBLEM STATEMENT

For any square matrix M, the symbols $\|M\|$, $\|M\|_2$ and $\lambda_i\{M\}$ denote any generic norm, the spectral norm and the eigenvalues of M respectively. Denoting by $m_{i,j}$, the generic element of M, the matrix norms $\|M\|_\infty$

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and $\|M\|_1$ are defined as $\max_{1\le i\le n}\sum_{j=1}^{n}|m_{i,j}| \triangleq \|M\|_\infty$ and $\max_{1\le j\le n}\sum_{i=1}^{n}|m_{i,j}| \triangleq \|M\|_1$ respectively. The notation $M \ge 0$ ($M \le 0$) means that M is positive (negative) semidefinite, while $M > 0$ ($M < 0$) means that M is positive (negative) definite. Given two $(n \times m)$ matrices M and N, the notation $M \preceq N$ ($M \succeq N$) means $m_{i,j} \le n_{i,j}$ ($m_{i,j} \ge n_{i,j}$), $i = 1, \cdots, n$, $j = 1, \cdots, m$. If the symbol $\prec$, ($\succ$), is used, the strictly inequality holds. The notation $M \in [M^-, M^+]$ means that M is an interval matrix satisfying $M^- \preceq M \preceq M^+$. The matrix $\bar{M}$ is defined by the elements $\bar{m}_{i,j} = \max\{|m^-_{i,j}|, |m^+_{i,j}|\}$, $i = 1, \cdots, n; j = 1, \cdots, m$. Clearly one has $M \preceq \bar{M}$ and if $M^- \succeq 0_{n,m}$, then $\bar{M} = M^+$. A time-varying matrix $M(\cdot)$ such that $M(\cdot) \in [M^-, M^+]$, is called an interval time-varying (ITV) matrix.

Consider the following discrete-time LPV system Σ

$$x(k+1) = A(\theta(k))x(k) + B(k)u(k), \tag{1}$$

$$y(k) = C(k)x(k), \tag{2}$$

where: $u(\cdot) \in \mathbb{R}^m$ is the control input, $x(\cdot) \in \mathbb{R}^n$ is the state, $y(\cdot) \in \mathbb{R}^q$ is the output, $\theta(\cdot) = [\theta_1(\cdot), \cdots, \theta_p(\cdot)]^T$ is a vector of "a priori" unknown time varying parameters which are assumed to be measurable in real time. It is also assumed that A1): there exists an infinite increasing sequence S of integers $\{k_m\}$, $k_0 = 0$, such that $0 < \tau \le k_{m+1} - k_m < \infty$, for some $\tau \in \mathbb{Z}^+$, $\forall m \in \mathbb{Z}^+$, and over each time interval $I'_m \triangleq [k_m, k_{m+1})$ one has $\theta(k) = \theta_\ell(k) = [\theta_{\ell,1}(k), \cdots, \theta_{\ell,p}(k)]^T \in \Theta_\ell$, $\forall k \in I'_m$ and for some fixed ℓ in the range $[1, \cdots, \tilde{\ell}]$, $\tilde{\ell} < \infty$; A2) Θ_ℓ is the hyperbox containing all the vectors $\theta_\ell(\cdot)$ such that $\theta_{\ell,i}(\cdot) \in [\theta^-_{\ell,i}, \theta^+_{\ell,i}]$, $i = 1, \cdots, p$, A3): the vectors $\theta^-_\ell \triangleq [\theta^-_{\ell,1}, \cdots, \theta^-_{\ell,p}]^T$, and $\theta^+_\ell \triangleq [\theta^+_{\ell,1}, \cdots, \theta^+_{\ell,p}]^T$ are "a priori" known, A5): the elements $a^{(\ell)}_{i,j}(\theta_\ell(\cdot))$ of $A_\ell(\theta_\ell(\cdot))$ are uniformly bounded functions of θ_ℓ, $\forall \theta_\ell \in \Theta_\ell$, A6): $B(k)$ and $C(k)$ are "a priori" known matrices such that $B(k) = B_\ell, C(k) = C_\ell$, $\forall k \in [k_m, k_{m+1}), \ell = 1, \cdots, \tilde{\ell}, m \in \mathbb{Z}^+$.

From the above assumptions it follows that Σ can be viewed as a time varying system with mode switch dynamics, each mode being described by the triplet $\Sigma_\ell \equiv (C_\ell, A_\ell(\theta_\ell(k)), B_\ell)$, $\ell = 1, \cdots, \tilde{\ell}$, and $A_\ell(\theta_\ell(\cdot))$ is an ITV matrix such that $A_\ell(\theta_\ell(\cdot)) \in I_{A_\ell} \triangleq [A^-_\ell, A^+_\ell]$ for suitably defined A^-_ℓ and A^+_ℓ. It follows that to each $A_\ell(\theta_\ell(k))$ the corresponding extremal matrix $\bar{A}_\ell$ can be associated. The following final assumption is now introduced: A7): each triplet $\bar{\Sigma}_\ell \equiv (C_\ell, \bar{A}_\ell, B_\ell)$, $\ell = 1, \cdots, \tilde{\ell}$, is controllable and observable. By now on, for simplicity of notation, the explicit dependence of the time-varying $A_\ell(\theta_\ell(\cdot))$ and of its elements on the subscript ℓ which identifies Θ_ℓ will be omitted. Hence, in the following, the dynamical matrix and its elements will be denoted by $A(\theta_\ell(\cdot))$ and $a_{i,j}(\theta_\ell(\cdot))$ respectively. The same simplified notation will be also adopted for the closed loop time varying dynamical matrices.

System Σ_ℓ is said uniformly, exponentially, γ_ℓ-stable if its state transition matrix

$$\|\Phi_\ell(k, \bar{k})\| = \|A(\vartheta_\ell(k-1))...A(\vartheta_\ell(\bar{k}))\|,$$

is such that,

$$\|\Phi_\ell(k, \bar{k})\| \le m_\ell \gamma_\ell^{(k-\bar{k})},$$

$\forall k, \bar{k} \in \mathbb{Z}^+, k \ge \bar{k}$, for some $m_\ell > 0$ and $0 < \gamma_\ell < 1$.

The stabilization problem considered consists in finding (if it exists) a dynamic output controller Σ_c, scheduled by the parameter measurements, yielding an uniformly exponentially stable closed-loop system Σ_f. The solution proposed is given by the connection of a family $\mathcal{F}$ of time-varying controllers $\Sigma_{c,\ell}$, $\ell = 1, \cdots, \tilde{\ell}$, with a switching policy inside $\mathcal{F}$. Each $\Sigma_{c,\ell}$ has an observer based form where the observer and the feedback gains are predetermined off-line on the basis of the extremal plants $\bar{\Sigma}_\ell, \ell = 1, \cdots, \tilde{\ell}$. LMI conditions are given for each $\Sigma_{c,\ell}$ to stabilize the corresponding triplet $(C_\ell, A(\theta_\ell(\cdot)), B_\ell)$, $\forall \theta_\ell(\cdot) \in \Theta_\ell$. This allows each $\Sigma_{c,\ell}$ to be kept acting as long as $\theta(\cdot) \in \Theta_\ell$. The switching inside $\mathcal{F}$ is driven by the current parameter measurements and the closed-loop stability is proved under the assumption that each I'_m be sufficiently long.

3. THE CONTROLLER DESIGN PROCEDURE

The following preliminary result is recalled Orsini (2006).

Lemma *If*

$$|\lambda_i\{\bar{A}_\ell\}| < \gamma_\ell < 1, \quad i = 1, \cdots, n, \tag{3}$$

then the corresponding Σ_ℓ is uniformly exponentially γ_ℓ-stable independently of the way the elements $a_{i,j}(\theta_\ell(\cdot))$ of $A(\theta_\ell(\cdot))$ vary inside their respective intervals. Moreover $\|\Phi_\ell(k, \bar{k})\|_2 \le \bar{m}_{2,\ell}\gamma_\ell^{(k-\bar{k})}$, $\forall k, \bar{k} \in \mathbb{Z}^+, k \ge \bar{k}$, with $\bar{m}_{2,\ell} = (\bar{m}_{\infty,\ell}\bar{m}_{1,\ell})^{\frac{1}{2}}$, where $\bar{m}_{1,\ell}$ and $\bar{m}_{\infty,\ell}$ are positive constants such that $\|\bar{A}^k_\ell\|_1 \le \bar{m}_{1,\ell}\gamma^k_\ell, \|\bar{A}^k_\ell\|_\infty \le \bar{m}_{\infty,\ell}\gamma^k_\ell$.

With reference to $\Sigma \equiv \Sigma_\ell$, consider the following observer-like based controller $\Sigma_{c,\ell}$

$$z_\ell(k+1) = (A(\theta_\ell(k)) + L_\ell C_\ell)z_\ell(k) + B_\ell u_\ell(k) - L_\ell y_\ell(k) \tag{4}$$

$$u_\ell(k) = K_\ell z_\ell(k), \tag{5}$$

where $z_\ell(\cdot) \in \mathbb{R}^n$ is the state of $\Sigma_{c,\ell}$. The feedback connection $\Sigma_{f,\ell}$ of $\Sigma_{c,\ell}$ with the ITV plant $\Sigma_\ell \equiv (C_\ell, A(\theta_\ell(k)), B_\ell)$ is described by the pair $(C_{f,\ell}, A_f(\theta_\ell(k)))$, with

$$A_f(\theta_\ell(k)) = \begin{bmatrix} A(\theta_\ell(k)) & B_\ell K_\ell \\ -L_\ell C_\ell & A(\theta_\ell(k)) + L_\ell C_\ell + B_\ell K_\ell \end{bmatrix}, \tag{6}$$

$$C_{f,\ell} = [\, C_\ell \;\; 0_{q,n} \,]. \tag{7}$$

The state transition matrix of $\Sigma_{f,\ell}$ is denoted by $\Phi_{f,\ell}(\cdot,\cdot)$. Applying the transformation matrix $T = \begin{bmatrix} I_n & 0_n \\ I_n & -I_n \end{bmatrix}$, one has $\Sigma_{f,\ell} \equiv (\hat{C}_{f,\ell}, \hat{A}_f(\theta_\ell(k)))$ with

$$\hat{A}_f(\theta_\ell(k)) = \begin{bmatrix} A(\theta_\ell(k)) + B_\ell K_\ell & -B_\ell K_\ell \\ 0_n & A(\theta_\ell(k)) + L_\ell C_\ell \end{bmatrix}, \tag{8}$$

$$\hat{C}_{f,\ell} = [\, C_\ell \;\; 0_{q,n} \,]. \tag{9}$$

The gain matrices K_ℓ and L_ℓ of $\Sigma_{c,\ell}$ are designed to assign the desired eigenvalues $\lambda_i\{\bar{A}_\ell + B_\ell K_\ell\} \cup \lambda_i\{\bar{A}_\ell + L_\ell C_\ell\}$, $i = 1, \cdots, n$, to the following extreme matrix

$$\hat{\hat{A}}_{f,\ell} = \begin{bmatrix} \bar{A}_\ell + B_\ell K_\ell & -B_\ell K_\ell \\ 0_n & \bar{A}_\ell + L_\ell C_\ell \end{bmatrix}. \quad (10)$$

This is surely possibly by A7). Let $\rho_{1,\ell}$ and $\rho_{2,\ell}$ be two arbitrarily fixed scalars such that $0 \leq \rho_{2,\ell} \leq \rho_{1,\ell} \leq 1$ and let $\rho_\ell = \max\{\rho_{1,\ell}, \rho_{2,\ell}\}$. The following theorem holds.

Theorem *There exists a controller $\Sigma_{c,\ell}$ given by (4) and (5) such that $\Sigma_{f,\ell}$ is λ_ℓ exponentially stable (for some $\lambda_\ell < \rho_\ell$) if there exist two matrices $U_{1,\ell}$ and $U_{2,\ell}$ and two diagonal matrices $S_{1,\ell} \succ 0_n$ and $S_{2,\ell} \succ 0_n$, such that the following LMIs are satisfied,*

$$\begin{bmatrix} S_{1,\ell} & \frac{1}{\rho_{1,\ell}}(\bar{A}_\ell S_{1,\ell} + B_\ell U_{1,\ell})^T \\ \frac{1}{\rho_{1,\ell}}(\bar{A}_\ell S_{1,\ell} + B_\ell U_{1,\ell}) & S_{1,\ell} \end{bmatrix} > 0 \quad (11)$$

$$\begin{bmatrix} S_{2,\ell} & \frac{1}{\rho_{2,\ell}}(\bar{A}_\ell^T S_{2,\ell} + C_\ell^T U_{2,\ell})^T \\ \frac{1}{\rho_{2,\ell}}(\bar{A}_\ell^T S_{2,\ell} + C_\ell^T U_{2,\ell}) & S_{2,\ell} \end{bmatrix} > 0 \quad (12)$$

$$\bar{A}_\ell S_{1,\ell} + B_\ell U_{1,\ell} \succeq 0_n, \quad \bar{A}_\ell^T S_{2,\ell} + C_\ell^T U_{2,\ell} \succeq 0_n,$$
$$-B_\ell U_{1,\ell} \succeq 0_n. \quad (13)$$

$$-\bar{A}_\ell S_{1,\ell} - 2B_\ell U_{1,\ell} - A_\ell^- S_{1,\ell} \preceq 0_n, \quad (14)$$
$$-\bar{A}_\ell^T S_{2,\ell} - 2C_\ell^T U_{2,\ell} - A_\ell^{-T} S_{2,\ell} \preceq 0_n, \quad (15)$$

The gain matrices K_ℓ and L_ℓ of $\Sigma_{c,\ell}$ are given by

$$K_\ell = U_{1,\ell} S_{1,\ell}^{-1}, \quad L_\ell = (U_{2,\ell} S_{2,\ell}^{-1})^T. \quad (16)$$

Proof Putting $U_{1,\ell} S_{1,\ell}^{-1} \stackrel{\triangle}{=} K_\ell$ and applying the congruence transformation $W_{1,\ell} = \mathrm{diag}[S_{1,\ell}^{-1}, S_{1,\ell}^{-1}]$, condition (11) can be rewritten as

$$\begin{bmatrix} S_{1,\ell}^{-1} & \frac{1}{\rho_{1,\ell}}(\bar{A}_\ell + B_\ell K_\ell)^T S_{1,\ell}^{-1} \\ \frac{1}{\rho_{1,\ell}} S_{1,\ell}^{-1}(\bar{A}_\ell + B_\ell K_\ell) & S_{1,\ell}^{-1} \end{bmatrix} > 0, \quad (17)$$

using the Schur complement and putting $S_{1,\ell}^{-1} \stackrel{\triangle}{=} P_{1,\ell}$, one has

$$P_{1,\ell} - \frac{1}{\rho_{1,\ell}^2}\left(\bar{A}_\ell + B_\ell K_\ell\right)^T P_{1,\ell}\left(\bar{A}_\ell + B_\ell K_\ell\right) > 0. \quad (18)$$

As $P_{1,\ell} > 0$, condition (18) means that $|\lambda_i\{\bar{A}_\ell + B_\ell K_\ell\}| < \rho_{1,\ell} \leq 1$. Moreover, as $S_{1,\ell}$ is diagonal and strictly positive and $U_{1,\ell} S_{1,\ell}^{-1} = K_\ell$, the first of conditions (13) implies $\bar{A}_\ell + B_\ell K_\ell \succeq 0_n$.

Putting $U_{2,\ell} S_{2,\ell}^{-1} \stackrel{\triangle}{=} L_\ell^T$ and arguing as before, it follows that (12) and the second of conditions (13) imply $|\lambda_i\{\bar{A}_\ell + L_\ell C_\ell\}| < \rho_{2,\ell} \leq 1$ and $\bar{A}_\ell + L_\ell C_\ell \succeq 0_n$, respectively. The third of conditions (13) implies $-B_\ell K_\ell \succeq 0_n$ because $S_{1,\ell}$ is diagonal and strictly positive. By (10) it follows that (11)-(13) and (16) give

$$|\lambda_i\{\hat{\hat{A}}_{f,\ell}\}| < \rho_\ell, \quad \hat{\hat{A}}_{f,\ell} \succeq 0_{2n}. \quad (19)$$

Moreover, by (14) and (15) one has

$$|A(\theta_\ell(k)) + B_\ell K_\ell| \preceq \bar{A}_\ell + B_\ell K_\ell,$$
$$|A(\theta_\ell(k)) + L_\ell C_\ell| \preceq \bar{A}_\ell + L_\ell C_\ell,$$
$$\forall A(\theta_\ell(k)) \in [A_\ell^-, A_\ell^+], \ k \in \mathbb{Z}^+.$$

Hence, by (8) and (10) one has: $|\hat{A}_f(\theta_\ell(k))| \preceq \hat{\hat{A}}_{f,\ell}$, $\forall A(\theta_\ell(k)) \in [A_\ell^-, A_\ell^+]$, $k \in \mathbb{Z}^+$. By lemma and (19), the uniform exponential λ_ℓ-stability (for some $0 < \lambda_\ell < \rho_\ell$) of $\hat{A}_f(\theta_\ell(k))$ follows from the analogous property of $\hat{\hat{A}}_{f,\ell}$, and the uniform, exponential λ_ℓ-stability of $A_f(\theta_\ell(k))$ follows from $A_f(\theta_\ell(k)) = T^{-1}\hat{A}_f(\theta_\ell(k))T$. △

The requirement that $S_{1,\ell} = P_{1,\ell}^{-1}$ and $S_{2,\ell} = P_{2,\ell}^{-1}$ be diagonal is not restrictive. In fact if $\bar{A}_\ell + B_\ell K_\ell \succeq 0_n$, then $|\lambda_i\{\bar{A}_\ell + B_\ell K_\ell\}| < \rho_{1,\ell} \leq 1$, if only if the matrix $P_{1,\ell}$ satisfying (18) is diagonal, L.Farina and Rinaldi (2000). An analogous consideration holds for $S_{2,\ell} = P_{2,\ell}^{-1}$.

By the theorem one has $\|\hat{\Phi}_{f,\ell}(k,\bar{k})\| \stackrel{\triangle}{=} \|\hat{A}_f(\theta_\ell(k-1))\cdots\hat{A}_f(\theta_\ell(\bar{k}))\| \leq \hat{m}_{f,\ell}\lambda_\ell^{(k-\bar{k})}$, where $\hat{m}_{f,\ell}$ is such that $\|(\hat{\hat{A}}_{f,\ell})^k\| \leq \hat{m}_{f,\ell}\lambda_\ell^k$, and $\|\Phi_{f,\ell}(k,\bar{k})\| = \|A_f(\theta_\ell(k-1))\cdots A_f(\theta_\ell(\bar{k}))\| \leq m_{f,\ell}\lambda_\ell^{(k-\bar{k})}$, where $m_{f,\ell} \leq \|T\|\|T^{-1}\|\,\hat{m}_{f,\ell}$.

The use of two different scalars $\rho_{1,\ell}$ and $\rho_{2,\ell}$ in (11) and (12) introduces more flexibility in the synthesis procedure. For example if $\rho_{2,\ell} < \rho_{1,\ell}$, an observer dynamics faster than the feedback compensator dynamics is obtained. If the values $\rho_{1,\ell} = \rho_{2,\ell} = 1$, are chosen, the assumption of a reachable and observable $\bar{\Sigma}_\ell \equiv (C_\ell, \bar{A}_\ell, B_\ell)$ can be relaxed to that of input-output stabilizability.

The above theorem implies that if conditions (11)-(15) are satisfied, then the controller $\Sigma_{c,\ell}$ not only stabilizes all the ITV matrices $A(\theta_\ell(\cdot)) \in [A_\ell^-, A_\ell^+]$, but the wider class of ITV matrices $A(\theta_\ell(\cdot))$ such that: $|A(\theta_\ell(\cdot)) + B_\ell K_\ell| \preceq \bar{A}_\ell + B_\ell K_\ell$, and $|A(\theta_\ell(\cdot)) + L_\ell C_\ell| \preceq \bar{A}_\ell + L_\ell C_\ell$, or equivalently: $-\bar{A}_\ell - 2B_\ell K_\ell \preceq A(\theta_\ell(\cdot)) \preceq \bar{A}_\ell$ and $-\bar{A}_\ell - 2L_\ell C_\ell \preceq A(\theta_\ell(\cdot)) \preceq \bar{A}_\ell$.

Once a family $\mathcal{F}$ of stabilizing pairs (K_ℓ, L_ℓ) has been computed off-line (if any), the supervisor drives the switching inside $\mathcal{F}$ according to the parameter measurements and no extra calculation has to be performed on line to implement the control algorithm given by (4) and (5).

4. THE "POSITIVIZABILITY" NOTION

A limit of the design procedure given in the previous section is that condition (14) of the theorem can not be satisfied if $\bar{A}_\ell$ is unstable and if the ITV matrix $A(\theta_\ell(\cdot)) \in [A_\ell^-, A_\ell^+]$ is negative, namely if $A_\ell^+ \preceq 0_n$. As shown in the proof of the theorem, condition (14) implies $|A(\theta_\ell(\cdot)) + B_\ell K_\ell| \preceq \bar{A}_\ell + B_\ell K_\ell$, $\forall A(\theta_\ell(\cdot)) \in [A_\ell^-, A_\ell^+]$, whence

$$-\bar{A}_\ell - 2B_\ell K_\ell \preceq A_\ell^-. \quad (20)$$

If $A_\ell^+ \preceq 0_n$, then $\bar{A}_\ell = -A_\ell^-$, and condition (20) can be rewritten as

$$A_\ell^- - 2B_\ell K_\ell \preceq A_\ell^-,$$

which can not be satisfied because, as shown in the proof of the theorem, the third of condition (13) implies $-B_\ell K_\ell \succeq 0_n$ and $B_\ell K_\ell \neq 0$ by the instability of $\bar{A}_\ell$.

To overcome this limit, the notion of "output positivizable system" is introduced here. Given a negative ITV matrix

$A(\theta_\ell(\cdot))$, the system $\Sigma_\ell \equiv (C_\ell, A(\theta_\ell(\cdot)), B_\ell)$ is said output positivizable if there exists a (possibly null) matrix G_ℓ such that

$$A(\theta_\ell(\cdot)) + B_\ell G_\ell C_\ell \succ 0_n, \quad \forall A(\theta_\ell(\cdot)) \in [A_\ell^-, A_\ell^+]. \tag{21}$$

It is clear that condition (21) can be satisfied if and only if there exists a matrix G_ℓ solution of the following LMI

$$A_\ell^- + B_\ell G_\ell C_\ell \succ 0_n. \tag{22}$$

If such a matrix G_ℓ exists, it can be seen as an internal static output gain giving the following "output positivized system" $\Sigma_{p,\ell} \equiv (C_\ell, A_p(\theta_\ell(\cdot)), B_\ell)$, where $A_p(\theta_\ell(\cdot)) = A(\theta_\ell(\cdot)) + B_\ell G_\ell C_\ell$. The new extremal matrix is given by $\bar{A}_{p,\ell} = A_\ell^+ + B_\ell G_\ell C_\ell$ and the corresponding extremal plant $\bar{\Sigma}_{p,\ell} \equiv (C_\ell, \bar{A}_{p,\ell}, B_\ell)$ is reachable and observable by assumption A7).

In conclusion, for systems $\Sigma_\ell \equiv (C_\ell, A(\theta_\ell(\cdot)), B_\ell)$ with a negative ITV matrix $A(\theta_\ell(\cdot))$, the design procedure of the stabilizing $\Sigma_{c,\ell}$ (if any) consists of the two following steps: 1) find an internal static output feedback G_ℓ solving the LMI (22), 2) apply the same design procedure of the previous section to the positivized system $\Sigma_{p,\ell}$.

5. NOISY PARAMETER MEASUREMENTS

Assume that some parameter vectors $\theta_\ell(\cdot)$, for some $\ell \in [1, \cdot, \tilde{\ell}]$, are measured according to,

$$\tilde{\theta}_\ell(\cdot) = \theta_\ell(\cdot) + v_\ell(\cdot), \tag{23}$$

where the unknown observation noise $v_\ell(\cdot) = [v_{\ell,1}(\cdot), \cdots, v_{\ell,p}(\cdot)]^T$ is such that $|v_{\ell,i}(\cdot)| \leq \bar{v}_\ell$, $i = 1, \cdots, p$. It is also assumed that A8): $[\theta_{\ell,i}^- - \bar{v}_\ell, \theta_{\ell,i}^+ + \bar{v}_\ell] \cap [\theta_{m,i}^- - \bar{v}_m, \theta_{m,i}^+ + \bar{v}_m] = \emptyset$, for at least one value of $i \in [1, \cdots, p]$, and $1 \leq \ell, m \leq \tilde{\ell}, \ell \neq m$, A9) the elements $a_{i,j}(\tilde{\theta}_\ell(\cdot))$ of the dynamical matrix $A(\tilde{\theta}_\ell(\cdot))$ corresponding to the noisy measures are uniformly bounded functions of $\tilde{\theta}_\ell(\cdot)$, $\forall \tilde{\theta}_\ell(\cdot) \in \tilde{\Theta}_\ell \supseteq \Theta_\ell$, where $\tilde{\Theta}_\ell$ is the hyperbox containing all the vectors $\tilde{\theta}_\ell(\cdot)$ such that $\tilde{\theta}_{\ell,i}(\cdot) \in [\theta_{\ell,i}^- - \bar{v}_\ell, \theta_{\ell,i}^+ + \bar{v}_\ell]$, $i = 1, \cdots, p$. Assumption A8) guarantees that each mode can be identified by the supervisor without ambiguity, A9) implies that $A(\tilde{\theta}_\ell(\cdot))$ is an ITV matrix such that $A(\tilde{\theta}_\ell(\cdot)) \in \tilde{I}_{A_\ell} \triangleq [\tilde{A}_\ell^-, \tilde{A}_\ell^+]$, with $\tilde{I}_{A_\ell} \supseteq I_{A_\ell}$, because its elements $a_{i,j}(\tilde{\theta}_\ell(\cdot))$ vary over $\tilde{\Theta}_\ell \supseteq \Theta_\ell$. Denoting by $\bar{A}'_\ell$ the extremal matrix of $A(\tilde{\theta}_\ell(\cdot))$, it follows that $\bar{A}'_\ell \succeq \bar{A}_\ell$.

As the controller is scheduled by the measured parameters, matrix $A(\theta_\ell(\cdot))$ must be replaced by $A(\tilde{\theta}_\ell(\cdot))$ in equation (4). Arguing as in the Section 3, it is easily seen that matrices $A_f(\theta_\ell(\cdot))$ and $\hat{A}_f(\theta_\ell(\cdot))$ are consequently replaced by

$$A_f(\tilde{\theta}_\ell(\cdot)) = \begin{bmatrix} A(\theta_\ell(\cdot)) & B_\ell K_\ell \\ -L_\ell C_\ell & A(\tilde{\theta}_\ell(\cdot)) + L_\ell C_\ell + B_\ell K_\ell \end{bmatrix},$$

$$\hat{A}_f(\tilde{\theta}_\ell(\cdot)) = \begin{bmatrix} A(\theta_\ell(\cdot)) + B_\ell K_\ell & -B_\ell K_\ell \\ \Delta A(\tilde{\theta}_\ell(\cdot), \theta_\ell(\cdot)) & A(\tilde{\theta}_\ell(\cdot)) + L_\ell C_\ell \end{bmatrix},$$

respectively, where $\Delta A(\tilde{\theta}_\ell(\cdot), \theta_\ell(\cdot)) = A(\theta_\ell(\cdot)) - A(\tilde{\theta}_\ell(\cdot))$.

For each fixed $\theta_\ell(\cdot)$, consider the hyperbox $\tilde{\Theta}_\ell$ containing all the vectors $\tilde{\theta}_\ell(\cdot)$ given by (23) and define Δ_ℓ as

$$\Delta_\ell \triangleq \max_{(\theta_\ell(\cdot), \tilde{\theta}_\ell(\cdot)) \in \Theta_\ell \times \tilde{\Theta}_\ell} |\Delta A(\tilde{\theta}_\ell(\cdot), \theta_\ell(\cdot))|. \tag{24}$$

It follows that $|\hat{A}_f(\tilde{\theta}_\ell(\cdot))| \preceq \hat{\bar{A}}'_{f,\ell}$, with

$$\hat{\bar{A}}'_{f,\ell} = \begin{bmatrix} \bar{A}_\ell + B_\ell K_\ell & -B_\ell K_\ell \\ \Delta_\ell & \bar{A}'_\ell + L_\ell C_\ell \end{bmatrix}. \tag{25}$$

Hence $\hat{\bar{A}}'_{f,\ell}$ can be considered the analogous of the extremal closed loop matrix $\hat{\bar{A}}_{f,\ell}$ given by (10). The idea is to apply the procedure of Section 3 (or 4 if necessary) to the unperturbed matrix $\hat{\bar{A}}^u_{f,\ell}$ obtained from (25) assuming $\Delta_\ell = 0$. The corresponding set of LMIs is obtained from (11)-(15) with minor changes relative to the observer. It is enough to replace $\bar{A}_\ell$ with $\bar{A}'_\ell$ in (12), in the second of (13) and in (15), and to replace A_ℓ^- with $\tilde{A}_\ell^-$ in (15). If this new set of LMIs is satisfied, then $\hat{\bar{A}}^u_{f,\ell}$ is λ_ℓ-stable so that $\|(\hat{\bar{A}}^u_{f,\ell})^k\| \leq \hat{m}^u_{f,\ell} \lambda_\ell^k$ for some $\hat{m}^u_{f,\ell} > 0$. It is clear that the stability of $\hat{\bar{A}}'_{f,\ell}$ is preserved if $\|\Delta_\ell\|$ is sufficiently small. Applying the method reported in Jetto and Orsini (2007) and based on the Bellman-Gronwall Lemma, one has that $\|(\hat{\bar{A}}'_{f,\ell})^k\| \leq \hat{m}^u_{f,\ell} \alpha_\ell^k$, where $\alpha_\ell = (\lambda_\ell + \hat{m}^u_{f,\ell} \|\Delta_\ell\|)$. This implies that, for any fixed $\rho'_\ell \in (\lambda_\ell, 1)$, the perturbed closed-loop matrix $\hat{\bar{A}}'_{f,\ell}$ is α_ℓ-stable for some $\lambda_\ell < \alpha_\ell \leq \rho'_\ell$, if $\|\Delta_\ell\| \leq \delta_\ell \triangleq (\rho'_\ell - \lambda_\ell) \big/ \hat{m}^u_{f,\ell}$. As $|\hat{A}_f(\tilde{\theta}_\ell(\cdot))| \preceq \hat{\bar{A}}'_{f,\ell}$, the stability of $\Sigma_{f,\ell}$ follows from the lemma and from $A_f(\tilde{\theta}_\ell(\cdot)) = T^{-1} \hat{A}_f(\tilde{\theta}_\ell(\cdot)) T$. More precisely, in the noisy case one has: $\|\hat{\Phi}_{f,\ell}(k, \bar{k})\| = \|\hat{A}_f(\tilde{\theta}_\ell(k-1)) \cdots \hat{A}_f(\tilde{\theta}_\ell(\bar{k}))\| \leq \hat{m}^u_{f,\ell} \alpha_\ell^{(k-\bar{k})}$ and $\|\Phi_{f,\ell}(k, \bar{k})\| = \|A_f(\tilde{\theta}_\ell(k-1)) \cdots A_f(\tilde{\theta}_\ell(\bar{k}))\| \leq m'_{f,\ell} \alpha_\ell^{(k-\bar{k})}$, where $m'_{f,\ell} \leq \|T\| \|T^{-1}\| \hat{m}^u_{f,\ell}$. Hence the above synthesis procedure can be applied if the bound $\bar{v}_\ell$ on each $v_{\ell,i}(\cdot)$, $i = 1, \cdots, p$, is such that the corresponding $\|\Delta_\ell\|$ is sufficiently small, for example if it is overbounded by the above estimate δ_ℓ. A comprehensive overview of methods to estimate the maximum perturbation preserving stability is given in de Ambreu-Garcia et al. (1998).

In conclusion, in the case of noisy parameter measures, the design procedure of the controller family $\mathcal{F}$ consists of the three following steps: 1) consider the matrix $\hat{\bar{A}}^u_{f,\ell}$ obtained from (25) assuming $\Delta_\ell = 0$ and check if the corresponding set of LMIs defined as explained in this section is satisfied, 2) if the set is satisfied, choose $\rho'_\ell \in (\lambda_\ell, 1)$ and compute the above bound δ_ℓ on $\|\Delta_\ell\|$ preserving the α_ℓ-stability of the perturbed matrix $\hat{\bar{A}}'_{f,\ell}$, 3) exploiting the knowledge of matrix Δ_ℓ and the bound $\bar{v}_\ell$ on each $v_{\ell,i}(\cdot)$, $i = 1, \cdots, p$ derive the exact value of $\|\Delta_\ell\|$. If $\|\Delta_\ell\| \leq \delta_\ell$, the controller design procedure can be applied.

6. STABILITY ANALYSIS

The stability analysis of this section refers to both exact and noisy parameter measurements, provided that $\|\Delta_\ell\| \leq \delta_\ell$, $\ell = 1, \cdot, \tilde{\ell}$. By the theorem, inside each I'_m, the norm of $\Phi_{f,\ell}(\cdot, \cdot)$ is bounded as $\|\Phi_{f,\ell}(k, k_m)\| \leq \nu_\ell \omega_\ell^{(k-k_m)}$ for some $\nu_\ell > 0$ and $\omega_\ell < 1$, where $\nu_\ell = m_{f,\ell}$ and $\omega_\ell = \lambda_\ell$ in the noise free case, while $\nu_\ell = m'_{f,\ell}$ and $\omega_\ell = \alpha_\ell$, in the noisy measurements case.

Let $\Phi_f(\cdot, \cdot)$ be the state transition matrix of Σ_f. For each $j, i \in \mathbb{Z}^+$, with $j \geq i$, one has

$$
\begin{aligned}
\|\Phi_f(j,i)\| &\le c \prod_{l=\ell_0(i)}^{\bar{\ell}(j)-1} \|\Phi_{f,h(l)}(k_{l+1},k_l)\| \\
&\le c \prod_{l=\ell_0(i)}^{\bar{\ell}(j)-1} \nu_{h(l)}\omega_{h(l)}^{(k_{l+1}-k_l)}, \qquad (26)
\end{aligned}
$$

where: the empty product is taken as 1, $k_{\ell_0(i)}$ is the minimum $k_m \in S$ such that $k_{\ell_0(i)} \ge i$, $k_{\bar{\ell}(j)}$ is the maximum $k_m \in S$ such that $k_{\bar{\ell}(j)} \le j$, $c = \|\Phi_{f,\ell_1}(k_{\ell_0(i)},i)\|\|\Phi_{f,\ell_2}(j,k_{\bar{\ell}(j)})\|$ if $j \ge k_{\bar{\ell}_0(i)}$, otherwise $c = \|\Phi_{f,\ell_1}(j,i)\|$, for some $\ell_1,\ell_2 \in [1,\cdots,\tilde{\ell}]$, $h(\cdot)$ is a function such that $h(\cdot) : \mathbb{Z}^+ \to [1,\cdots,\tilde{\ell}]$. It directly follows that the uniform closed loop asymptotic stability is guaranteed if $\nu_\ell\omega_\ell^{(k_{m+1}-k_m)} < 1$, $\ell = 1,\cdots,\tilde{\ell}$, $\forall m \in \mathbb{Z}^+$, namely if $k_{m+1} - k_m \ge \tau$, $\forall m \in \mathbb{Z}^+$, where τ is such that

$$
\tau > \tau_{\min} \stackrel{\triangle}{=} \max_{\ell\in[1,\tilde{\ell}]} (\ln \nu_\ell / \ln(1/\omega_\ell)). \qquad (27)
$$

By the equivalence of uniform asymptotic and exponential stability, condition (27) also implies that Σ_f is uniformly exponentially $\bar{\omega}$ stable for some $0 \le \bar{\omega} < 1$. The value $\bar{\omega}$ can be computed as a function of the ω_ℓ $\ell = [1,\cdots,\tilde{\ell}]$, in the following way. Let ν and q_ℓ be defined as $\nu \stackrel{\triangle}{=} \max_{\ell\in[1,\tilde{\ell}]} \nu_\ell$, and $q_\ell \stackrel{\triangle}{=} \nu_\ell\omega_\ell^\tau < 1$, respectively. Consider the $\tilde{\ell}$ functions $\bar{\omega}_\ell^k$, $\ell = 1,\cdots,\tilde{\ell}$, where $\bar{\omega}_\ell$ is computed imposing the condition $\bar{\omega}_\ell^{(\tau+1)} = q_\ell$, and define $\bar{\omega} \stackrel{\triangle}{=} \max_\ell \bar{\omega}_\ell$. Taking into account that i) $0 < \bar{\omega} < 1$, ii) $c \le \nu_{\ell_1}\omega_{\ell_1}^{(k_{\ell_0(i)}-i)}\nu_{\ell_2}\omega_{\ell_2}^{(j-k_{\bar{\ell}(j)})}$, it is easily seen that the way the function $\nu\bar{\omega}^k$ is defined implies that the r.h.s. of (26) is upperly bounded by $\nu^3\bar{\omega}^{(j-i)}$, $\forall j \ge i$.

7. A NUMERICAL EXAMPLE

Example Consider the LPV system Σ with the following mode switch dynamics $\Sigma_\ell = (C_\ell, A(\theta_\ell(\cdot)), B_\ell)$, $\ell = 1,2$,

$$
C_1 = [\,0.3 \;\; 0.4\,], B_1 = \begin{bmatrix} 0.5 \\ 0.3 \end{bmatrix},
$$

$$
A(\theta_1(\cdot)) = \begin{bmatrix} \theta_{11}(\cdot)\,\theta_{12}(\cdot) & 1 \\ 0.5 & \theta_{11}(\cdot) + \theta_{13}^2(\cdot) \end{bmatrix},
$$

$$
C_2 = [\,0.5 \;\; 0.5\,], B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},
$$

$$
A(\theta_2(\cdot)) = \begin{bmatrix} -0.5 & \theta_{22}(\cdot) \\ -1 & \theta_{21}(\cdot)\,\theta_{23}(\cdot) \end{bmatrix},
$$

where $\theta_{11}(\cdot) \in [0,1]$, $\theta_{12}(\cdot) \in [0,1.2]$, $\theta_{13}(\cdot) \in [-0.6,0]$, $\theta_{21}(\cdot) \in [0.5,1]$, $\theta_{22}(\cdot) \in [-1,-0.6]$, $\theta_{23}(\cdot) \in [-1,-0.8]$. It follows that

$$
A(\theta_1(\cdot)) = \begin{bmatrix} [0,1.2] & 1 \\ 0.5 & [0,1.36] \end{bmatrix},
$$

$$
A(\theta_2(\cdot)) = \begin{bmatrix} -0.5 & [-1,-0.6] \\ -1 & [-1,-0.4] \end{bmatrix}.
$$

It is easy to see that $\bar{\Sigma}_\ell$, $\ell = 1,2$ satisfies A7). It is also assumed that $k_{m+1} - k_m \ge \tau = 40, m \in \mathbb{Z}^+$, and that $\theta_2(\cdot) = [\theta_{21}(\cdot),\theta_{22}(\cdot),\theta_{23}(\cdot)]^T$ is observed under an additive measurement noise $v_2(\cdot)$ such that $|v_{2,i}(\cdot)| \le \bar{v}_2 = 0.005$, $i = 1,2$, and $v_{2,3}(\cdot) = 0$. This implies that A8) is satisfied. As the ITV matrix $A(\theta_2(\cdot))$ is negative and $\bar{A}_2 = \begin{bmatrix} 0.5 & 1 \\ 1 & 1 \end{bmatrix}$ is unstable, the positivation procedure of Section 4 must be applied. Using the internal static output feedback $G_2 = 2.5$ as a possible solution of (22), the output positivized system $\Sigma_{p,2}$ results to be $\Sigma_{p,2} \equiv (C_2, A_p(\theta_2(\cdot)), B_2)$ with

$$
A_p(\theta_2(\cdot)) = (A(\theta_2(\cdot)) + B_2G_2C_2) \in \begin{bmatrix} 0.75 & [0.25,0.65] \\ 0.25 & [0.25,0.85] \end{bmatrix}.
$$

Choosing $\rho_{1,1} = \rho_{2,1} = \rho_1 = 0.8$ and $\rho_{1,2} = \rho_{2,2} = \rho_2 = 0.61$, it is found that, as for Σ_1, the set of LMIs (11)-(15) admits the solution $K_1 = [-1.1632,-1.9817]$, $L_1 = [-1.9817,-1.5883]^T$. As for Σ_2, the set of LMIs defined as explained in Section 4 has to be considered. It is found that the matrix

$$
\hat{\bar{A}}_{f,2}^u = \begin{bmatrix} \bar{A}_{p,2} + B_2K_2 & -B_2K_2 \\ 0_2 & \bar{A}'_{p,2} + L_2C_2 \end{bmatrix},
$$

with

$$
\bar{A}_{p,2} = \begin{bmatrix} 0.75 & 0.65 \\ 0.25 & 0.85 \end{bmatrix}, \quad \bar{A}'_{p,2} = \begin{bmatrix} 0.75 & 0.655 \\ 0.25 & 0.854 \end{bmatrix},
$$

is stabilized by the pair (K_2,L_2) with $K_2 = [-0.2265, -0.4263]$, $L_2 = [-0.8719,-0.498]^T$. Once the pairs (K_ℓ,L_ℓ), $\ell = 1,2$, have been computed, the matrix $\hat{\bar{A}}_{f,1}$ can be obtained by (10) and $\hat{\bar{A}}_{f,2}^u$ by (25), assuming $\Delta_2 = 0$. The maximum modulus eigenvalues of $\hat{\bar{A}}_{f,1}$ and $\hat{\bar{A}}_{f,2}^u$ are $\lambda_1 = 0.7743$ and $\lambda_2 = 0.6057$, respectively. It is found that $\left\|\left(\hat{\bar{A}}_{f,1}\right)^k\right\|_2 \le \hat{m}_{f,1}\lambda_1^k = (199.72)\cdot(0.7743)^k$ and $\left\|\left(\hat{\bar{A}}_{f,2}^u\right)^k\right\|_2 \le \hat{m}_{f,2}^u\lambda_2^k = (36.2385)\cdot(0.6057)^k$.

Choosing $\rho'_2 = 0.9$, the Bellman-Gronwall based approach described in Jetto and Orsini (2007), shows that, for any $\alpha_2 \in (\lambda_2,\rho'_2)$, also the perturbed closed-loop matrix $\hat{\bar{A}}'_{f,2}$ given by (25) is α_2-stable if $\|\Delta_2\|_2 \le \delta_2 = (\rho'_2 - \lambda_2)\big/\hat{m}_{f,2}^u = (0.9-0.6057)/36.2385 = 0.0081$. For the given $A(\theta_2(\cdot))$ and $v_2(\cdot)$, (24) gives

$$
\Delta_2 = \begin{bmatrix} 0 & \max_k |v_{2,2}(k)| \\ 0 & \max_k |v_{2,1}(k)|\cdot\max_k|\vartheta_{2,3}(k)| \end{bmatrix} = \begin{bmatrix} 0 & 0.005 \\ 0 & 0.005 \end{bmatrix},
$$

to which the value $\|\Delta_2\|_2 = 0.0071$ corresponds. This gives $\alpha_2 = (\lambda_2 + \hat{m}_{f,2}^u\|\Delta_2\|_2) = (0.6057 + 36.2385\cdot 0.0071) = 0.863$, so that $\left\|\left(\hat{\bar{A}}'_{f,2}\right)^k\right\|_2 \le \hat{m}_{f,2}^u\cdot\alpha_2^k = (36.2385)\cdot(0.863)^k$. By the theorem one has $\left\|\Phi_{f,1}(k,\bar{k})\right\|_2 \le m_{f,1}\cdot\lambda_1^{(k-\bar{k})} = (373.8)\cdot(0.7743)^{(k-\bar{k})}$ and $\left\|\Phi_{f,2}(k,\bar{k})\right\|_2 \le m'_{f,2}\cdot\alpha_2^{(k-\bar{k})} = (61.9131)\cdot(0.863)^{(k-\bar{k})}$, from which the value $\tau_{\min} = 29$ is found. Hence condition (27) is satisfied and Σ_f is $\bar{\omega}$-stable. Applying the procedure given in Section 6, the value $\bar{\omega} = 0.9578$ is obtained. In conclusion, the mode-switch LPV system considered in this example can be really stabilized using the present approach.

8. CONCLUSIONS

The stabilization problem for a discrete-time, LPV plant with mode-switch dynamics has been considered. Conditions for problem solvability have been established in terms of LMIs which only involve the extremal plants A_ℓ^- and $\bar{A}_\ell$. The solution (if any) is given by a family of observer like based controllers with constant gain matrices. This makes the method very appealing from the numerical point of view because the set of LMIs to be checked is independent of the number of time-varying parameters and all the calculations can be performed off-line. Another interesting feature is that the method proposed is amenable to deal with noisy parameter measurements. This is a key point which is often neglected in the literature though it represent almost all cases of a practical interest. The extension of the present approach to the tracking problem only requires the definition of a proper error system.

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