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A Discrete-Time Adaptive ILC for Systems with Random Initial Condition and Iteration-Varying Trajectory

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Abstract: A discrete-time adaptive ILC scheme is presented for systems with time-varying parametric uncertainties. Using the analogy between the discrete-time axis and the iterative learning axis, the new AILC can incorporate a recursive Least-Squares algorithm, hence the learning gain can be tuned iteratively along the learning axis and pointwisely along the time axis. When the initial states are random and the reference trajectory is iteration-varying, the new AILC can achieve the pointwise convergence over a finite time interval asymptotically along the iterative learning axis. An extension of the new AILC is also developed by using nonlinear data weighting to systems without assuming any growth conditions on the nonlinearity.

1. INTRODUCTION

In control practice, many control tasks will end in a finite interval and repeat, e.g. the track following control of a hard disk drive and the temperature or pressure control in a batch reactor. By the time-varying nature of parameters and the finite time interval, the well-established adaptive control methods (Goodwin & Sin, 1984; Kanellakopoulos, 1994) are not applicable. While, in such a circumstance iterative learning control (ILC) methods, evolved over the past two decades (Arimoto *et al.*, 1984; Saab, 1995; Sun & Wang, 2003; Xu, 1997), are most suitable. However, majority of ILC schemes developed hitherto focus on the systems with non-parametric uncertainties without making full use of the prior knowledge about the parameterization, and as a result still require the identical conditions on the initial value and target trajectory.

Recently several adaptive iterative learning control schemes have been developed (Narushima et al, 1995; Fukuda & Shin, 1998; Choi & Lee, 2000; Xu & Viswanathan, 2000; Norrlöf, 2002), which introduce parametric adaptation law in the learning process. A major issue that remains is still the requirement of the identical conditions on the initial states and the reference trajectory, if the pointwise tracking performance is to be obtained. Without these two identical conditions, what ILC can guarantee is often a bounded tracking performance over the finite interval.

In this work we investigate the possibility of extending the well-established discrete-time adaptive control method to cope with the iterative learning control tasks with time-varying parametric uncertainties. Using the analogy between the discrete time axis and the iterative learning axis, a new adaptive iterative learning control approach – adaptive ILC (AILC) – is presented. A Least-Squares algorithm is used to update the AILC parameters iteratively along the learning axis. Comparing with the discrete-time adaptive control, the new AILC has two distinct features: 1) achieving a pointwise convergence over a finite interval, and 2) coping with time-varying parameters. Comparing with existing discrete-time ILC, the new discrete-time AILC also has two distinct features: 1) the strict identical initial condition is not required, and 2) the reference trajectory can vary from iteration to iteration.

Furthermore, we present an extension of the new AILC to more general systems with nonsector nonlinearities. As a result, a modified AILC algorithm is presented by using nonlinear data weighting and also achieves the almost perfect tracking performance with random initial system states and iteration-varying reference.

It is also worth mentioning that the most interesting feature of the new AILC is the direct heritage of discrete-time adaptive control in controller configurations, analysis methods, and convergence properties. These novelties are achieved via replacing the discrete time axis with the iteration axis, in conjunction with appropriate time domain modification, as we will demonstrate later in this work.

This paper is organized as follows. Section 2 presents the problem formulation and the new AILC design. Section 3 shows the learning convergence. Section 4 gives an extension to more general systems with nonsector nonlinearities. An illustrative example is provided in Section 5. Finally, some conclusions are given in Section 6.

2. LEAST-SQUARES ADAPTIVE ILC

2.1 Problem Formulation

To focus on and clearly demonstrate the underlying idea and method, we consider a simple discrete-time system with only one unknown parameter

$$x_k(t+1) = \theta(t)\xi(x_k(t),t) + u_k(t), \quad t \in \{0,1,\cdots,T\} \tag{1}$$

where $x_k(t) \in R$ is the measurable system state, $u_k(t) \in R$ is the system control input, $\theta(t)$ is an unknown time-varying parameter, and $\xi(x_k(t),t)$ is a known scalar nonlinear function which is bounded for bounded $x_k(t)$. The subscript, $k \in \{0,1,\cdots\}$, denotes the learning iterations.

Denote the target trajectory at the k-th iteration as $r_k(t)$, $t \in \{0,1,\cdots,T\}$, which belongs to a compact set over all iterations.

Define the tracking error $e_k(t+1) = x_k(t+1) - r_k(t+1)$, $t \in \{0,1,\cdots,T\}$, we have

$$\begin{aligned} e_k(t+1) &= x_k(t+1) - r_k(t+1) \\ &= \theta(t)\xi(x_k(t),t) + u_k(t) - r_k(t+1). \end{aligned} \tag{2}$$

The objective of AILC is to find an appropriate control input sequence $u_k(t)$, $t \in \{0,1,\cdots,T-1\}$, such that the system output $x_k(t)$ follows the target trajectory $r_k(t)$, i.e., the tracking error $e_k(t)$, $t \in \{1,\cdots,T\}$, converges to zero as the iteration number k approaches infinity.

Remark 1: Note that we assume the initial states $x_k(0)$, cannot be manipulated via any control signals. Thus the initial error $e_k(0)$, shall be excluded from the learning control objective.

2.2 New Adaptive ILC Design

The presented learning control law at the k-th iteration is

$$u_k(t) = r_k(t+1) - \hat{\theta}_k(t)\xi(x_k(t),t), \; t \in \{0,1,\cdots,T-1\} \tag{3}$$

where $\hat{\theta}_k(t)$ is to learn the time-varying parameter $\theta(t)$ and updated iteratively as follows

$$\hat{\theta}_k(t) = \hat{\theta}_{k-1}(t) + P_{k-1}(t)\xi(x_{k-1}(t),t)e_{k-1}(t+1), \tag{4}$$

$P_{k-1}(t)$ is a learning gain updated iteratively as below

$$P_{k-1}(t) = P_{k-2}(t) - \frac{P_{k-2}{}^2(t)\xi^2(x_{k-1}(t),t)}{1 + P_{k-2}(t)\xi^2(x_{k-1}(t),t)}. \tag{5}$$

The initial values of $\hat{\theta}_0(t)$, $t \in \{0,1,\cdots,T\}$, can be chosen arbitrarily, e.g. zero if no prior knowledge about $\theta_k(t)$ is available. Similarly, we can choose the initial values $P_{-1}(t) = P_0 > 0$, $\forall t \in \{0,1,\cdots,T\}$, with P_0 a sufficiently large scalar.

Remark 2: Because of the time-varying parameters and finite time tracking, adaptive control approaches are not suitable.

Remark 3: The adaptive learning law (4) and (5) is processed along the iterative learning axis k, not along the time axis t. Nevertheless, the time index plays an important role, because the process under control is dynamical in the time domain.

Remark 4: Looking into the parameter updating law (4), the non-causal form is required, that is when computing $\hat{\theta}_k(t)$, $e_{k-1}(t+1)$ is used. The non-causal term plays a key role in convergence analysis, as we will show in subsequent section.

Remark 5: In standard ILC, the control updating law is in essence a linear integrator along the iteration axis and often with an iteration-invariant gain. In contrast, the AILC law (3)-(5) provides a more generic and nonlinear updating law along the iteration axis, and further provides a nonlinear gain updating law.

3. LEARNING CONVERGENCE ANALYSIS

To restrict our discussion, the following assumptions are exposed on system (1).

Assumption 1: The nonlinear function $\xi(x_k(t),t)$ satisfies linear growth condition, i.e., $\forall t$ and $\forall k$,

$$|\xi(x_k(t),t)| \le c_1^0 + c_2^0 |x_k(t)|, \tag{6}$$

where $0 < c_1^0 < \infty$ and $0 < c_2^0 < \infty$.

Assumption 2: The unknown time-varying parameter $\theta(t)$, the target trajectory $r_k(t)$, and the initial states $x_k(0)$, are uniformly bounded for all $t = 0,1,\cdots,T$ and $k = 0,1,\cdots$.

Remark 6: Note that in Assumption 2, we only assume the existence of such bounds, without requiring the exact values.

The convergence property of the presented AILC is summarized in the following theorem.

Theorem 1: For system (1) under assumptions 1 and 2, the AILC law (3)-(5) guarantee that the parameter estimation error is bounded and the tracking error converges to zero pointwisely over the finite time interval $\{1,2,\cdots,T\}$ as k approaches to infinity.

Proof: Define the parametric estimation error $\phi_k(t) = \theta(t) - \hat{\theta}_k(t)$, substituting the control law (3) into the error dynamics (2) yields

$$e_k(t+1) = \phi_k(t)\xi(x_k(t),t). \tag{7}$$

Define a non-negative function $V_k(t) = P_{k-1}^{-1}(t)\phi_k^2(t)$, the difference of the function $V_k(t)$ along the iteration axis is

$$\begin{aligned} V_k(t) - V_{k-1}(t) &= P_{k-1}^{-1}(t)\phi_k^2(t) - P_{k-2}^{-1}(t)\phi_{k-1}^2(t) \\ &= [P_{k-1}^{-1}(t) - P_{k-2}^{-1}(t)]\phi_{k-1}^2(t) \\ &\quad - 2\phi_{k-1}(t)\xi(x_{k-1}(t),t)e_{k-1}(t+1) \\ &\quad + P_{k-1}(t)\xi^2(x_{k-1}(t),t)e_{k-1}^2(t+1). \end{aligned} \tag{8}$$

From (5), we obtain

$$P_{k-1}^{-1}(t) = P_{k-2}^{-1}(t) + \xi^2(x_{k-1}(t), t). \quad (9)$$

Using (9) and the error dynamics (7), we have

$$[P_{k-1}^{-1}(t) - P_{k-2}^{-1}(t)]\phi_{k-1}^2(t) = \xi^2(x_{k-1}(t), t)\phi_{k-1}^2(t) = e_{k-1}^2(t+1), \quad (10)$$

and

$$-2\phi_{k-1}(t)\xi(x_{k-1}(t), t)e_{k-1}(t+1) = -2e_{k-1}^2(t+1). \quad (11)$$

Substituting (10) and (11) into (8) yields

$$V_k(t) - V_{k-1}(t) = -e_{k-1}^2(t+1)[1 - P_{k-1}(t)\xi^2(x_{k-1}(t), t)]. \quad (12)$$

In terms of (9), we can derive

$$1 - P_{k-1}(t)\xi^2(x_{k-1}(t), t) = 1 - \frac{P_{k-2}(t)\xi^2(x_{k-1}(t), t)}{1 + P_{k-2}(t)\xi^2(x_{k-1}(t), t)} = \frac{1}{1 + P_{k-2}(t)\xi^2(x_{k-1}(t), t)}. \quad (13)$$

From (13), (12) can be rewritten as

$$V_k(t) - V_{k-1}(t) = -\frac{e_{k-1}^2(t+1)}{1 + P_{k-2}(t)\xi^2(x_{k-1}(t), t)} \le 0 \quad (14)$$

Thus $V_k(t)$ is non-increasing, implying that $\phi_k(t)$ is bounded.

Summing (14) from 0 to k leads to

$$V_k(t) = V_0(t) - \sum_{i=1}^{k} \frac{e_{i-1}^2(t+1)}{1 + P_{i-2}(t)\xi^2(x_{i-1}(t), t)}. \quad (15)$$

Consider that $V_k(t)$ is nonnegative, $V_0(t)$ is finite in the interval $\{0, 1, \cdots, T\}$, thus according to the convergence theorem of the sum of series, we have

$$\lim_{k\to\infty} \frac{e_k(t+1)}{[1 + P_{k-1}(t)\xi^2(x_k(t), t)]^{1/2}} = 0, \quad (16)$$

Now let us derive the asymptotic learning convergence of $e_k(t+1)$ in terms of (16) along the iterative learning axis. From assumptions 1 and 2, it is straightforward to derive $|e_k(0)| \le |x_k(0)| + |r_k(0)| \in l^\infty$, and

$$|\xi(x_k(t), t)| \le c_1^0 + c_2^0 |x_k(t)| \le c_1^0 + c_2^0 |e_k(t)| + c_2^0 |r_k(t)|.$$

Since $|r_k(t)|$ is known bounded, there exist appropriate constants $c_1^* = c_1^0 + c_2^0 \max_{t\in[0,T]} |r_k(t)|$ and $c_2^* = c_2^0$ such that

$$|\xi(x_k(t), t)| \le c_1^* + c_2^* |e_k(t)|, \quad (17)$$

then

$$\begin{aligned} |\xi(x_k(t), t)| &\le c_1^* + c_2^* |e_k(t)| \\ &\le c_1^* + c_2^* \left(|e_k(0)| + \max_{i\in[0,t]} |e_k(i+1)| \right) \\ &\le c_1 + c_2 \max_{i\in[0,t]} |e_k(i+1)|, \end{aligned} \quad (18)$$

where $c_1 = c_1^* + c_2^* |e_k(0)|$ and $c_2 = c_2^*$ are constants.

Therefore, $[1 + P_{k-1}(t)\xi^2(x_k(t), t)]^{1/2}$ satisfies the linear growth condition. By virtue of the *Key Technical Lemma* (Goodwin & Sin, 1984), the convergence property (16) together with the linear growth condition (18) implies the asymptotical convergence of $e_k(t)$ over the entire finite time interval $\{1, 2, \cdots, T\}$ along the iteration axis k. □

4. EXTENSION TO MORE GENERAL CASES WITH NONSECTOR NONLINEARITIES

It shall be noted that it has to impose linear growth conditions on the nonlinearities to guarantee the convergence property of the presented AILC in Section 3. If, on the other hand, the nonlinear function $\xi(\cdots)$ is not sector-bounded, then $[1 + P_{k-1}(t)\xi^2(x_k(t), t)]^{1/2}$ is not satisfied with the linear growth condition. Thus the *Key Technical Lemma* is not applicable. To solve this problem, we present a modified Least-Squares algorithm by using nonlinear data weighting in this section. The almost perfect tracking performance is also achieved without assuming any growth conditions on the nonlinearities.

The modified Least-Squares algorithm is presented as

$$\hat{\theta}_k(t) = \hat{\theta}_{k-1}(t) + \alpha_{k-1}(t)P_{k-1}(t)\xi(x_{k-1}(t), t)e_{k-1}(t+1), \quad (19)$$

$$P_{k-1}(t) = P_{k-2}(t) - \frac{\alpha_{k-1}(t)P_{k-2}^2(t)\xi^2(x_{k-1}(t), t)}{1 + \alpha_{k-1}(t)P_{k-2}(t)\xi^2(x_{k-1}(t), t)}, \quad (20)$$

where $\alpha_{k-1}(t)$ is a nonnegative weighting coefficients.

Subtracting $\theta(t)$ from both sides of (19) and in terms of the definition of $\phi_k(t)$, we have

$$\phi_k(t) = \phi_{k-1}(t) - \alpha_{k-1}(t)P_{k-1}(t)\xi(x_{k-1}(t), t)e_{k-1}(t+1). \quad (21)$$

From (20), it is easy to derive

$$P_{k-1}^{-1}(t) = P_{k-2}^{-1}(t) + \alpha_{k-1}(t)\xi^2(x_{k-1}(t), t). \quad (22)$$

Still definite $V_k(t) = P_{k-1}^{-1}(t)\phi_k^2(t)$. Using (21) and (22), the difference of $V_k(t)$ with respect to iteration axis is

$$\begin{aligned} \Delta V_k &= V_k(t) - V_{k-1}(t) = P_{k-1}^{-1}(t)\phi_k^2(t) - P_{k-2}^{-1}(t)\phi_{k-1}^2(t) \\ &= P_{k-1}^{-1}(t)[\phi_{k-1}(t) - \alpha_{k-1}(t)P_{k-1}(t)\xi(x_{k-1}(t), t)e_{k-1}(t+1)]^2 \\ &\quad - P_{k-2}^{-1}(t)\phi_{k-1}^2(t) \\ &= [P_{k-1}^{-1}(t) - P_{k-2}^{-1}(t)]\phi_{k-1}^2(t) \\ &\quad - 2\alpha_{k-1}(t)\phi_{k-1}(t)\xi(x_{k-1}(t), t)e_{k-1}(t+1) \\ &\quad + \alpha_{k-1}^2(t)P_{k-1}(t)\xi^2(x_{k-1}(t), t)e_{k-1}^2(t+1) \\ &= \alpha_{k-1}(t)\xi^2(x_{k-1}(t), t)\phi_{k-1}^2(t) - 2\alpha_{k-1}(t)e_{k-1}^2(t+1) \\ &\quad + \alpha_{k-1}^2(t)P_{k-1}(t)\xi^2(x_{k-1}(t), t)e_{k-1}^2(t+1) \\ &= -\alpha_{k-1}(t)(1 - \alpha_{k-1}(t)P_{k-1}(t)\xi^2(x_{k-1}(t), t))e_{k-1}^2(t+1) \\ &= -\frac{\alpha_{k-1}(t)e_{k-1}^2(t+1)}{1 + \alpha_{k-1}(t)P_{k-2}(t)\xi^2(x_{k-1}(t), t)} \le 0. \end{aligned} \quad (23)$$

Thus $V_k(t)$ is non-increasing, implying that $\phi_k(t)$ is bounded.

Following the same steps that lead to (16) in Theorem 1, we conclude that

$$\lim_{k\to\infty} \frac{\alpha_k(t)e_k^2(t+1)}{1+\alpha_k(t)P_{k-1}(t)\xi^2(x_k(t),t)} = 0. \quad (24)$$

If we can choose $\alpha_k(t)$ such that $\forall k = 0,1,2,\cdots$ and $\forall t \in \{0,1,\cdots,T\}$, $\frac{\alpha_k(t)}{1+\alpha_k(t)P_{k-1}(t)\xi^2(x_k(t),t)} > d > 0$, then we can acquire that $\lim_{k\to\infty} e_k(t+1) = 0$.

To show the learning convergence, we need introduce the following Lemma.

Lemma 1: There must exist a constant $d_1 > 0$ satisfies that for $\forall k = 0,1,2,\cdots$ and $\forall t \in \{0,1,\cdots,T\}$,

$$\frac{P_{-1}^{-1}(t)}{\xi^2(x_k(t),t)} + \xi^2(x_{k-1}(t),t) + \frac{\xi^2(x_{k-1}(t),t)}{\xi^2(x_k(t),t)} > \frac{1}{d_1}. \quad (25)$$

Proof: We arbitrarily choose a positive constant δ_0 and examine the following two cases.

(1) When $\xi^2(x_k(t),t) \ge \xi^2(x_{k-1}(t),t) > \delta_0$ or $\xi^2(x_{k-1}(t),t) \ge \xi^2(x_k(t),t) > \delta_0$, then (25) is satisfied with $d_1 > \delta_0^{-1}$.

(2) When $\xi^2(x_k(t),t) \le \xi^2(x_{k-1}(t),t) \le \delta_0$ or $\xi^2(x_{k-1}(t),t) \le \xi^2(x_k(t),t) \le \delta_0$, then (25) is satisfied with $d_1 > \delta_0 P_0$.

The above discussion shows that (25) is satisfied for all $k = 0,1,2,\cdots$ and $t \in \{0,1,\cdots,T\}$ with $d_1 > \max\{\delta_0^{-1},\ \delta_0 P_0\}$

Remark 7: It shall be noted that in Lemma 1, we only need to show the existence of d_1, without requiring the exact value.

Theorem 2: If we choose that $\alpha_k(t) = 1 + \xi^2(x_{k+1}(t),t)$, then for system (1) under Assumption 2, the modified AILC algorithm (3), (19) and (20) guarantees that the parameter estimation error is bounded and the tracking error converges to zero pointwisely over the finite time interval $\{1,2,\cdots,T\}$ as k approaches to infinity.

Remark 8: Note that we allow $\alpha_k(t)$ to be a positive nonlinear function of all measured variables up to and including the time instant *t* of the (*k*+1)-th iteration. This does not affect the causality of the algorithm since $x_{k+1}(t)$ is needed for computation of $e_{k+1}(t+1)$.

Proof: The boundedness of parameter estimation error has been shown from (23). Now we show the learning convergence of the tracking error. From (22) and (25), we can derive

$$\begin{aligned}\frac{P_{k-1}^{-1}(t)}{\xi^2(x_k(t),t)} &= \frac{P_{k-2}^{-1}(t) + \alpha_{k-1}(t)\xi^2(x_{k-1}(t),t)}{\xi^2(x_k(t),t)} \\ &\ge \frac{P_{-1}^{-1}(t) + (1+\xi^2(x_k(t),t))\xi^2(x_{k-1}(t),t)}{\xi^2(x_k(t),t)} \\ &= \frac{P_{-1}^{-1}(t)}{\xi^2(x_k(t),t)} + \xi^2(x_{k-1}(t),t) + \frac{\xi^2(x_{k-1}(t),t)}{\xi^2(x_k(t),t)} \\ &\ge \frac{1}{d_1},\end{aligned} \quad (26)$$

that is $P_{k-1}(t)\xi^2(x_k(t),t) \le d_1$, so we have

$$\begin{aligned}\frac{1+\alpha_k(t)P_{k-1}(t)\xi^2(x_k(t),t)}{\alpha_k(t)} &= \frac{1}{\alpha_k(t)} + P_{k-1}(t)\xi^2(x_k(t),t) \\ &\le \frac{1}{\alpha_k(t)} + d_1 \le 1 + d_1,\end{aligned} \quad (27)$$

this implies

$$\begin{aligned}e_k^2(t+1) &= \frac{1+\alpha_k(t)P_{k-1}(t)\xi^2(x_k(t),t)}{\alpha_k(t)} \\ &\quad \times \frac{\alpha_k(t)e_k^2(t+1)}{1+\alpha_k(t)P_{k-1}(t)\xi^2(x_k(t),t)} \\ &\le (1+d_1)\frac{\alpha_k(t)e_k^2(t+1)}{1+\alpha_k(t)P_{k-1}(t)\xi^2(x_k(t),t)}.\end{aligned} \quad (28)$$

Thus

$$0 \le \lim_{k\to\infty} e_k^2(t+1) \le (1+d_1)\lim_{k\to\infty}\frac{\alpha_k(t)e_k^2(t+1)}{1+\alpha_k(t)P_{k-1}\xi^2(x_k(t),t)} = 0. \quad (29)$$

From (29), clearly we can conclude that $\lim_{k\to\infty} e_k(t+1) = 0$. □

5. ILLUSTRATIVE EXAMPLE

Consider a numerical example

$$x(t+1) = \theta(t)\xi(x(t),t) + u(t),$$

where $\theta(t) = 2\sin(4t\pi/3)$ is a time-varying parameter; $t = 0,1,\cdots,100$ is the tracking interval; $k = 0,1,2\cdots$ is the iteration number. The initial state value $x_k(0)$ is randomly varying in the interval $(0,\ 1]$ when the iteration *k* evolves. Fig. 1 shows the initial state $x_k(0)$ over 100 iterations.

The desired trajectories are chosen as the following two functions.

Class 1, if *k* is odd

$$r_k(t+1) = \tau(k) * \begin{cases} 0.5\times(-1)\text{^}round(t/10), & 0 \le t \le 30 \\ 0.5\sin(t\pi/10) + 0.3\cos(t\pi/5), & 30 < t \le 70 \\ 0.5\times(-1)\text{^}round(t/10), & 70 < t \le 100 \end{cases}$$

Class 2, if k is even

$$r_k(t+1) = \tau(k) * \begin{cases} 0.5\sin(t\pi/10) + 0.3\cos(t\pi/5), & 0 \le t \le 30 \\ 0.5 \times (-1)\text{^}\,round(t/10), & 30 < t \le 70 \\ 0.5\sin(t\pi/10) + 0.3\cos(t\pi/5), & 70 < t \le 100 \end{cases}$$

where $\tau(k)$ is also randomly varying in the interval $(0,\ 1]$ as k evolves. Fig. 2 shows the target trajectory at the first four iterations.

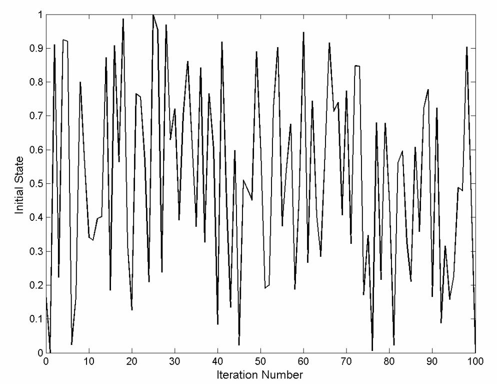

Fig. 1. The profile of random initial state value $x_k(0)$.

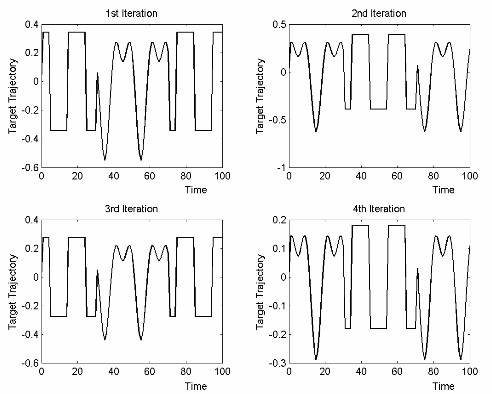

Fig. 2. The target trajectory at the first four iterations

Case 1: The nonlinear function $\xi(x_k(t), t) = \sin(x_k(t))$. Note that the linear growth condition (Assumption 1) is satisfied, thus the standard adaptive ILC law (3)-(5) can be used. Since $e_k(0)$ is not learnable, we will check the interval $t \in \{1, \cdots, 100\}$. Fig. 3 shows the learning convergence w.r.t. the iteration k. The horizon is the iteration number and the vertical axis is the maximum absolute tracking error, $e^*_{\max,k} = \sup_{t\in\{1,\cdots,100\}} |r_k(t) - x_k(t)|$.

It shall be noted that the modified AILC (3), (19) and (20) is also applicable for *case 1*. Let $\alpha_{k-1}(t) = 1 + \sin^2(x_k(t))$. Checking the interval $t \in \{1, \cdots, 100\}$, the simulation result is given in Fig. 4.

We can see from figures 1-4 that despite the random initial values and the random variations of the target trajectory along the iteration axis, the tracking error over the interval $t \in \{1, \cdots, 100\}$ converges asymptotically to zero along the iteration axis, except for the initial instant that is not learnable.

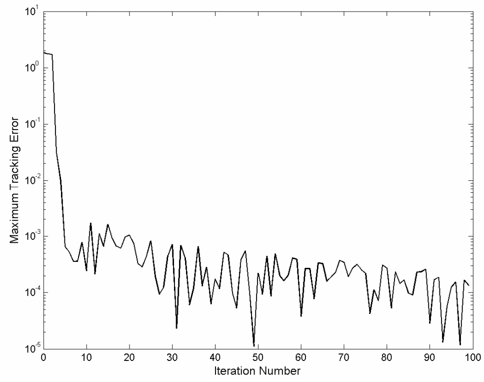

Fig. 3. The asymptotic convergence of the tracking error by means of the presented AILC (3)-(5)

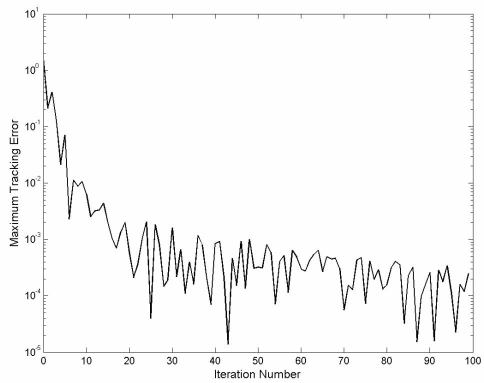

Fig. 4. The asymptotic convergence of the tracking error by means of the modified AILC (3), (19), (20).

For comparison, a P-type ILC $u_k(t) = u_{k-1}(t) + 0.01 \times e_{k-1}(t+1)$ is applied. The best performance is shown in Fig. 5, which is however only bounded instead of asymptotic convergence.

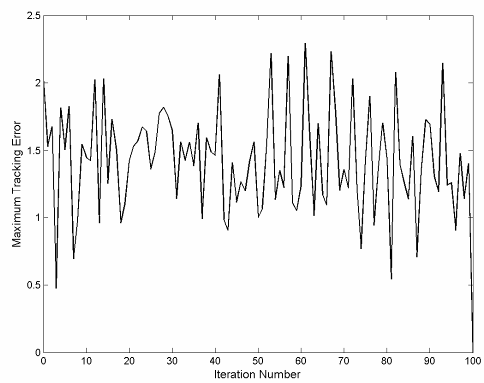

Fig. 5. The bounded convergence of the tracking error by means of the P-type ILC.

Case 2: The nonlinear function $\xi(x_k(t),t) = x_k^2(t)$. Note that the linear growth condition is not satisfied and the nonlinear function is just bounded for bounded $x_k(t)$. Thus the adaptive ILC law (3)-(5) is not applicable and a finite escape phenomenon, as shown in Fig. 6, along the iterative learning axis *k* will arise. Instead, applying the modified AILC law (3), (19) and (20) with $\alpha_{k-1}(t) = 1 + x_k^4(t)$, Fig. 7 shows the learning convergence w.r.t. the iteration *k*.

From figures 6 and 7, we can see that the modified algorithm can cope with more general systems without assuming any growth conditions on the nonlinearities and still achieves the almost perfect tracking performance with random conditions on the initial values (Fig. 1) and the target trajectories (Fig. 2) along the iteration axis. While the presented standard AILC (3)-(5) cannot.

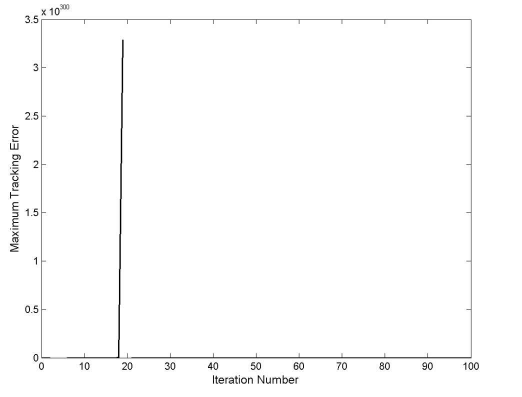

Fig. 6. Finite escape phenomenon with nonsector nonlinearities by means of the presented AILC (3)-(5).

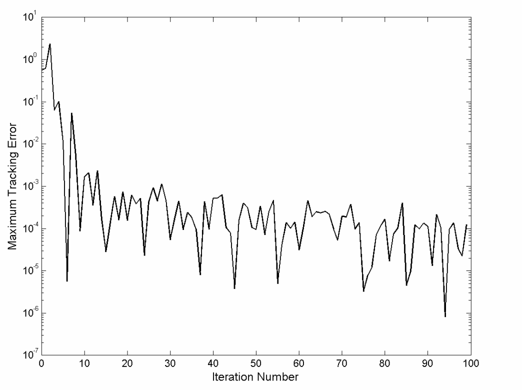

Fig. 7. The asymptotic convergence of the tracking error for systems with nonsector nonlinearities by means of the modified AILC (3), (19), (20).

6. CONCLUSIONS

A new discrete-time adaptive ILC approach is presented. By parameterization, the new AILC constitutes a nonlinear iterative learning mechanism, and incorporates a RLS in the learning mechanism to update the parametric learning gain. Comparing with the existing ILC, the new AILC and its modification can perform well when the initial state value and the target trajectory are varying along the iteration axis. Meanwhile, an extension of the new AILC is presented by using nonlinear data weighting such that it can cope with systems without assuming any growth conditions on the nonlinearities. Both the theoretical analysis and simulation results confirm the effectiveness of the presented AILC method and its extension.

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