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Testing the Covariance Matrix of the Innovation Sequence in Application to Aircraft Sensor Fault Detection

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Abstract: Operative methods of testing the covariance matrix of the innovation sequence of the Kalman filter are proposed. The quadratic form of the random Wishart matrix is used in this process as monitoring statistic, and the testing problem is reduced to the classical problem of minimization of a quadratic form on the unit sphere. As a result, two algorithms for testing the covariance matrix of the innovation sequence are proposed. In the first algorithm, the sum of all the elements of the matrix is used for the scalar measure of the Wishart matrix being tested, while in the second algorithm the maximal eigenvalue of this matrix is used. In the simulations, the longitudinal and lateral dynamics of the F-16 aircraft model is considered, and detection of pitch rate gyro failures, which affect the covariance matrix of the innovation sequence, are examined. Some recommendations for the fastest detection of failure are given.

Keywords: Fault detection and diagnosis, Estimation and filtering, Statistical methods/signal analysis for FDI, Aerospace applications

1. INTRODUCTION

The problem of detection, in real time, of faults in systems of estimation appears in many problems of navigation and control (Golovan and Mironovskii, 1993; Chen and Patton, 1999; Hajiyev and Caliskan, 2003). Abnormal measurements, sudden shifts appearing in the measuring channel, faultiness of measuring devices, changes in statistical characteristics of noises of an object or of measurements, malfunctions in the computer, and also a sharp change in the trajectory of a control process, etc. should be enumerated among these faults.

In real situations of exploiting an object, the problem occurs of operative detection of such changes in order to subsequently correct estimators or to make timely decisions on the necessity and character of control actions with respect to the process of technical exploitation of the object. Under this process, different methods of control and diagnostics are used, a brief survey of these methods are given in (Hajiyev and Caliskan, 2003).

In this direction of studies, it is necessary to mention the theory of diagnostics of a dynamic system by the innovation sequence of the Kalman filter, which has been extensively developed in recent years. The advantages of these methods are as follows: they provide the supervision of the correctness of the result obtained by current working input actions, they do not require a priori information on the values of changes in the statistical characteristics of the innovation sequence in the case of fault; they allow one to solve the fault detection problem in real time; they require small computational expenditures for their realizations since they do not increase, in contrast to the most algorithmic methods, the dimension of the initial problem.

As is known (Mehra and Peschon , 1971) in the case when a system is normally operated, the normalized innovation sequence in the Kalman filter compatible with the model of dynamics is the white Gaussian noise with zero mean and identity covariance matrix. The faults appearing in the system of estimations lead to the changes in these statistical characteristics of the normalized innovation sequence. Therefore, in this case, the fault detection problem is reduced to the problem of fastest detection of the deviation of these characteristics from nominal.

The methods of testing the correspondence between the innovation sequence and the white noise and of revealing the change of its expectation are based on the classical statistical methods and are considered in detail in the literature (Willsky, 1976; Himmelblau, 1978) therefore, it shall not be concentrated on testing these characteristics.

Testing, in real time, the covariance matrix of the innovation sequence of the Kalman filter turns out to be very complicated and not well developed, since there are difficulties in the determination of the confidence domain for a random matrix. Moreover, the existing methods of high-dimensional statistical analysis (Kendall and Stuart, 1976; Anderson, 1984) usually lead to asymptotic distributions; this sharply diminishes the operativeness of these methods. In practice, therefore, one makes use of a scalar measure of this matrix such as the trace (Mehra and Peschon, 1971), the sum of the matrix elements, generalized variance (determinant), the maximal eigenvalue of a matrix, etc., each characterizing one or another geometrical parameter of the correlation ellipsoid. Although the trace of the sample covariance matrix is the easiest to check, it might lead to incorrect decisions at detection of faults, because it disregards the off-diagonal matrix elements. The method of testing the covariance matrix of the innovation sequence proposed in (Gadzhiev, 1992) on the basis of using the statistics of the ratio of two quadratic forms, whose matrices are reversed sample and theoretical covariance matrices, is free from the above-mentioned shortcoming. Nevertheless, the results obtained in (Gadzhiev, 1992) are valid only in the case where the reversed matrices which enter the expression of the controlled statistics are nonsingular.

In the present paper the sum of all the elements and the maximal eigenvalue of the matrix are used for the scalar measure of the Wishart matrix being tested. In contrast to (Gadzhiev, 1992), the procedure of matrix reversing is excluded from the testing algorithm.

2. STATEMENT OF THE PROBLEM

Let us consider a class of systems described by differential equations of the form

$$\begin{aligned} x(k+1) &= \Phi(k+1,k)x(k) + G(k+1,k)w(k) \\ z(k) &= H(k)x(k) + V(k)\,, \end{aligned} \tag{1}$$

where $x(k)$ is the n-dimensional state vector of the system, $\Phi(k+1,k)$ is the transition matrix of order nxn of the system, $w(k)$ is the random n-dimensional vector of system noises, $G(k+1,k)$ is the transition matrix of system noises of order nxn, $z(k)$ is the s-dimensional measurement vector, $H(k)$ is the measurement matrix of the system of order sxn, $v(k)$ is the random s-dimensional vector of measurement noises. It is assumed that the random vectors $w(k)$, $v(k)$, and $x(0)$ are mutually independent white Gaussian processes with zero expectations and covariance matrices defined by the relations: $E[w(k)w^T(j)] = Q(k)\delta(kj)$, $E[v(k)v^T(j)] = R(k)\delta(kj)$, $E[x(0)x^T(0)] = P(0)$, where $\delta(kj)$ is the Kronecker symbol.

Under the above-mentioned a priori information, the estimator $\hat{x}(k/k)$ of the state vector and the covariance matrix of errors $P(k/k)$ are found with the help of the optimal Kalman filter (Sage and Melsa, 1971). Moreover, if the optimal filter is normally operating, then the normalized innovation sequence

$$\begin{aligned} \tilde{v}(k) &= [H(k)P(k/k-1)H^T(k) + R(k)]^{-1/2} \\ &\times [z(k) - H(k)x(k/k-1)] \end{aligned} \tag{2}$$

is a white Gaussian noise with zero mean and identity covariance matrix (Mehra and Peschon , 1971): $E[\tilde{v}(k)] = 0$, $E[\tilde{v}(k)\tilde{v}^T(j)] = P_{\tilde{v}} = I\delta(kj)$,

where $\hat{x}(k/k-1)$ is the extrapolation value by one step,

$P(k/k-1) = \Phi(k,k-1)P(k-1/k-1)\Phi^T(k,k-1)$
$+G(k,k-1)Q(k-1)G^T(k,k-1)$

is the covariance matrix of extrapolation errors, $P(k-1/k-1)$ is the covariance matrix of estimation errors in the preceding step, I is the identity matrix.

The changes in the properties of the system or characteristics of perturbations (faults of measuring devices, abnormal measurements, changes in statistical characteristics of noises of the object or of measurements, etc) leading to a change in the covariance matrix of the innovation sequence (2) are considered.

It is of interest to develop an operative method of testing the covariance matrix of sequence (2).

3. ALGORITHM OF SOLUTION

Let us introduce the following two hypotheses: γ_o, the estimation system is normally operating; γ_1, there is a fault. Let us write the expression for the sample covariance matrix of the sequence $\tilde{v}(k)$:

$$\hat{S}(k) = \frac{1}{M-1}\sum_{j=k-M+1}^{k}[\tilde{v}(j) - \bar{\tilde{v}}(k)][\tilde{v}(j) - \bar{\tilde{v}}(k)]^T \tag{3}$$

where

$$\bar{\tilde{v}}(k) = \frac{1}{M}\sum_{j=k-M+1}^{k}\tilde{v}(j) \tag{4}$$

is the sample mean; M is the number of realizations used (the width of the sliding window).

As is known (Anderson, 1984), under the validity of the hypotheses γ_o, the random matrix

$$A(k) = (M-1)\hat{S}(k) \tag{5}$$

has the Wishart distribution with M degrees of freedom and is denoted by $W_s(M, P_{\tilde{v}})$:

$$A \sim W_s(M, P_{\tilde{v}})\,, \tag{6}$$

where s and $P_{\tilde{v}}$ dimension and covariance matrix of the normalized innovation sequence $\tilde{v}$ respectively. In testing statistical hypotheses, the testing of the Wishart statistics (6) is complicated and not well developed in view of the difficulty of constructing the confidence domain for a random matrix. In practice, one of the scalar measures of the above-mentioned matrix is usually applied for testing random matrices. The choice of one or another scalar measure as the monitoring statistic for a particular problem being solved depends on the basic indicators of supervision (the sensitivity, the inertia, the volume of computational expenditures, etc) The experience and intuition of a researcher is also of importance. In what follows, the construction of confidence intervals for the sum of all elements of the matrix $A(k)$ and also for the maximal eigenvalue of this matrix are considered. Given below are new results for distribution (6).

Theorem 1. Let $A \sim W_s(M, P_{\tilde{V}})$. Then the ratio of the sums of all the elements of the matrices A and $P_{\tilde{V}}$ is distributed as $\chi^2(k)$, that is

$$\frac{\sum_{i=1}^{s}\sum_{j=1}^{s} a_{ij}}{\sum_{i=1}^{s}\sum_{j=1}^{s} \sigma_{ij}} \sim \chi_M^2 \tag{7}$$

where a_{ij} and σ_{ij} are elements of the matrices A and $P_{\tilde{V}}$, respectively.

Proof. As is known (Rao, 1965), if a random matrix A obeys the Wishart distribution $W_s(M, P_{\tilde{V}})$ and L is a fixed vector, then the quadratic form

$$L^T AL \sim \sigma_L^2 \chi_M^2 , \tag{8}$$

where $\sigma_L^2 = L^T P_{\tilde{V}} L$.

The unit vector $I_s^T = (1,1,...1)$ is used as the vector L. In this case expression (8) can be represented in the form

$$\frac{I_s^T A I_s}{I_s^T P_{\tilde{V}} I_s} \sim \chi_M^2 \tag{9}$$

Allowing for $I_s^T A I_s = \sum_{i=1}^{s}\sum_{j=1}^{s} \alpha_{ij}$ and $I_s^T P_{\tilde{V}} I_s = \sum_{i=1}^{s}\sum_{j=1}^{s} \sigma_{ij}$, the sought expression (7) follows from Eq. (9).

Corollary 1. If $A \sim W_s(M, I)$, where I is the unit matrix, then the sum of all the elements of the matrix A is

$$\sum_{i=1}^{s}\sum_{j=1}^{s} a_{ij} \sim s\chi_M^2 \tag{10}$$

Proof. Substituting the unit matrix I for $P_{\tilde{V}}$ in (9) with allowance for $I_s^T I I_s = s$, the required expression (10) is obtained.

Theorem 2. If a random matrix A obeys the Wishart distribution $W_s(M, P_{\tilde{V}})$ and L_{opt} is the eigenvector corresponding to its maximum value, then the maximum eigenvalue $\lambda_{\max}(A)$ of this matrix obeys the following distribution law:

$$\lambda_{\max}(A) \sim L_{opt}^T P_{\tilde{V}} L_{opt} \chi_M^2 \tag{11}$$

Proof. As is known (Rao, 1965), if $A \sim W_s(M, P_{\tilde{V}})$ and L is a fixed vector, then the following relation holds

$$l = L^T AL \sim L^T P_{\tilde{V}} L \chi_M^2 \tag{12}$$

The vector L that maximizes l subject to the condition $L^T L = 1$ is sought, that is, we pose the problem of maximizing the quadratic form (11) on the unit sphere. The solution to this problem is achieved on the eigenvectors of the matrix of the quadratic form, that is, in this case the optimum vector L_{opt} is the eigenvector of the matrix A corresponding to its maximum value. Since matrix A is symmetric, the maximum value of the quadratic form $L^T AL$ (on condition that $L^T L = 1$) is equal to the largest eigenvalue of the matrix A (Horn and Johnson, 1986), that is

$$\max_{L^T L=1} L^T AL = \lambda_{\max}(A) \tag{13}$$

Invoking (12) and (13), we see that $\lambda_{\max}(A) \sim L_{opt}^T P_{\tilde{V}} L_{opt} \chi_M^2$, which was to be proved.

Corollary 2. If $A \sim W_s(M, I)$, then the maximum eigenvalue of the matrix A obeys the distribution law χ_M^2, that is,

$$\lambda_{\max}(A) \sim \chi_M^2 \tag{14}$$

Proof. Indeed, substituting the unit matrix I for $P_{\tilde{V}}$ in (11) and allowing for the normalization condition $L^T L = 1$, we obtain the sought expression (14).

By selecting α level of significance as,

$$P\{\chi^2 > \chi_{\alpha,M}^2\} = \alpha ; \quad 0 < \alpha < 1$$

So from the equation above, the threshold value $\chi_{\alpha,M}^2$ will be determined. Under the validity of the hypotheses γ_1, the left hand side of expression (14) tends to exceed the threshold value $\chi_{\alpha,M}^2$. Then the decision rule on the current state of the system of estimation with respect to the introduced hypotheses will be written in the form

$$\gamma_o : l_{opt}(k) = \lambda_{\max}(k) \le \chi_{\alpha,M}^2 \qquad \text{fault free}$$

$$\gamma_1 : \lambda_{\max}(k) > \chi_{\alpha,M}^2 \qquad \text{with fault} \tag{15}$$

where $\lambda_{\max}(k)$ is the maximal eigenvalues of the matrix $A(k)$. Since $A(k)$ is a positive definite matrix, its maximal eigenvalues is equal to the spectral radius $\rho_A(k)$:

$$\rho_A(k) = \max\{|\lambda_i(k)|, i = \overline{1, s}\} = \lambda_{\max}(k) \tag{16}$$

Thus, from the computational point of view the supervision method of the estimation system considered is reduced to comparing the values of the maximal eigenvalues (spectral radius) of the above-introduced Wishart matrix (5) calculated on the basis of a representative sample and $\chi^2_{\alpha,M}$, and making a decision on the basis of the decision rule (15).

On the basis of the results obtained, two fault detection algorithms are proposed. The first algorithm is based on the calculation of statistic (10) i.e., for the fault detection the sum of all elements of the matrix $A(k)$ is used. The second fault detection algorithm is based on the computation of the maximal eigenvalue of the matrix $A(k)$.

4. EKF FOR THE F-16 AIRCRAFT MODEL ESTIMATION

The technique for failure detection is applied to an unstable multi-input multi-output model of an AFTI/F-16 fighter. The fighter is stabilized by means of a linear quadratic optimal controller. The control gain brings all the eigenvalues that are outside the unit circle, inside the unit circle. It also keeps the mechanical limits on the deflections of control surfaces. The model of the fighter is as follows (Lyshevski, 1997):

$$x(k+1) = Ax(k) + Bu(k) + F(x(k)) + w(k) \tag{17}$$

The state variables are: $x = [v, \alpha, q, \theta, \beta, p, r, \phi, \psi]^T$,
where, v is the forward velocity, α is the angle of attack, q is the pitch rate, θ is the pitch angle, β is the side-slip angle, p is the roll rate, r is the yaw rate, ϕ is the roll angle, and ψ is the yaw angle, $w(k)$ is the system noise with zero mean and the correlation matrix $E[w(k)w^T(j)]=Q(k)\delta(kj)$. The fighter has six control surfaces and hence six control inputs are: $u = [\delta_{HR}, \delta_{HL}, \delta_{FR}, \delta_{FL}, \delta_C, \delta_R]$, where δ_{HR} and δ_{HL} are the deflections of the right and left horizontal stabilizers, δ_{FR} and δ_{FL} are the deflections of the right and left flaps, δ_C and δ_R are the canard and rudder deflections. A, B, and $F(x)$ are calculated for the sampling period of 0.03 s.

Let us define the estimated vector as:

$$x^T(k) = [v(k), \alpha(k), q(k), \theta(k), \beta(k), p(k), r(k), \phi(k), \psi(k)]$$

and apply the Kalman filter to estimate this vector. The measurement equations can be written as:

$$z(k)=Hx(k)+v(k), \tag{18}$$

where H is the measurement matrix, which is 9×9 unit matrix, $v(k)$-measurement noise with zero mean and the correlation matrix $E[v(k)v^T(j)]=R(k)\delta(kj)$. By using quasi-linearization method let us linearize the equation (17):

$$x(k) = A\hat{x}(k-1) + B\hat{u}(k-1) + F(\hat{x}(k-1)) + A[x(k-1) - \hat{x}(k-1)] + F_x(k-1)[x(k-1) - \hat{x}(k-1)] + B[u(k-1) - \hat{u}(k-1)] + w(k-1) \tag{19}$$

where $F_x = \left[\frac{\partial F}{\partial x}\right]_{\hat{x}(k-1)}$.

The following recursive EKF algorithm for the state vector estimation of the F-16 fighter motion is obtained in (Caliskan and Hajiyev, 2003):

$$\begin{aligned}
&\hat{x}(k) = A\hat{x}(k-1) + B\hat{u}(k-1) + \\
&F(\hat{x}(k-1)) + P(k)H^T R^{-1}(k) \times v(k) \\
&v(k) = z(k) - H[A\hat{x}(k-1) + B\hat{u}(k-1) + F(\hat{x}(k-1))] \\
&P(k) = M(k) - M(k)H^T [R(k) + HM(k)H^T]^{-1} HM(k) \\
&M(k) = AP(k-1)A^T + BD_u(k-1)B^T + \\
&F_x(k-1)P(k-1)F_x^T(k-1) + GQ(k-1)G^T
\end{aligned} \tag{20}$$

where $M(k)$ is the covariance matrix of the extrapolation error, D_u is the covariance matrix of the control input error.

5. SENSOR FAULT DETECTION SIMULATION RESULTS

Let us show that, on the basis of the algorithms for testing the covariance matrix of the innovation sequence proposed in this paper, one can in a timely manner detect the faults appearing in the measuring channel. Measurements were processed using the Extended Kalman filter (20). The expressions for the innovation sequence and the normalized innovation sequence of EKF respectively are:

$$v(k) = z(k) - H\left[A\hat{x}(k-1) + B\hat{u}(k-1) + F\left(\hat{x}(k-1)\right)\right] \tag{21}$$

$$\tilde{v}(k) = \left[R(k) + HM(k)H^T\right]^{-1/2} v(k) \tag{22}$$

To detect failures changing the covariance matrix of the innovation sequence the above statistics (10) and (14) are used. In the simulations, $M = 20$, s=9, and $\alpha = 0.05$ are taken, and the threshold values $\chi^2_{\alpha,M}$ and $s\chi^2_{\alpha,M}$ are found as 31.41 and 282.69. Obtained results are presented in Fig.1-6. Figures 1 and 2 shows respectively admissible bounds of the statistics $\Sigma_a(k) = \sum_{i=1}^{s}\sum_{j=1}^{s} a_{ij}(k)$ and $\lambda_{\max}(k)$ and the plots of their behaviors in the case of normal functioning of the all measurement channels. As is expected, at all points, $\Sigma_a(k)$ <282.69 and $\lambda_{\max}(k)$ < 31.41. The corresponding normalized innovation sequence in the third measurement channel (pitch rate gyroscope channel) $\tilde{v}_q(k)$ is shown in Fig. 3. The graphs of the normalized innovation sequences in the other measurement channels are very similar to the ones in Fig.3.

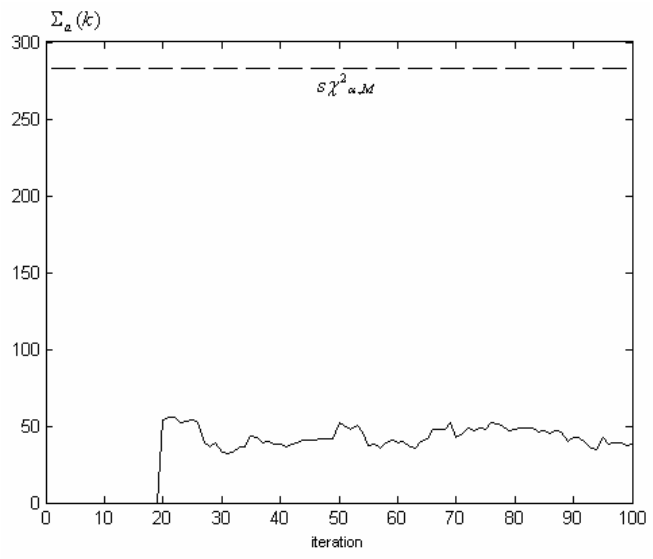

Fig.1. Graph of the statistic $\Sigma_a(k)$ for normal operating of the measurement channels

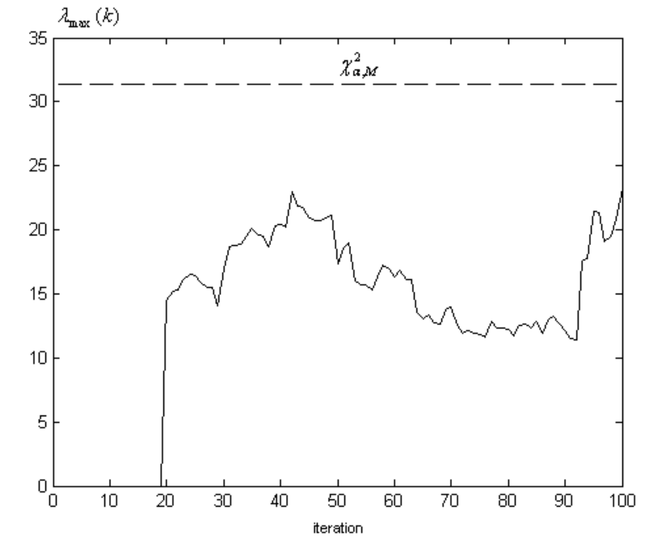

Fig.2. Graph of the statistic $\lambda_{\max}(k)$ for normal operating of the measurement channels

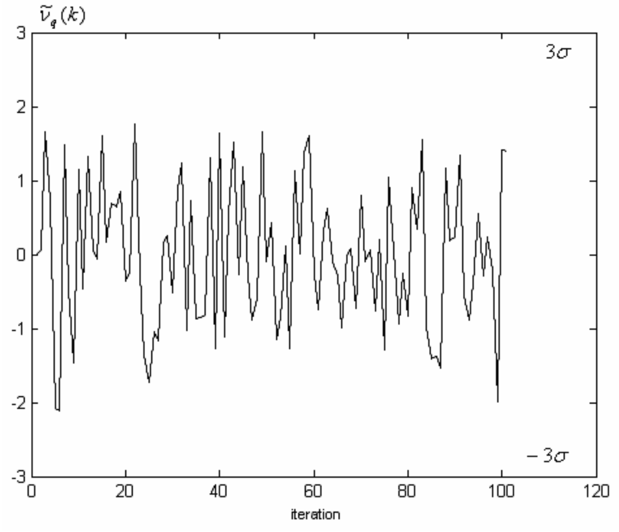

Fig.3. Behaviour of the $\tilde{v}_q(k)$ in the case of normal operating of the measurement channels

To verify efficiency of proposed algorithms, beginning from the step k=30, a fault in the third measurement channel (pitch rate gyroscope fault) is simulated (the noise variance in the pitch rate gyroscope is changed). The simulation results corresponding to this case are presented in Fig.4-6. Figure 4 shows that the value of $\Sigma_a(k)$ increases after the 30th step and intersects its threshold at the step *k=95*. As a result, based on the decision rule (16), estimation system failure is noted.

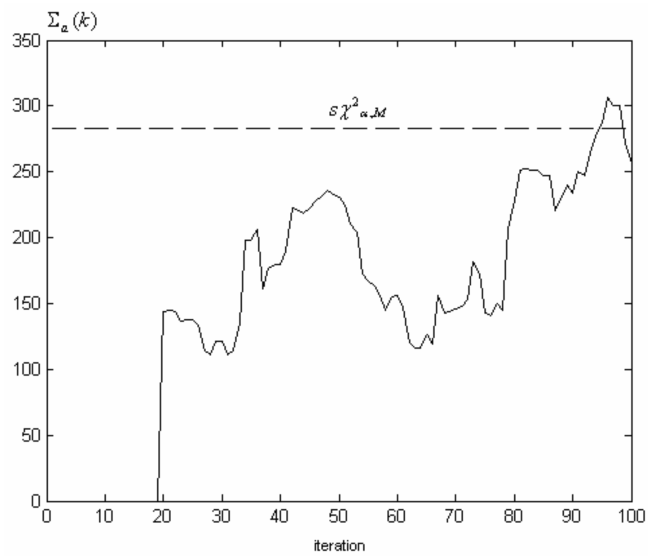

Fig.4. Behaviour of the statistic $\Sigma_a(k)$ in case of changes in noise variance in the pitch rate gyroscope

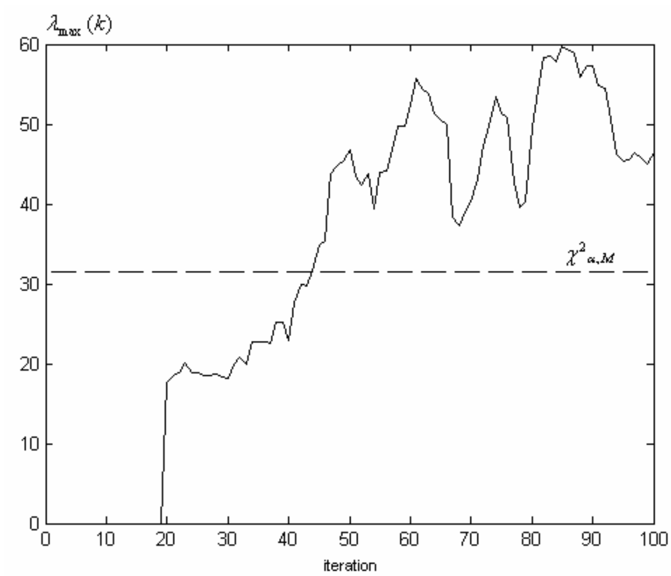

Fig.5. Behaviour of the statistic $\lambda_{\max}(k)$ in case of changes in noise variance in the pitch rate gyroscope

Figure 5 shows behaviour of the statistic $\lambda_{\max}(k)$ in case of changes at the 30th step in noise variance in the pitch rate gyroscope. Plots show that the value of the statistic $\lambda_{\max}(k)$ in this case after the 30th step grows abruptly, and at the step *k=44* it exceeds its admissible bound. As a result, using the decision rule (15) the fault in the estimation system is detected. The behavior of the appropriate normalized innovation sequences $\tilde{v}_q(k)$ is presented in the Fig. 6.

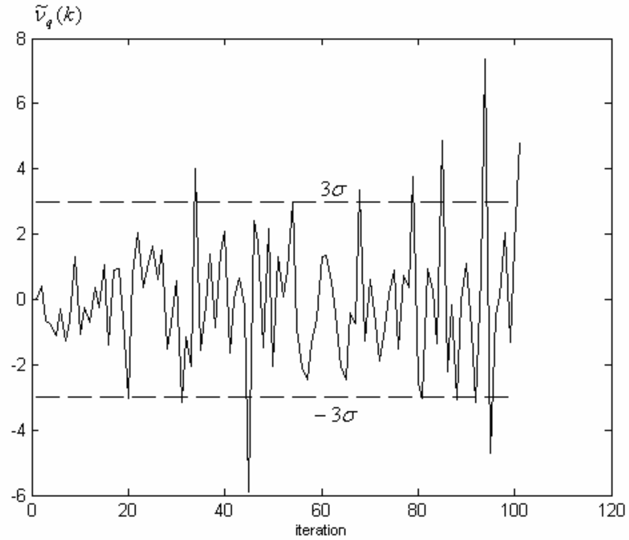

Fig.6. Behavior of the normalized innovation sequence $\tilde{v}_q(k)$ in case of changes in noise variance in the pitch rate gyroscope

The analysis of the results obtained shows that the statistic $\lambda_{\max}(k)$ is more effective in the sense of the fastest detection of a fault.

The introduction of developed fault detection algorithms does not distort the results of the estimates of the filter and has no influence on their accuracy. On the whole, the simulation results justify the theoretical calculation obtained and show the practical applicability of the proposed fault detection algorithms.

6. CONCLUSION

In this paper operative methods of testing the covariance matrix of the innovation sequence of the Kalman filter are proposed. The approach proposed under this process is based on the use of a quadratic form of the random Wishart matrix (5), which under the normal operation of the filter has the χ^2 distribution. The optimization of quality of testing is reduced to the classical problem of maximization of a quadratic form on the unit sphere. The solution of this problem is attained on the eigenvector of the matrix of this quadratic form (in this case, the Wishart matrix) corresponding to the maximal eigenvalues of this matrix.

An extended Kalman filter has been developed for nonlinear flight dynamic estimation of an F-16 fighter. Failures in the sensors affect the characteristics of the innovation sequence of the EKF. The failures that affect the variance of the innovation sequence have been considered. The theoretical results are confirmed by the simulations carried out on a nonlinear dynamic model of the F-16 aircraft.

A lag (delay time) of fault detection in this case depends on the number M of realizations used; this characteristic being deteriorating with its increase. On the other hand, a too small value of M causes often false failures. The value of is specified heuristically, since a theoretically substantiated technique for choosing M does not exist at present.

The algorithms proposed in this paper do not require a priori information on the change of the covariance matrices of the innovation sequences in the case of a fault and can be used in the problems of fault detection and fault diagnosis of dynamic systems.

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