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Global discrete-time stabilization of the PVTOL aircraft based on fast predictive control

Ahmed Chemori * **Nicolas Marchand** **

* *LIRMM, Univ. Montpellier 2 - CNRS, 161, rue Ada, 34392 Montpellier Cedex 05, France*
** *GIPSA-lab, Control Systems Departement (former Laboratoire d'Automatique de Grenoble), CNRS-Grenoble, ENSIEG BP 46, Domaine Universitaire, 38400 Saint Martin d'Heres - France*

Abstract: This paper deals with stabilization control of a non-minimum phase under-actuated Planar Vertical Take-Off and Landing (PVTOL) aircraft. The proposed control approach, inspired from that proposed in Poulin et al. (2007), is based on a discrete time model of the PVTOL and receding horizon technique to take into account constraints on the control inputs (positivity and boundedness). State constraints can also be handled. The computational cost is reduced by decoupling the optimization problem into two QP problems of reduced dimensions. The minimized cost functions proposed here extends the previous work by allowing state and control weight. The proposed control approach is illustrated through simulation case studies including stabilization and robustness towards parameters uncertainties.

Keywords: PVTOL aircraft, under-actuation, non-minimum phase, receding horizon control, global stabilization, quadratic optimization.

1. INTRODUCTION

Planar and Vertical Take Off and Landing (PVTOL) aircrafts are those flying machines which are able to take-off and land vertically like a helicopter, but have the ability to fly with the efficiency of an airplane. Their simplified dynamics, introduced by Hauser et al. (1992a), with three degree of freedom and two control inputs has become a benchmark of non-minimum phase control problems. It can be seen as the projection of a six degree of freedom flying body into a vertical plane attached to the body. This dynamics includes many difficulties that explain the popularity of this model. It is for instance the *under-actuation* (three degrees of freedom for only two controls), or the *non-minimum phase* property (unstable zero dynamics). This system also concentrates all the difficulties of the well known Brockett's integrator (also referred the unicycle) that one gets by neglecting the coupling factor and the gravity. Within this context a great number of methods have been proposed to control it. The proposed control approaches can be classified into two families: *trajectory tracking control* approaches and *stabilization control* approaches.

In the case of trajectory tracking two subclasses exist : linearization based approaches and decomposition based approaches. Within the first sub-class, Hauser et al. (1992a) have proposed a feedback control strategy based on an approximate input/output linearization, where the non-minimum phase system is approximated by a minimum phase one. The proposed approach results in bounded tracking and stabilization for V/STOL aircrafts. Martin et al. (1996) have proposed an extension of Hauser's approach. The proposed control strategy uses Huygens center of oscillation as a flat output of the VTOL model, then a dynamic state feedback is used to exactly linearize the model. The advantage of such a method is that it works for both small and large coupling parameter. Inspired from the approach of Okou et al. (1999), Huang and Yuan (2002) have proposed a control scheme that generates an input/output linearization while insuring with Lyapunov arguments. In decomposition based approaches AL-Hiddabi et al. (1999) have proposed a scheme where the model is decomposed into a minimum and a non-minimum phase parts, then a dynamic inversion is used for the former, while a robust stabilizing feedback control, based on LQR approach, is applied to the linearized model around the equilibrium point for the latter.

Stabilization is aimed to take the system states from some initial condition to the origin. The proposed approaches to solve this problem could be classified into two sub-classes: unbounded controls and bounded controls. In unbounded controls sub-class, Saeki and Sakaue (2001) have proposed a design method which makes use of the center of oscillation and a two-step linearization, to stabilize the aircraft. Olfati-Saber (2002) has addressed the problem of global configuration stabilization of the PVTOL aircraft with strong input coupling using smooth static state feedback. Lin et al. (1999) have proposed a robust stabilization of the PVTOL aircraft, where the objective is formulated as an optimal control problem. Within bounded controls sub-class, the output constraints as well as input control constraints are taken into account in the design of the control approach. For instance, Castillo et al. (2002) propose a control approach that takes into account inputs constraints and does not have any singularities. The control algorithm has been obtained by imposing a desired dynamics to the x and y sub-systems such that both corresponding linear acceleration and velocities are bounded. Zavala et al. (2003); Fantoni et al. (2002) have proposed a global stabilizing control based on the use of nonlinear combinations of linear saturation functions bounding the thrust input and the rolling moment to arbitrary saturation limits. The convergence of these approaches is considerably improved in (Hably et al., 2006).

In this paper a fast model predictive control is proposed for stabilization of the PVTOL aircraft. The terminology "fast" means that the structure of the system is used in order to design an optimization scheme very suitable for computation (Alamir, 2006). As in (Poulin et al., 2007), an exact discrete-time model is given. The PVTOL aircraft system can then be considered as two interconnected linear systems resulting in two convex optimization problem that can be solved efficiently and successively with quadratic programming algorithms (QP). This paper extends (Poulin et al., 2007) by allowing classical quadratic cost functions of the control and the states. This reveals better robustness performances.

The outline of the paper is as follows. In Section 2 the dynamic model of the PVTOL aircraft is described. Section 3 is devoted to detail the proposed control approach. Numerical simulations are proposed in Section 4 to illustrate the control strategy. Finally, concluding remarks are drawn in Section 5.

2. DYNAMICAL MODEL OF THE PVTOL AIRCRAFT

Consider the widely used simplified PVTOL dynamics first introduced by Hauser et al. (1992b):

$$\ddot{x} = -\sin(\theta)u + \varepsilon\cos(\theta)v \quad (1)$$

$$\ddot{y} = \cos(\theta)u + \varepsilon\sin(\theta)v - 1 \quad (2)$$

$$\ddot{\theta} = v \quad (3)$$

where x and y represent Cartesian positions of the center of mass of the aircraft as shown in Figure 1. θ is the roll angle that the aircraft makes with the horizon. The control inputs u and v represent normalized quantities related to the vertical thrust directed upwards with respect to the aircraft and the rolling moment. ε denotes the coupling parameter between the rolling moment and the lateral acceleration of the aircraft. The constant term $'-1'$ in (2) corresponds to the normalized gravity.

3. PROPOSED CONTROL SCHEME

3.1 Open-loop control problem formulation

The proposed control law is based on the classical framework of predictive control. This consist, at any time t, in finding a control function $(u(\tau), v(\tau))$, $\tau \in [t, t+t_f]$ defined on a time horizon t_f that drives the state of the system from $z(t) = (x(t), \dot{x}(t), y(t), \dot{y}(t), \theta(t), \dot{\theta}(t))$ to some desired final state $z(t+t_f) = (x(t+t_f), \dot{x}(t+t_f), y(t+t_f), \dot{y}(t+t_f), \theta(t+t_f), \dot{\theta}(t+t_f))$. The update at each time t of this control sequence yield a feedback. If many formulations in continuous time exists, one way to solve this problem is to discretize it with some sampling period T such that $t_f = nT$, assuming constant control over sampling periods, in order to find a control sequence

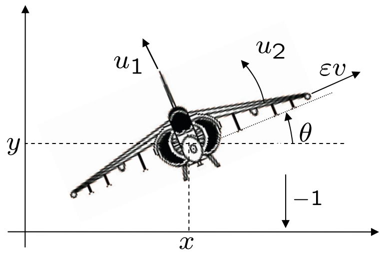

Fig. 1. View of the PVTOL in the frontal plane

$$U(t) = [u_0 \; u_1 \; \cdots \; u_{n-1}]^T, V(t) = [v_0 \; v_1 \; \cdots \; v_{n-1}]^T$$

that drives the system to its final desired configuration. In the case of the PVTOL aircraft system, the controls must in addition verify the constraints $0 \le u_k \le u^{\max}$ and $v^{\min} \le v_k \le v^{\max}$ with for obvious controllability reasons $u^{\max} > 1$. In the remainder of the paper, the subscript k will denote the value at time $t + kT$. The basic idea of the control approach is to subdivide the control problem into two subproblems. In the first one, the control v is used to stabilize θ and $\dot{\theta}$. While the second one deals with stabilization of x, y and their derivatives whatever the choice made for v. This is only possible if the evolution of θ is sufficiently *"rich"*. By rich is inferred two main phenomena. First, due to the positivity of the control u, θ must evolves positively and negatively on the time interval $[t, t+nT]$ in order to preserve the controllability of the rest of the system even in the case of null coupling. For this, the dynamic (3) is completed with $\dot{I}_\theta = \theta$. If I_θ starts and ends at the origin, the controllability of the $(x, \dot{x}, \theta, \dot{\theta})$ will be preserved. The second possible obstruction that we will discuss later on corresponds to a loss of controllability if θ vanishes.

Rotational stabilization Consider now the extended subsystem $\Theta := (I_\theta, \theta, \dot{\theta})^T$ that corresponds to a third order chain of integrators and takes its discretization:

$$\Theta_{k+1} = A_\theta \Theta_k + B_\theta v_k$$

By recurrence, Θ_k can be expressed as:

$$\Theta_k = A_\theta^k \Theta_0 + \left(A_\theta^{k-1} B_\theta \; A_\theta^{k-2} B_\theta \; \cdots \; B_\theta\right) \begin{pmatrix} v_0 \\ \vdots \\ v_{k-1} \end{pmatrix} \quad (4)$$

Let $\tilde{V}(n, \Theta(t), \Theta_f)$ denote, when it exists, the solution of the constrained QP problem expressed for the sake of simplicity in its null-stabilization formulation:

$$\mathcal{P}_v : \; \tilde{V} := \operatorname{Arg}\min_V \sum_{k=1}^{n} \Theta_k^T Q \Theta_k + r v_k^2 \quad (5)$$

(with $Q, r > 0$ are classical weight terms) subject to the constraints (4), $\Theta_0 = \Theta(t)$, $\Theta_n = \Theta_f$, and:

$$v^{\min} \le V_i \le v^{\max} \quad \text{for} \quad i = 0, \ldots, n-1$$

Any solution of $\mathcal{P}_v$ brings after n periods the rotational subsystem from $\Theta(t)$ to Θ_f with $v^{\min} \le v_i \le v^{\max}$. By convention, if $n = 0$, $\mathcal{P}_v$ admits the empty vector $V = (\cdot)$ as a solution. All classical arguments of linear optimal control apply especially the following useful property: if n is sufficiently large, $\mathcal{P}_v$ necessarily admits a non empty solution (Gauthier and Bornard, 1983).

Translational stabilization Let $\zeta := (x, \dot{x}, y, \dot{y})^T$ denote the translational variables. Then, based on (1) and (2) and after tedious computations one has:

$$\begin{aligned}\zeta_k = &\begin{pmatrix} 0 \\ 0 \\ -\frac{(kT)^2}{2} \\ -kT \end{pmatrix} + \begin{pmatrix} 1 & kT & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & kT \\ 0 & 0 & 0 & 1 \end{pmatrix} \zeta_0 \\ &+ (Q_1\Gamma_k + TQ_3\Gamma_k D_k) \begin{pmatrix} u_0 \\ \vdots \\ u_{k-1} \end{pmatrix} \\ &+ \varepsilon(Q_2\Gamma_k + TQ_4\Gamma_k D_k) \begin{pmatrix} v_0 \\ \vdots \\ v_{k-1} \end{pmatrix}\end{aligned} \tag{6}$$

where ε is the coupling parameter, Q_i's are permutation matrices, $D_k = \operatorname{diag}(k-1, \ldots, 1, 0)$. Using the fact the second time derivative of θ is constant, one has $\theta(\tau) = \alpha_k + \beta_k[\tau - (t+kT)] + \gamma_k[\tau - (t+kT)]^2$ with $\tau \in [t+kT, t+(k+1)T]$ and $\alpha_k := \theta(t+kT)$, $\beta_k := \dot{\theta}(t+kT)$, $\gamma_k := \frac{1}{2}v(t+kT)$. All the terms depending upon θ can be gathered in Γ_k that writes:

$$\Gamma_k = \begin{pmatrix} c_0 & c_1 & \ldots & c_{k-1} \\ s_0 & s_1 & \ldots & s_{k-1} \\ C_0 & C_1 & \ldots & C_{k-1} \\ S_0 & S_1 & \ldots & S_{k-1} \end{pmatrix}$$

with $s_k := \int_0^T \sin(\theta(\tau))d\tau$, $c_k := \int_0^T \cos(\theta(\tau))d\tau$, $S_k := \int_0^T \int_0^{\tau_2} \sin(\theta(\tau))d\tau_1 d\tau_2$ and $C_k := \int_0^T \int_0^{\tau_2} \cos(\theta(\tau))d\tau_1 d\tau_2$. These parameters can be very efficiently computed using the Fresnel integrals $s(z) = \int_0^z \sin\left(\frac{\pi}{2}\nu^2\right) d\nu$ and $c(z) = \int_0^z \cos\left(\frac{\pi}{2}\nu^2\right) d\nu$. For instance, the routine proposed by Zhang and Jin (1996) computes on a 1.8GHz Pentium with Matlab these two integrals in an average of 5.10^{-5} s with 22 iterations and a precision of 10^{-20}.

To find an open-loop bounded control sequence $U(t)$, one needs to invert (6). This requires that $Q_1\Gamma + TQ_3\Gamma D_n$ be full rank, which will not be proved here. This property, can formally be proved to hold if $n > 4$, $\theta(t) \neq \theta(t + t_f)$ and T sufficiently small (Poulin et al., 2007). However, the condition for small T can probably be relaxed since Γ, columnwise composed of linearly independent functions, should also be full rank. Let the control sequence $\tilde{U}(n, \tilde{V}, \eta(t), \zeta_f)$ denote the solution, when it exists, a solution of the following QP problem with weight terms $O, p > 0$:

$$\mathcal{P}_u : \tilde{U} := \operatorname{Arg}\min_U \sum_{k=1}^{n} \zeta_k^T O \zeta_k + p u_{k-1}^2 \tag{7}$$

subject to (6), $\zeta_0 = \zeta(t)$, $\zeta_n = \zeta_f$, and:

$$0 \leq U_i \leq u^{\max} \quad \text{for} \quad i = 0, \ldots, n-1$$

Any solution of $\mathcal{P}_u$ brings after n periods the translational subsystem from $\zeta(t)$ to ζ_f with $0 \leq u_i \leq u^{\max}$. By convention, if $n = 0$ and therefore V is empty, $U = \{\cdot\}$ will be considered as a solution of $\mathcal{P}_u$.

3.2 Open-loop singularities avoidance

The idea is to place the system in a configuration that insures the existence of solutions to (5) and (7). Two closely related problems are possible. The first one deals with the singular initial condition $\theta(t) = \theta(t + nT) = 0$, resulting in a loss of controllability on x. This problem is classical in nonholonomic control when the linearized system is not controllable. Unfortunately, handling this case as a particular case, as often proposed in the literature on chained form systems, is not satisfactory. Indeed, if $\theta(t)$ and $\theta(t + t_f)$ are too close to each other, problem (5) will fail to have a bounded solution. The second problem already mentioned above occurs when the constraint on u is too restrictive with respect to the one on v. If for all n sufficiently large, one can insure with classical linear arguments that (5) admits a solution, the problem on ζ persists. Typically the optimal open-loop trajectory of $\theta(t)$ is to be first of the same sign as $\theta(0)$, then of opposite sign until θ joins the origin where it remains to the end of the predictive horizon. Hence, if θ vanishes "too fast" because of non restrictive constraints $(v^{\min}, v^{\max})$, it may be impossible to drive ζ to the origin with a too restrictive constraint $u^{\max}$ and increasing the horizon does not solve this problem. Hence, the control scheme allows to first move laterally by forcing θ to be nonzero before going to the origin. This insures that the system necessary falls in a configuration from which it can be brought to the origin by means of an a priori bounded control. Indeed, if for instance the system starts with some $x(0) > 0$, bringing θ to some $\theta_d > 0$ with v and taking u in order to compensate the gravity will produce a translation towards x negative (left). At some distance from the axis $x = 0$ that depends upon v_{max} and v_{min}, there will exists some control law that brings the system on the axis $x = 0$ where $\mathcal{P}_u$ and $\mathcal{P}_v$ obviously have a solution. For this, take (Yang et al. (1997); Marchand et al. (2007)):

$$y = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{T^2}{2} & \frac{3T^2}{2} \\ T & T \end{pmatrix}^{-1} \begin{pmatrix} \theta - \theta_d \\ \dot{\theta} \end{pmatrix}$$

and the following control with $\bar{v} := \min(v^{max}, -v^{min})$ and $0 < \gamma_1 < \gamma_2 < 1$:

$$v = \frac{-\bar{v}}{\gamma_1 + \gamma_2} \left[\gamma_1 \operatorname{sat}\left(\frac{y_1(\gamma_1 + \gamma_2)}{\bar{v}}\right) + \gamma_2 \operatorname{sat}\left(\frac{y_2(\gamma_1 + \gamma_2)}{\bar{v}}\right) \right] \tag{8}$$

The sign of θ_d must clearly be the same as x and $|\theta_d|$ must be not to large in order to be able to compensate the weight using:

$$u = \max\left(0, \operatorname{sat}_{u^{\max}}\left(\frac{1 - \varepsilon \sin\theta}{\cos\theta}\right)\right) \tag{9}$$

New control sequences $\hat{U}(n, \delta, \hat{V}, z(t), z_f)$ and $\hat{V}(n, \delta, \Theta(t), \Theta_f)$ follow when the controls (8-9) are first applied during δ sampling period before the control sequences $\tilde{U}(n, \tilde{V}, \zeta(t + \delta T), \zeta_f)$ et $\tilde{V}(n, \Theta(t + \delta T), \Theta_f)$ defined as above.

3.3 The closed-loop stabilizing feedback

The main result is introduced in the following theorem:

Theorem 1. Let $(\zeta_f, \Theta_f) = (x_f, 0, y_f, 0, 0, 0, 0)$ be some desired final configuration. Then the following algorithm, applied at each sampling period to $(\zeta(kT), \Theta(kT))$, globally asymptotically stabilizes the PVTOL aircraft to (ζ_f, Θ_f):

Algorithm:

- **Step 0**: Take a prediction horizon $t_f := NT$
- **Step 1**: Compute with $\mathcal{P}_u$ and $\mathcal{P}_v$ the set $\mathcal{C}$ of admissible control profiles defined by:

$$\mathcal{C}(\zeta(kT), \Theta(kT), \zeta_f, \Theta_f, N, \Delta) = \left\{ \begin{pmatrix} \hat{U}(n, \delta, \hat{V}, \zeta(kT), z_f) \\ \hat{V}(n, \delta, \Theta(kT), \Theta_f) \end{pmatrix}, \delta \in \{0, \ldots, \Delta\}, n \in \{0, \ldots, N\} \right\}$$

- **Step 2**: If $\mathcal{C} = \emptyset$ [1], which means that there is no control (U, V) fulfilling the constraints that drives the system from $(\zeta(kT), \Theta(kT))$ to (ζ_f, Θ_f) in the time NT, then increase N an go back to step 1. Otherwise, $\mathcal{C}$ contains the admissible open-loop control. Let $(U^{\text{opt}}, V^{\text{opt}})$ be defined by:

$$\begin{pmatrix} U^{\text{opt}}(kT) \\ V^{\text{opt}}(kT) \end{pmatrix} := \text{Arg} \min_{\binom{U}{V} \in \mathcal{C}} J(U, V) \tag{10}$$

  with $J(U, V) = U^T U + \mu V^T V$ and $\mu > 0$ and by convention $J((\cdot), (\cdot)) = 0$ if $(U, V) = (\cdot, \cdot)$.
- **Step 3**: The closed loop control is then given by:
  - ◇ if $(U^{\text{opt}}, V^{\text{opt}})$ are empty vectors:

$$\begin{cases} u(\zeta(kT), \Theta(kT)) := 1 \\ v(\zeta(kT), \Theta(kT)) := 0 \end{cases}$$

  - ◇ otherwise, the control to apply is the first element of $(U^{\text{opt}}, V^{\text{opt}})$:

$$\begin{cases} u(\zeta(kT), \Theta(kT)) := u_0^{\text{opt}} \\ v(\zeta(kT), \Theta(kT)) := v_0^{\text{opt}} \end{cases}$$

Note that J can be chosen linear in U instead of quadratic. As mentioned above, this choice highly depends on the relation between the thrust and the electrical fuel or any other used energy.

Proof : The key element of the proof consists in establishing the Bellman's invariance principle of the proposed scheme. In other words, we will check that for all k, the optimal solution of (10) at time kT, shifted by one sampling period, remains in the set of admissible control profiles $\mathcal{C}(\zeta((k+1)T), \Theta((k+1)T), \zeta_f, \Theta_f, N, \delta^{\max}, \beta)$ where the optimal solution at time $(k+1)T$ is searched. This property enables then to conclude on the stability of the proposed scheme.

Invariance principle: Let $(U^{\text{opt}}(kT), V^{\text{opt}}(kT))$ be the solution of (10) at time kT and n^{opt} the corresponding prediction horizon. Let $(U_s^{\text{opt}}(kT), V_s^{\text{opt}}(kT))$ denote the control sequence $(U^{\text{opt}}(kT), V^{\text{opt}}(kT))$ without its first element that correspond to the time instant kT:

$$\begin{pmatrix} U_s^{\text{opt}}(kT) \\ V_s^{\text{opt}}(kT) \end{pmatrix} := \begin{pmatrix} U_1^{\text{opt}}(kT), \ldots, U_{n^{\text{opt}}}^{\text{opt}}(kT) \\ V_1^{\text{opt}}(kT), \ldots, V_{n^{\text{opt}}}^{\text{opt}}(kT) \end{pmatrix}$$

Clearly $(U_s^{\text{opt}}(kT), V_s^{\text{opt}}(kT))$ steers the system from $(\zeta((k+1)T), \Theta((k+1)T))$ to (ζ_f, Θ_f) in $n^{\text{opt}} - 1$ sampling period and with the control constraints fulfilled. Moreover, $U_s^{\text{opt}}(kT)$ is a solution of $\mathcal{P}_u(T, n^{\text{opt}} - 1, V_s^{\text{opt}}(kT), \zeta((k+1)T), \zeta_f)$. We rule out the trivial case when $n^{\text{opt}} = 0$ meaning that the target final state is already reached, since the control law here clearly insures that the system will remain at the final state with an associated zero cost J. Then it follows clearly from the definition of $\mathcal{C}$ that $(U_s^{\text{opt}}(kT), V_s^{\text{opt}}(kT))$ belongs to $\mathcal{C}(\zeta((k+1)T), \Theta((k+1)T), \zeta_f, \Theta_f, N-1, \delta^{\max}, \beta)$ and hence to $\mathcal{C}(\zeta((k+1)T), \Theta((k+1)T), \zeta_f, \Theta_f, N, \delta^{\max}, \beta)$.

Stability: The stability follows from the above point. Indeed, taking J as Lyapunov function, one has:

- *J is necessarily decreasing*: This clearly follows from the above property. The cost J_{k+1} is necessarily lower to the cost associated to the shifted solution $(U_s^{\text{opt}}(kT), V_s^{\text{opt}}(kT))$ and therefore to the cost J_k.
- *N can not increase* on the closed loop trajectories of the system: Indeed, N is increased if and only if $\mathcal{C} = \emptyset$. If a solution exists at time k, then the shifted solution $(U_s^{\text{opt}}(kT), V_s^{\text{opt}}(kT))$ belongs to the set of admissible control at time $(k+1)T$ which is therefore necessarily not empty.
- *J necessarily converges to 0*: From the definition of J, $J_k = J_{k+1}$ if and only if $(u_0^{\text{opt}}(kT), v_0^{\text{opt}}(kT)) = (0, 0)$. Therefore, if J remains constant, necessarily it implies that a null control is applied. Under this control, the aircraft falls under the influence of the gravity and therefore moves away from the target point. But a constant cost J also implies that it remains possible to join this target point a the finite time lower than $n^{\text{opt}}T$ with a bounded control. Therefore, invoking a Lasalle's invariance principle argument, J can not remain constant forever except if $J = 0$.

With the above items, one may conclude to stability. Ineed, once J has converged to zero, it is straightforward that (z, Θ) converges in less that NT to (z_f, Θ_f) where it remains.

4. ILLUSTRATIVE SIMULATIONS

This section is devoted to simulation results obtained by application of the proposed control approach. For that two simulations were considered, the first one deals with the stabilization of the aircraft, while the second one concerns the robustness study of the proposed control scheme towards parameters uncertainty. The used parameters are $\varepsilon = 0.25$ for the coupling parameter, $t_f = 20$ s as prediction horizon, $T = 0.5$ s as sampling period, $u^{\max} = 4$ and $v^{\max} = -v^{\min} = 0.5$ as control bounds, $\delta = 1$ as singularity avoidance parameter. The weight on Θ in (5) is $Q_k = I_{3\times3}$, on V in (5) is $R = 5I_{k\times k}$, on ζ in (7) is $O_k = I_{4\times4}$, and the weight on U in (7) is $P = 5I_{k\times k}$. The initial conditions are as in (Fantoni et al., 2002) $(x(0), \dot{x}(0), y(0), \dot{y}(0), \theta(0), \dot{\theta}(0)) = (30, 0, 20, 0, \frac{3\pi}{5}, 0)$. Since θ exceeds $\frac{\pi}{2}$ many other methods like (Hauser et al., 1992b; Olfati-Saber, 2002) can not be used.

Scenario 1 : stabilization Figures 2-6 display the evolution of the system in a stabilization scheme. Figures 2 and 3 respectively show the translational and rotational movment of the PVTOL aircraft. The corresponding controls are plotted on figure 4, and the evolution of the cost function J on Figure 5. Finally, strobe snapshots of the moving aircraft in the plane are depicted on Figure 6. The convergence takes approximatively 13 s. In terms of energy

[1] If $U = (\cdot)$ and $V = (\cdot)$ are solutions of $\mathcal{C}$, we will consider that $\mathcal{C} \neq \emptyset$

consumption, the integral of u^2 needed to reach a ball of radius 0.05 around the final state is 65 % less than that of Fantoni et al. (2002). This fact is related to the needed time to join the proposed neighborhood, which is half with the proposed scheme ($\approx 13sec$).

Scenario 2 : robustness test As mentioned by Hauser et al. (1992b), the coupling parameter is generally not well known. Furthermore, in some flying machines (such as biomimetic robots with flapping wings), this parameter may not be constant. Therefore, the robustness with respect to an uncertainty on this parameter was tested using to compute the control a coupling parameter $\varepsilon_u = \varepsilon + \Delta\varepsilon$; $\Delta\varepsilon = 50\%$. Simulation results are displayed on figures 7-10, for both nominal and uncertain system. It is worth to note that the obtained trajectories for the uncertain system are very close to those of the nominal one, despite the big amount of uncertainty considered (50% of nominal value). This interesting result reflect clearly the robustness of the proposed control scheme.

5. CONCLUSION

In this paper, a global stabilizing receding horizon control approach is proposed for a PVTOL aircraft. The proposed scheme takes into account bounds on both control inputs,namely the thrust and the rolling moment, moreover, it enables stabilization to any point in the space. An other interesting feature of the proposed scheme lies in the very low computational cost, which is obtained by formulation of the control problem as two quadratic optimization problems of reduced dimensions. Since the optimization function is convex, a convergence to the optimum is insured. The stability analysis of the closed-loop system under the proposed control approach is formally established. Illustrative simulations are proposed to show the stabilization of the PVTOL aircraft. Furthermore to attest the robustness of the proposed control scheme, a simulation case study for stabilization of the system under parameter uncertainties is proposed.

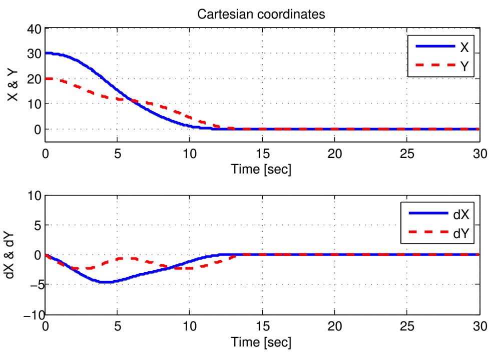

Fig. 2. Evolution of cartesian coordinates and velocities versus time

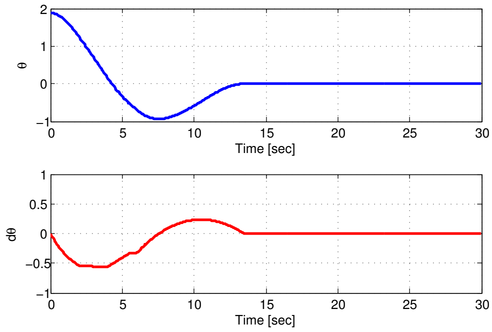

Fig. 3. Evolution of θ position and velocity versus time

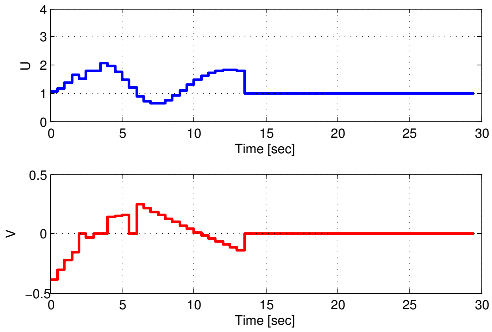

Fig. 4. Evolution of the control inputs versus time

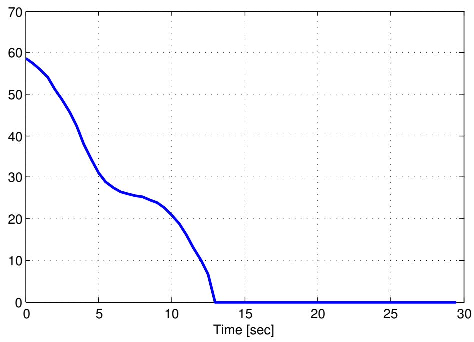

Fig. 5. Evolution of the cost function versus time

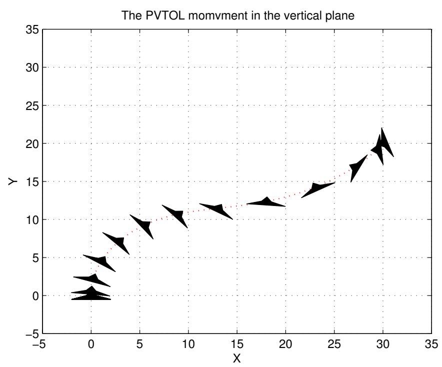

Fig. 6. Stick figures of the aircraft movement in the plane

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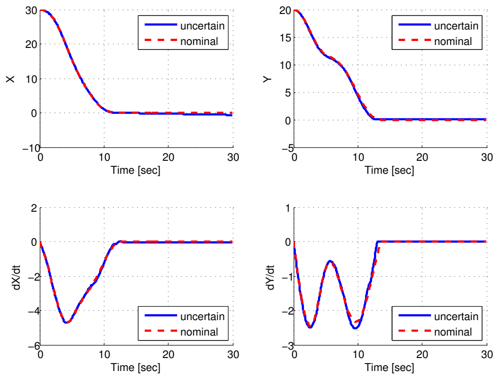

Fig. 7. Evolution of cartesian coordinates and velocities versus time

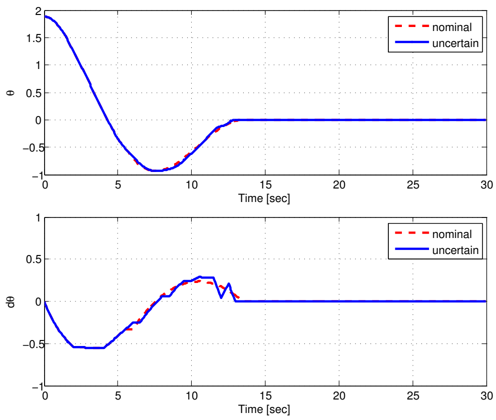

Fig. 8. Evolution of θ position and velocity versus time

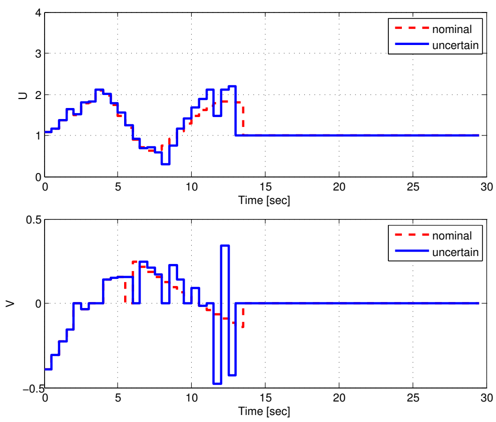

Fig. 9. Evolution of the control inputs versus time

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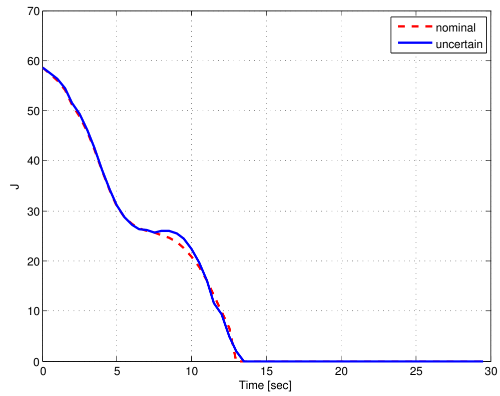

Fig. 10. Evolution of the cost function versus time

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