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Tuning of PID-controller Based on the External Disturbance Spectrum

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Abstract: The paper presents the tuning method of digital PID-controllers based on the solution of parametric optimization problem under uncertain process model and given power spectrum densities (PSD) of stochastic disturbance and set point signals. The new approach for determination of set of stabilizing PID values is issued. It was used in the optimization procedure in order to check closed loop stability conditions during the search iterations.

1. INTRODUCTION

The proportional-integral-derivative (PID) controller remains as popular control algorithm in industry due to its simplicity of realization. There is a great deal of PID-tuning rules (Astrom and Hagglund, 1995; Ko and Edgar, 2004) but only a few of them takes into account random disturbances. The survey of PID-controllers optimization techniques in the presence of stochastic signals is given by Huang and Huang (2004). They also proposed the algorithm for optimal PID parameters calculation based on the LMI-approach and covariance criterion. The increase of LMI computational complexity should be noted in case of it application for the uncertain plant models. Furthermore, the simultaneous influence of the disturbance and set point signals having stochastic nature was not considered in the analysis of closed loop performance whereas this is a wide spread case in practice of PID-controller operation in a cascade mode. Also, the description of PID-controller is often assumed in a continuous form (Toscano, 2005; Hwang and Hsiao, 2002; Yaniv and Nagurka, 2004). However, the modern controllers are implemented in the Distributed Control System (DCS) and must be represented using discrete time models.

In the present work the design of optimal PID-controller is considered as parametric optimization problem in the sense of minimum variance criterion under uncertain plant model. The PSD of both unmeasured disturbance and set point signals are incorporated by the design method. This gives possibility to take into account the various signals nature whether it will be filtered white noise or harmonic process in white noise of any order (Hayes, 1996). The proposed approach is distinguished by the simple method of stability domain determination for closed loop with digital PID-controller and plant models with delay. It was integrated in the framework of optimization procedure.

2. SET OF PROCESS MODEL PARAMETERS

There are two types of process models covering the control problems of many industrial process units (for example, flowrate, temperature, pressure and level control):

$$G^I(s) = K\frac{1}{Ts+1}e^{-\tau s};\quad G^{II}(s) = K\frac{(T_1 s+1)}{s(Ts+1)}e^{-\tau s}, \qquad (1)$$

where $G(s)$ – transfer function (TF) of plant model.

The model parameters in (1) are unknown exactly in practice. We can only suppose that these values belong to the certain interval (i.e. the low (min) and upper (max) bounds are known). In that case for model $G^I(s)$ we get a set of parameters $\mathbf{P}^I$ and for $G^{II}(s)$ is $\mathbf{P}^{II}$, respectively. In turn, the $\mathbf{P}^I$ or $\mathbf{P}^{II}$ contains the vectors of plant model parameters P_i: $P^I_i=\{\tau_i,\ T_i,\ K_i\}$, $i=1,\dots,2^3$; $P^{II}_i=\{\tau_i,\ T_{1i},\ T_i,\ K_i\}$, $i=1,\dots,2^4$. Table 1 illustrates the example of parameters set for $\mathbf{P}^I$:

Table 1. $\mathbf{P}^I$ set

P^I_i	τ_i	T_i	K_i
P^I_1	$\tau^{\min}$	$T^{\min}$	$K^{\min}$
P^I_2	$\tau^{\min}$	$T^{\min}$	$K^{\max}$
P^I_3	$\tau^{\min}$	$T^{\max}$	$K^{\min}$
P^I_4	$\tau^{\min}$	$T^{\max}$	$K^{\max}$
P^I_5	$\tau^{\max}$	$T^{\min}$	$K^{\min}$
P^I_6	$\tau^{\max}$	$T^{\min}$	$K^{\max}$
P^I_7	$\tau^{\max}$	$T^{\max}$	$K^{\min}$
P^I_8	$\tau^{\max}$	$T^{\max}$	$K^{\max}$

3. STATEMENT OF PROBLEM OF PARAMETRIC OPTIMIZATION FOR PID-CONTROLLER

Consider the closed loop on the Fig. 1 with TF of PID-controller

$$C(z) = K_1 + K_0 \frac{z}{z-1} + K_2 \frac{z-1}{z}. \qquad (2)$$

$H(s)$ – TF of zero-order hold. $S_g(\omega)$, $S_N(\omega)$ – the PSD of set point signal and unmeasured disturbance, respectively. It was assumed that $g(t)$ and $N(t)$ are uncorrelated.

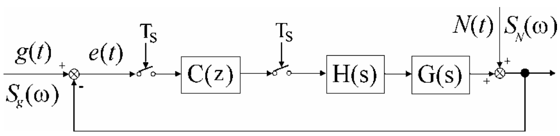

Fig. 1. Closed loop control system

The PSD of the error signal $e(t)$ (Fig.1) has the following form (Appendix 1)

$$S_e(\omega) = |F_2(j\omega)|^2 S_N(\omega) + |F_1(j\omega) - 1|^2 S_g(\omega), \qquad (3)$$

where $$F_2(j\omega) = \frac{1}{1 + C(j\omega)G'(j\omega)};$$

$F_1(j\omega)=F_2(j\omega)C(j\omega)\,G'(j\omega)$. The substitution $z = e^{j\omega T_S}$ is used (2). T_S – sampling interval. $G'(j\omega) = G(j\omega)H(j\omega)$.

The error variance D_e based on the (3) will be expressed by the equation

$$D_e = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |F_2(j\omega)|^2 S_N(\omega)d\omega + \frac{1}{2\pi}\int_{-\infty}^{+\infty} |F_1(j\omega) - 1|^2 S_g(\omega)d\omega \qquad (4)$$

The integral (4) is calculated numerically and integration limits are replaced by the finite numbers $+\omega_c$ and $-\omega_c$ $\left(\int_{-\infty}^{+}(\cdot) \to \int_{-\omega_c}^{+\omega_c}(\cdot)\right)$. $|\omega_c| = 5$ rad/(time unit.) satisfies for many practical applications.

The optimization problem of controller for uncertain plant (1) is formulated by

$$\sum_{i=1}^{k} D_{ei}(P_i) \underset{K_1, K_0, K_2 \in \Omega}{\longrightarrow} \min, \qquad (5)$$

under constrains

$$0 \le K_2 \le \min\{K_{2i}^{\max}\} \ \forall i, \qquad (6)$$

$$K_1^{\min} \le K_1 \le \min\{f_1(K_{2i})\} \ \forall i, \qquad (7)$$

$$K_0^{\min} \le K_0 \le \min\{f_2(K_{2i}, K_{1i})\} \ \forall i. \qquad (8)$$

4. INPUT SIGNAL PSD FUNCTION IMPACT ON VARIANCE VALUE

In this section we demonstrate the importance of input signal (disturbance) PSD function consideration during the design of optimal PID-controller. Let us consider the industrial process control problem example: flowrate control. The several disturbances sources exist (for example, pumping fluctuations, pressure variation inside the pipeline and so on) in real plant. Fig. 2 shows the flow measurements under fixed valve position.

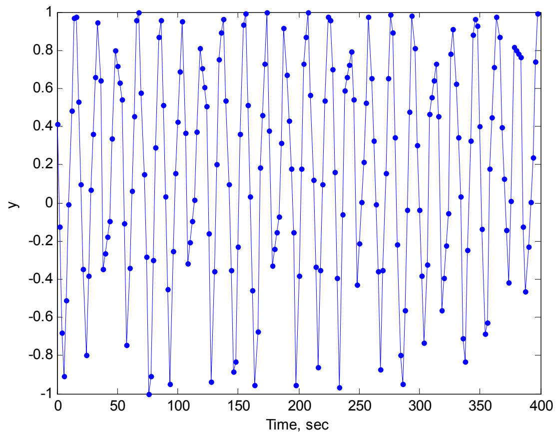

Fig.2. Normalized process variable (industrial data, T_s=2 sec)

This is obvious that the stochastic disturbance has harmonic nature and its description using conventional filtered white noise model (in term of polynomial $C(z)w$) is not valid for current case. The more suitable model is sum of sinusoids (or complex exponents) in white noise of certain order. In order to avoid problems with selection disturbance model structure the author propose to use a PSD function as more powerful approach for accurate handling disturbance or set point influence.

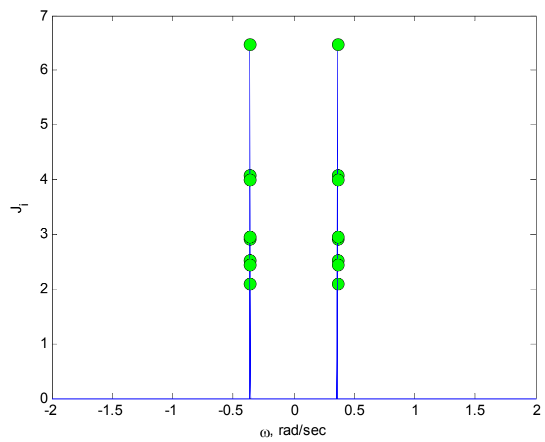

Fig.3. J_i values: impulse functions with different amplitudes

For the simplicity of further PSD function analysis assuming that disturbance on the Fig.2 is described by equation (neglecting white noise)

$$n(j) = A\sin(\omega_0 j + \phi).$$

ϕ is uniformly distributed between -π and π. It was estimated that ω_0=0.36 rad/sec and A≈1. The PSD function of sinusoid with random phase has the following form (Hayes, 1996):

$$S_N(\omega) = \frac{\pi}{2} A^2 [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)],$$

where δ - impulse function.

The nominal plant model for example on Fig.2 is

$$G(s) = 1.67 \frac{e^{-5s}}{20s+1} \text{ + 10\% of parametric uncertainty.}$$

The reduced criterion (5) has form

$$\mathrm{J} = \sum_{i=1}^{8} \Big(\int_{-\omega_c}^{+\omega_c} J_i(\omega) d\omega \Big), \text{ where } J_i(\omega) = | F_2^i(j\omega) |^2 \, S_N(\omega).$$

Let us investigate influence of K_2 on J under fixed K_1=2.2 and K_0=0.1. The J_i is depicted on the Fig. 3 and variation of J is shown on the Fig 4. It is obviously that parameter of PSD function strongly affects on the placement of K_2^{OPT} (three different optimal values). It was found for our case (ω_0=0.36 rad/sec) that introduction of differential term D into control law does not provide performance improvement.

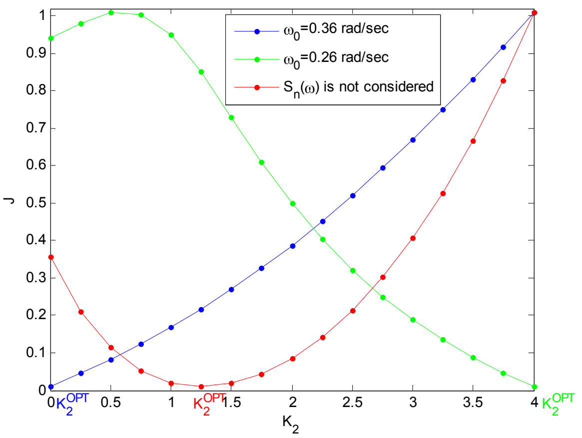

Fig.4. Variance as function from K_2

5. DETERMINATION OF PID-CONTROLLER STABILIZING PARAMETERS DOMAIN FOR CLOSED LOOP SYSTEM

The structure of design algorithm is depicted on the Fig.5. The SQP optimization technique is accompanied by the calculation blocks for checking closed loop stability for set of plant models. Therefore, in the present section we consider details required for obtaining low and upper bounds of inequalities (6)-(8).

The closest work in this area is the paper of Silva *et. al.*, 2001. It was analyzed continuous time PI-controller with the help of Hermit-Biehler Theorem. However, to extent such results on the digital PID-controller is extremely difficult because of the substitution $z=e^{j\omega}$ will not allow to issue the analytical constrains on the K_1. Here we are offering a more simple solution with extension on the digital PID-controller for delayed systems whereas the work of Xu *et. al.*, 2001 did not point how to handle transport delay.

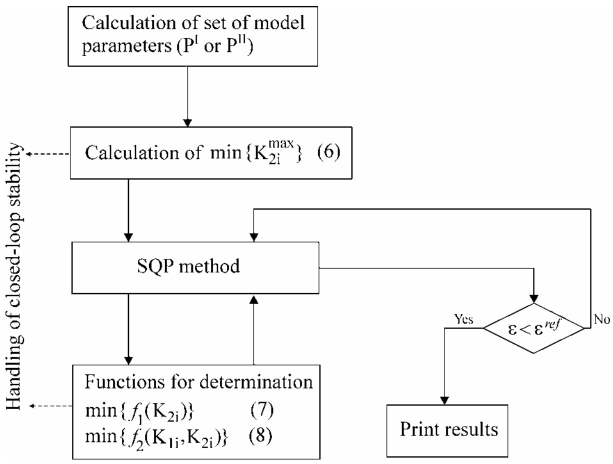

Fig.5. Design algorithm

For the convenient presentation of the proposed approach consider the following plant model example

$$G(s) = \frac{e^{-10s}}{60s+1}; \to G_d(z) = \frac{b_0}{z - a_0} z^{-d}, \tag{9}$$

where b_0=0.0165; a_0=0.9835; d=10. T_s=1 sec. $G_d(z)$ includes the extrapolator TF.

5.1 Obtaining the frequencies of closed loop undamped oscillations separately for P-, I- and D-controllers.

Consider the three independent control systems with P-, I- and controllers, respectively. The critical values of K_1^{cr}, K_0^{cr} and K_2^{cr} are existing and corresponding to stability boundaries of each closed loop having own frequencies ω_1^{cr}, ω_0^{cr} and ω_2^{cr} of undamped oscillations.
For each loop the stability criterion is holding

$$1 + K_1^{cr} G_d(z) = 0.$$

$$1 + K_0^{cr} \frac{z}{z-1} G_d(z) = 0. \tag{10}$$

$$1 + K_2^{cr} \frac{z-1}{z} G_d(z) = 0.$$

The subsequent solutions of (10) can be performed using the following technique. The closed loop equation

$$1 + C'(j\omega) G'(j\omega) = 0$$

is rewriting in the form

$$1 + A_{C'}(\omega) e^{\varphi_C(\omega)} A_{G'}(\omega) e^{\varphi_{G'}(\omega)} = 0.$$

The ω^{cr} is calculated from the phase equation

$$\varphi_C(\omega) + \varphi_{G'}(\omega) = 0\ .$$

Because of the expression of $C'(j\omega)$ by the constant term, then

$$\pi + \varphi_{G'}(\omega) = 0\ . \tag{11}$$

It can be shown that the root of (11) belongs to the interval $\omega \in [0;\pi]$ for (1). The values of K_1^{cr}, K_0^{cr} and K_2^{cr} are derived from the amplitude equation

$$A_{C'}(\omega) = 1 / A_{G'}(\omega)\ , \tag{12}$$

where $A_{C'}(\omega)$ is K_1^{cr}, K_0^{cr} or K_2^{cr}.

The critical frequencies for example (9) are presented in the Table 2.

Table 2. Calculated P, I and D critical gains and frequencies

	K_*^{cr}	ω^{cr}*,rad/sec*
K_1	9.6113	0.1595
K_0	0.1027	0.0397
K_2	60.0985	0.2908

5.2 Stabilizing gains of PD-part.

Consider the stability condition of closed loop system with PID-controller

$$1 + \left[K_1 + K_0 \frac{e^{j\omega}}{e^{j\omega}-1} + K_2 \frac{e^{j\omega}-1}{e^{j\omega}}\right] G_d(j\omega) = 0\ . \tag{13}$$

Introduce the following notations

$$V(\omega) = G_d(j\omega) = V_1(\omega) + jV_2(\omega) \tag{14}$$

$$X(\omega) = G_d(j\omega)\frac{e^{j\omega}}{e^{j\omega}-1} = X_1(\omega) + jX_2(\omega) \tag{15}$$

$$Y(\omega) = G_d(j\omega)\frac{e^{j\omega}-1}{e^{j\omega}} = Y_1(\omega) + jY_2(\omega) \tag{16}$$

The condition (13) is transformed into the system of equations

$$\begin{cases} K_1V_1(\omega) + K_0X_1(\omega) + K_2Y_1(\omega) = -1 \\ K_1V_2(\omega) + K_0X_2(\omega) + K_2Y_2(\omega) = 0 \end{cases}. \tag{17}$$

It follows from (17) that

$$K_1(\omega) = -\frac{-K_0X_2Y_1 + Y_2 + Y_2K_0X_1}{-V_2Y_1 + V_1Y_2}\ ; \tag{18}$$

$$K_2(\omega) = -\frac{-V_2 - V_2K_0X_1 + V_1K_0X_2}{-V_2Y_1 + V_1Y_2}\ . \tag{19}$$

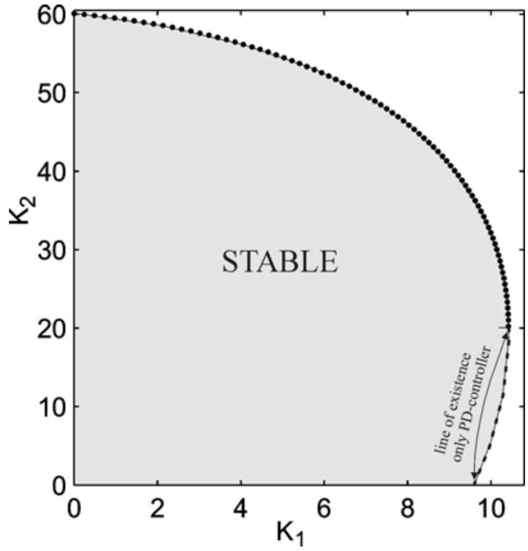

Fig. 6. Stability domain of PD-controller

If K_0=0 in (18)-(19) then it is possible to find stability domain for PD-controller. Figure 6 shows it for example (9) in the frequency range of $\omega \in [\omega_{cr}^P;\omega_{cr}^D]$. In order to check inequality (7) under given K_2 it needs to calculate the root $\omega = \omega^{cr}$ for (19). The substitution of ω^{cr} in (18) gives $K_1^{\max} = f_1(K_2)$.

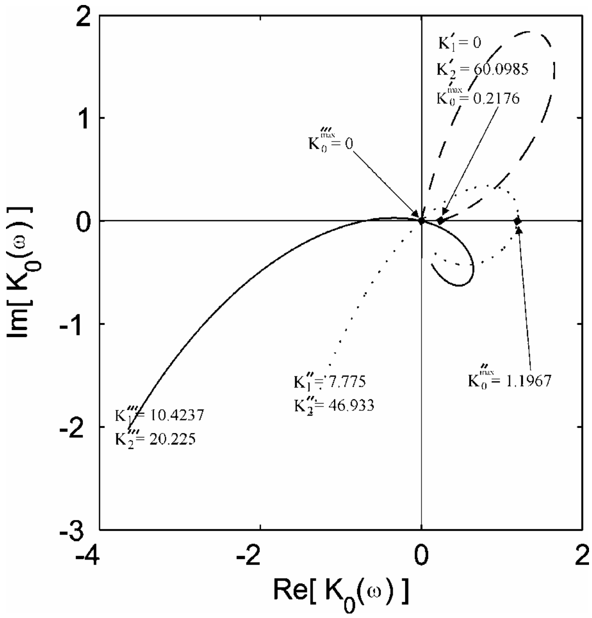

Fig. 7. Hodograph of K_0

It is easy to spread (17) on the PI-controller case expressing K_1 and K_0 by analogy with (18)-(19) and assuming that K_2=0.

5.3 Obtaining the upper bound of inequality (8).

Express the K_0 from (13) as

$$K_0(j\omega) = -\frac{e^{j\omega}(e^{j\omega}-1)}{G_d(j\omega)(e^{j\omega})^2} - -\frac{K_1(e^{j\omega}-1)}{e^{j\omega}} - \frac{K_2(e^{j\omega}-1)^2}{(e^{j\omega})^2} \tag{20}$$

The hodograph (20) is valid in the frequency range $\omega \in [\omega_{cr}^I;\omega_{cr}^D]$. The required value of K_0 is located on the right real axis (Im{K_0}=0, Re{K_0}≥0) as shown on the Fig. 7. The set of stabilizing gains of PD-controller (Fig.3) corresponds to the set of maximum K_0 values depicted on the Fig.8. For the values of K_2<20 and K_1>9.6 it was found that there is no stabilizing K_0 gain (i.e. the PD-controller only exists for that parameters range).

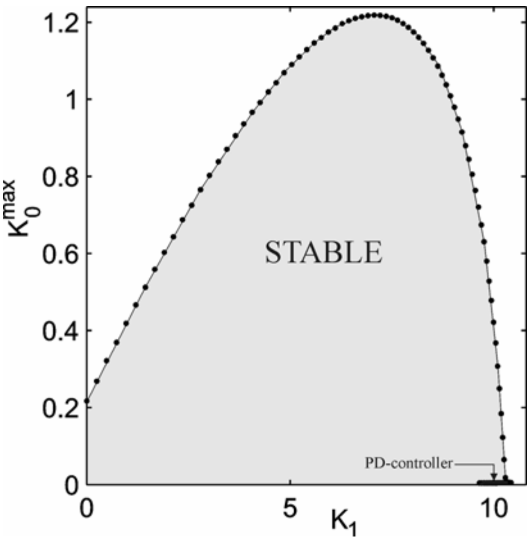

Fig. 8. Stability domain for $K_0^{\max}$

The stability domain plots are not standard (fig.6-8). The total PID stabilizing gains domain was shown (3D plot was presented as two 2D graphs) and demonstrates that adding integrator term to digital PD-controller will cause instability (dashed line in fig.6) under certain PD gains.

6. TUNING EXAMPLE

The optimization criterion (5) involves the external signals spectrums. Assume that *g(t)* and *N(t)* are having the following PSD

$$S_g(\omega) = a_g^2 T_g^2 \left(\frac{\sin\frac{\omega T_g}{2}}{\frac{\omega T_g}{2}} \right);\ S_N(\omega) = a_N^2 T_N^2 \left(\frac{\sin\frac{\omega T_N}{2}}{\frac{\omega T_N}{2}} \right). \tag{21}$$

The expressions (21) are reflecting the square pulse signals with amplitudes a_g, a_N and duration T_g and T_N in the random time instances.

The true parameters of (9) are unknown and by assumption lies in the 10% range from the nominal values. It was also accepted that a_g=1; T_g=2(mean(T)+mean(τ)); a_N=a_g/2; T_N=T_g/40. The results of optimization problem are depicted on the Fig.9-12.

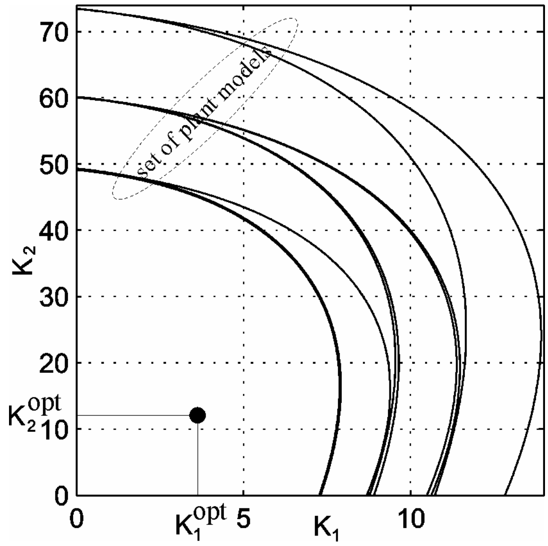

Fig. 9. The set of K_2, K_1 and optimal solution

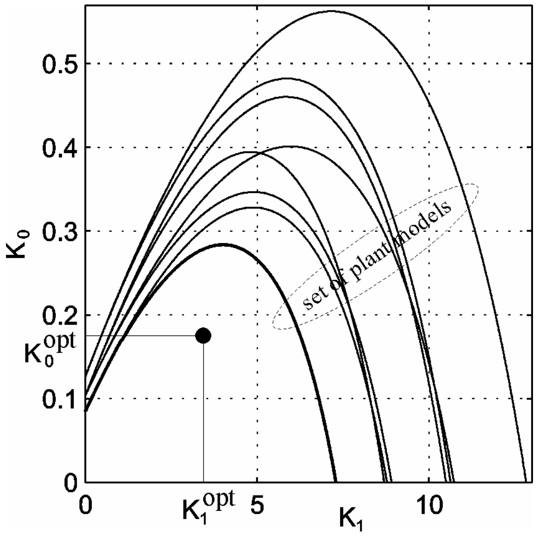

Fig. 10. The set of K_0, K_1 and optimal solution

7. CONCLUSION

The tuning of PID-controller was considered as optimization problem and solved in case of stochastic disturbance and set point signals under uncertain parameters of plant model. The simple numerical method is proposed for the determination and checking of closed loop stability. The author presents statements which may be considered as new results or contribution for PID tuning methods:

a) Proposed frequency domain minimum variance convolution criterion (5) is more valid for practice;
b) New method for robust stabilizing PID gains domain calculation during optimization trials by criterion (5) is derived. In general, there is no limitation for plant transfer function form for SISO case.

The application of SQP or GA (or another optimization technique) is not involved as novelty in the paper. The present paper also demonstrates that we can find easily global optimum without LMI (BMI and etc) solving. Moreover, the existing LMI applications for PID tuning rules are not handling stochastic disturbance model as sum of complex exponents in the white noise for discrete time systems.

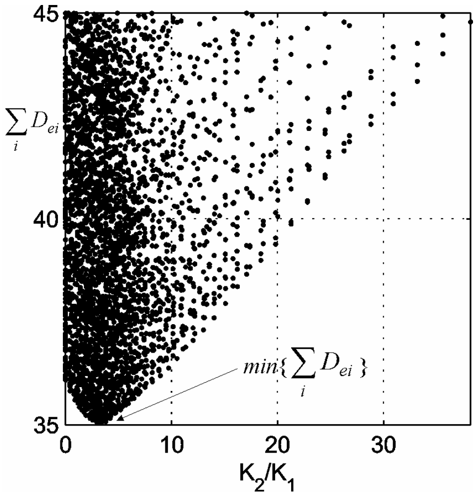

Fig.11. Criterion values and K_2/K_1

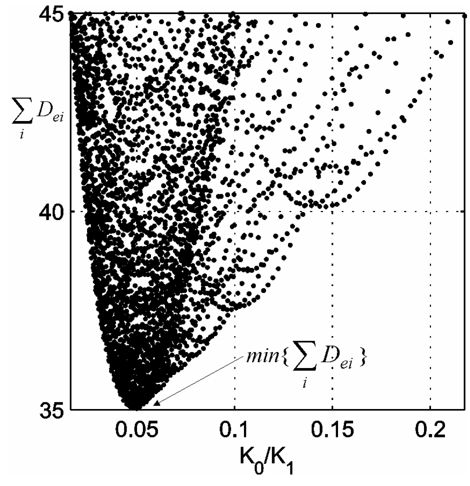

Fig.12. Criterion values and K_0/K_1

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Appendix A.

Assume that the random signals $x(t)$, $y(t)$, $v(t)$ and $f(t)$ are interconnected by the following equations

$$Y(j\omega) = W_1(j\omega)X(j\omega); V(j\omega) = W_2(j\omega)F(j\omega). \quad \text{(A.1)}$$

The realizations of each signal are known on the interval $2T$ and denoted by $x_T(t)$, $y_T(t)$, $v_T(t)$ and $f_T(t)$ or $X_T(t)$, $Y_T(t)$, $V_T(t)$ and $F_T(t)$, respectively. W_1 and W_2 are certain linear transformations. The cross PSD of $y(t)$ and $v(t)$ within the interval $2T$ can be expressed via correlation function R_{yv} (Jaffe, 1999)

$$\begin{aligned} S_{yvT}(j\omega) &= \int_{-\infty}^{\infty} e^{-j\omega\tau} R_{yv}(\tau)d\tau = \\ &= \int_{-\infty}^{\infty} e^{-j\omega\tau} E\left\{\frac{1}{2T}\int_{-T}^{T} y_T(t)v(t+\tau)dt\right\}d\tau, \end{aligned} \quad \text{(A.2)}$$

where E{.} – expectation.
The transformation of (A.2) gives

$$\begin{aligned} S_{yv}(j\omega) &= \frac{1}{2T}E\{\int_{-T}^{T} y_T(t)e^{j\omega t}dt \int_{-\infty}^{\infty} v_T(t+\tau)\times \\ &\times e^{-j\omega(t+\tau)}d(t+\tau)\} = \frac{1}{2T}E\{Y_T(-j\omega)V_T(j\omega)\}. \end{aligned} \quad \text{(A.3)}$$

Taking into account (A.1) for $Y_T(-j\omega)$ and $V_T(j\omega)$ in (A.3) the following equation can be obtained

$$\begin{aligned} S_{yv}(j\omega) &= W_1(-j\omega)W_2(j\omega)\frac{1}{2T}E\{X_T(-j\omega)\times \\ F_T(j\omega)\} &= W_1(-j\omega)W_2(j\omega)S_{xfT}(j\omega). \end{aligned} \quad \text{(A.4)}$$

Finally, (A.4) may be written in the form

$$S_y(\omega) = |W|^2 S_x(\omega). \quad \text{(A.5)}$$