Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Guaranteed Dominant Pole Placement with PID Controllers

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Abstract: Pole placement is a well-established design method for linear control systems. Note however that with an output feedback controller of low-order such as PID one cannot achieve arbitrary pole placement for a high-order or delay system, and then partially or hopefully, dominant pole placement becomes the only choice. To the best of the authors' knowledge, no method is available in the literature to guarantee dominance of the assigned poles in the above case. This paper proposes two simple and easy methods which can guarantee the dominance of the assigned two poles for PID control systems. They are based on Root-Locus and Nyquist plot, respectively. If a solution exists, the parametrization of all the solutions is explicitly given. Examples are provided for illustration.

Keywords: PID Controllers, Dominant Poles, Pole Placement, Root-Locus, Nyquist Plot.

1. INTRODUCTION

Pole placement in the state space and polynomial settings is very popular. For SISO plants, the equivalent output feedback control should be at least of the plant order minus one to achieve arbitrary pole placement. Arbitrary pole placement is otherwise difficult to achieve if one has to use a low-order output feedback controller for a high-order or time-delay plant. One typical example is that in process control, PID controller is used to regulate a plant with delay. To overcome this difficulty, the dominant pole design has been proposed. It is to choose and position a pair conjugate poles which represent the requirements on the closed-loop response, such as overshoot and settling time. Dominant pole design was first introduced by P. Persson Persson et al. (1992) and further explained in Astrom et al. (1995). Their methods are based on a simplified model of plants and thus cannot guarantee the chosen poles are indeed dominant in reality. In the case of high-order plants or plants with time delay, the conventional dominant pole design, if not well handled, could result in sluggish response or even instability of the closed-loop. To the best of the authors' knowledge, no method is available in the literature to guarantee the dominance of the assigned poles in the above case.

It is thus desirable to find out ways to ensure the dominance of chosen poles and also the closed-loop stability. This paper aims to solve this problem. The common idea behind our methods is that the chosen pair of poles give rise to two real equations which are solved for I and D terms via the proportional gain and the locations of all other closed-loop poles can then be studied with respect to this single variable gain by means of Root-locus or Nyquist techniques. Hence, two methods for guaranteed dominant pole placement with PID controller are naturally developed.

The rest of the paper is organized as follows. Section 2 states the problem and preliminary. Sections 3 and 4 each present a method along with illustrating examples. Section 5 is the conclusion.

2. PROBLEM STATEMENT AND PRELIMINARY

Consider a plant described by its transfer function,

$$G(s) = \frac{N(s)}{D(s)} e^{-sL}, \tag{1}$$

where $N(s)/D(s)$ is a proper and co-prime rational function. A PID controller in the form of

$$C(s) = K_P + \frac{K_I}{s} + K_D s$$

is used to control the plant in the conventional unity output feedback configuration as depicted in Figure 1. The closed-loop characteristic equation is

$$1 + C(s)G(s) = 0. \tag{2}$$

The closed-loop transfer function is

$$H(s) = \frac{N(s)\left(K_D s^2 + K_P s + K_I\right)}{D(s)s + N(s)e^{-Ls}\left(K_D s^2 + K_P s + K_I\right)} e^{-Ls}.$$

⋆ This work was sponsored by the Ministry of Education AcRF Tier 1 funding, R-263-000-306-112, Singapore. (Corresponding Author: Prof. Qing-Guo Wang, Email: elewqg@nus.edu.sg, Fax: 65-67791103).

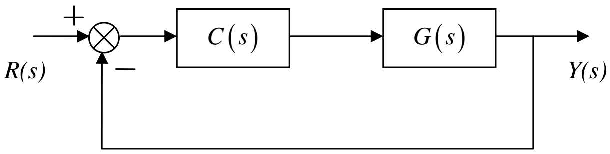

Fig. 1. Unity Output Feedback Control System

Suppose that the requirements of the closed-loop control performance in frequency or time domain are converted into a pair of conjugate poles Astrom et al. (1995): $\rho_{1,2} = -a \pm bj$. Their dominance requires that the ratio of the real part of any of other poles to $-a$ exceeds m (m is usually 3 to 5) and there are no zeros nearby. Thus, we want all other poles to be located at the left of the line of $s = -ma$, that is, the desired region as hatched in Figure 2. The problem of the guaranteed dominant pole placement is to find the PID parameters such that all the closed-loop poles lie in the desired region except the dominant poles, $\rho_{1,2}$.

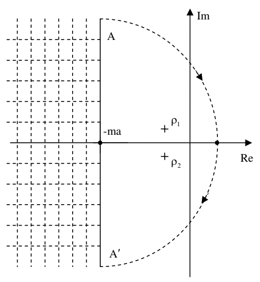

Fig. 2. Desired Region(hatched) of Other Poles

Substitute $\rho_1 = -a + bj$ into (2):

$$K_P + \frac{K_I}{-a+bj} + K_D(-a+bj) = -\frac{1}{G(\rho_1)},$$

which is a complex equation. Solving the two equations given by its real and imaginary parts for K_I and K_D in terms of K_P yields

$$\begin{cases} K_I = \dfrac{a^2+b^2}{2a}K_p - \left(a^2+b^2\right)X_1, \\ K_D = \dfrac{1}{2a}K_p + X_2, \end{cases} \tag{3}$$

where $X_1 = \frac{1}{2b}\mathbf{Im}\left[\frac{-1}{G(\rho_1)}\right] + \frac{1}{2a}\mathbf{Re}\left[\frac{-1}{G(\rho_1)}\right]$, $X_2 = \frac{1}{2b}\mathbf{Im}\left[\frac{-1}{G(\rho_1)}\right] - \frac{1}{2a}\mathbf{Re}\left[\frac{-1}{G(\rho_1)}\right]$. This simplifies the original problem to a one-parameter problem for which well known methods like Root-locus and Nyquist plot are applicable now.

3. ROOT-LOCUS METHOD

The root-locus method is to used to show movement of the roots of the characteristic equation for all values of a system parameter. We plot the roots of the closed-loop characteristic equation for all the positive values of K_P and determine the range of K_P such that the roots other than the chosen dominant pair are all in the desired region.

Substituting (3) into (2) yields

$$1 + X_2\frac{N(s)e^{-Ls}}{D(s)}s - \left(a^2+b^2\right)X_1\frac{N(s)e^{-Ls}}{D(s)s} \tag{4}$$

$$+ K_P\frac{N(s)e^{-Ls}}{D(s)}\frac{s^2+2as+(a^2+b^2)}{2as} = 0. \tag{5}$$

Dividing both sides by the terms without K_P gives:

$$1 + K_P\overline{G}(s) = 0, \tag{6}$$

where

$$\overline{G}(s) = \frac{N(s)\left[s^2+2as+(a^2+b^2)\right]e^{-Ls}}{2a[D(s)s + X_2N(s)s^2e^{-Ls} - (a^2+b^2)X_1N(s)e^{-Ls}]}. \tag{7}$$

It can be easily verified that the manipulation does not change the roots. If $G(s)$ has no time-delay term, $\overline{G}(s)$ is a proper rational transfer function since the degrees of its numerator and denominator of $\overline{G}(s)$ equal those of the closed-loop transfer function's numerator and denominator, respectively. The root locus of (6) can easily be drawn with Matlab as K_P varies. The interval of K_P for guaranteed dominant pole placement can be determined from the root locus. Example 1 shows the design procedure in detail.

Example 1: Consider a fourth-order process,

$$G(s) = \frac{1}{(s+1)^2(s+5)^2}.$$

If the overshoot is to be less than 5% and the rising time less than 2.5 s, the corresponding dominant poles are $\rho_{1,2} = -0.6136 \pm 0.6434j$. Equation (3) becomes

$$\begin{cases} K_I = 0.6442K_P - 0.1847, \\ K_D = 0.8149K_P - 12.4627. \end{cases}$$

And it follows from (7) that

$$\overline{G}(s) = \frac{s^2+1.227s+0.7905}{1.227s^5+14.73s^4+56.45s^3+58.33s^2+30.68s-0.2267}.$$

The root-locus of $\overline{G}(s)$ is exhibited in Figure 3 with the solid lines while the edge of the desired region with $m = 3$ is indicated with dotted lines. Note that $\overline{G}(s)$ is of 5-th order and has five branches of root loci, of which two are fixed at the dominant poles while the other three move with the gain. From the root-locus, two intersection points corresponding to root locus entering into and departing from the desired region are located and give the gain range of $K_P \in (36, 51)$, which ensures all other three poles in the desired region. Besides, the positiveness of K_D and K_I requires $K_P > 15.2935$. Taking the joint solution of these two, we have $K_P \in (36, 51)$. If $K_P = 50$ is chosen, the PID controller is

$$C(s) = 50 + \frac{32.0233}{s} + 28.2832s.$$

The zeros of the closed-loop system are at $s = -0.8839 \pm 0.5934j$, which are not near the dominant poles. Figure 4 shows the step response of the closed-loop system.

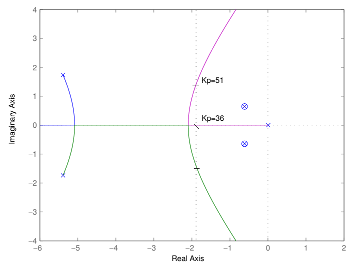

Fig. 3. Root-Locus for Example 1

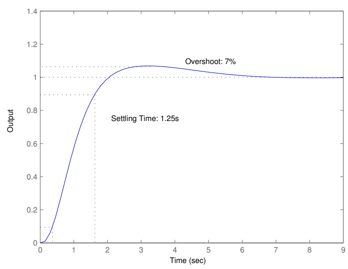

Fig. 4. Closed-loop Step Response for Example 1

4. NYQUIST PLOT METHOD

If $G(s)$ has time delay, so will be $\overline{G}(s)$. Then, drawing the root locus for it could be difficult and checking locations of infinite poles is a forbidden task. Note that the Nyquist plot works well for delay systems. The Nyquist stability criterion determines the number of unstable closed-loop poles based on the Nyquist plot and the open-loop unstable poles. We use the same idea but have to modify the conventional Nyquist contour. The Modified Nyquist contour is obtained by shifting the conventional Nyquist contour to the left by ma, as Figure 2 shows. The image of G(s) when s traverses the modified Nyquist contour is called the modified Nyquist plot. The number of poles located outside the desired region plays the same role as that of unstable poles in the standard Nyquist criterion.

Rewrite (6) as

$$\frac{1}{K_P} + \overline{G}(s) = 0. \tag{8}$$

It always has $\rho_{1,2}$ as its two roots by our construction. These two lie outside the desired region. We want no more to ensure dominant pole placement. Equivalently, we want the modified Nyquist plot of $\overline{G}(s)$ to have the number of clockwise encirclements with respect to $(-\frac{1}{K_P}, 0)$ equal to 2 minus the number of poles of $\overline{G}(s)$ outside the desired region. This condition will determine the interval of K_P such that roots of (8) other than two dominant poles are in the desired region.

To find the poles of $\overline{G}(s)$ located outside the desired region, note that they are simply the roots of its denominator. Thus, we construct another characteristic equation from the denominator of $\overline{G}(s)$ in (7) as follows:

$$1 + \overline{G}_o(s) = 0, \tag{9}$$

where $\overline{G}_o(s) = \frac{X_2 N(s)s^2 - (a^2+b^2)X_1 N(s)}{D(s)s} e^{-Ls}$. $\overline{G}_o(s)$ has its rational part with the degrees of its numerator and denominator being equal to those of the open-loop transfer function's numerator and denominator, respectively. The number of the roots of (9), that is, poles of $\overline{G}(s)$ lying outside the desired region, equals the number of clockwise encirclements of the modified Nyquist plot of $\overline{G}_o(s)$ with respect to $(-1, 0)$, plus the number of poles of $\overline{G}_o(s)$ located outside the desired region. The latter is easy to find from the known denominator of $\overline{G}_o(s)$, which is, $D(s)s$.

The design procedure is summarized as follows.

Step 1. Find the poles of $\overline{G}_o(s)$ (the roots of $D(s)s$) outside the desired region and name its total number as $P^+_{\overline{G}_o}$;

Step 2. Draw the modified Nyquist plot of $\overline{G}_o(s)$, count the number of clockwise encirclements with respect to the $-1 + j0$ point as $N^+_{\overline{G}_o}$, and obtain the number of poles of $\overline{G}(s)$ outside the desired region as $P^+_{\overline{G}} = N^+_{\overline{G}_o} + P^+_{\overline{G}_o}$;

Step 3. Draw the modified Nyquist plot of $\overline{G}(s)$ and find the range of K_P during which the clockwise encirclements with respect to the $(-\frac{1}{K_P}, 0)$ is 2-$P^+_{\overline{G}}$.

We now provide Example 2 to illustrate the design procedure in detail.

Example 2: Consider a highly oscillatory process,

$$G(s) = \frac{1}{s^2 + s + 5} e^{-0.1s}.$$

If the overshoot is to be not larger than 10% and the settling time to be less than 15 s, the dominant poles are $\rho_{1,2} = -0.2751 \pm 0.3754j$. Equation (3) becomes

$$\begin{cases} K_I = 0.3937K_P + 1.8773, \\ K_D = 1.8173K_P + 7.7760. \end{cases}$$

We have

$$\overline{G}_o(s) = \frac{7.776s^2 + 1.877}{s(s^2 + s + 5)} e^{-0.1s}.$$

Take $m = 3$. We have $ma = 0.8253$ and all three poles of $\overline{G}_o(s)$ outside the desired region and $P^+_{\overline{G}_o} = 3$. Figure 5 is the modified Nyquist plot of $\overline{G}_o(s)$ and there is one anti-clockwise encirclement of the point $(-1, 0)$, that is, $N^+_{\overline{G}_o} = -1$. Therefore, $\overline{G}(s)$ has two poles located in the desired region since $P^+_{\overline{G}} = N^+_{\overline{G}_o} + P^+_{\overline{G}_o} = 2$. It means the modified Nyquist plot of $\overline{G}(s)$ should have its clockwise encirclement with respect to the point $(-1/K_P, 0)$, equal to $2 - P^+_{\overline{G}} = 0$, that is zero net encirclement, for two assigned poles to dominate all others. Figure 6 shows the modified Nyquist plot of $\overline{G}(s)$, from which $-1/K_P \in (-\infty, -0.2851)$ is determined to have zero clockwise encirclement. A positive K_P could always make K_D and K_I positive. Therefore, we have the joint solution as $K_P \in (0, 3.5075)$. If $K_P = 1$ is chosen, the PID controller is

$$C(s) = 1 + \frac{2.2709}{s} + 9.5933s.$$

The zeros of the closed-loop system are at $s = -0.0521 \pm 0.4837j$, which are not near the dominant poles. Figure 7 shows the step response of the closed-loop system.

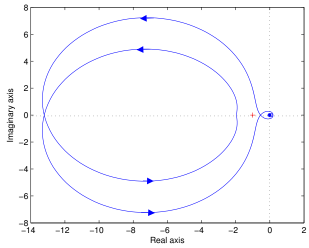

Fig. 5. Modified Nyquist Plot of $\overline{G}_o$ for Example 2

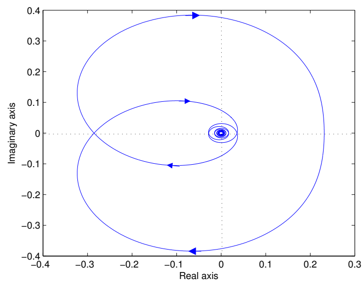

Fig. 6. Modified Nyquist Plot of $\overline{G}$ for Example 2

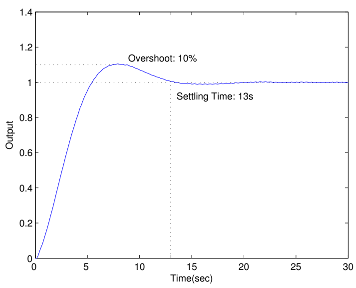

Fig. 7. Closed-loop Step Response for Example 2

5. CONCLUSION

Two simple yet effective methods have been presented for guaranteed dominant pole placement by PID, based on Root locus and Nyquist plot, respectively. Each method is demonstrated with examples. Obviously, the methods are not limited to PID controllers. They can be extended to other controllers where one controller parameter is used as the variable gain and all other parameters are solved in terms of this gain to meet the fixed pole requirements.

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