FUZZY OPTIMIZATION OF COST FUNCTION IN PRODUCT MIX SELECTION PROBLEM

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Abstract: The modern trend in industrial application problem deserves modeling of all relevant vague or fuzzy information involved in a real decision making problem. In this paper the usefulness of the proposed S-curve membership function is established using a real life industrial production planning of a chocolate manufacturing unit. The unit produces 8 products using 8 raw materials; mixed in various proportions by 9 different processes under 29 constraints. A solution to this problem establishes the usefulness of the suggested membership function for decision making in industrial production planning. The objective of this paper is to find the optimal cost to produce 8 products using modified S-curve membership function as a methodology. The fuzzy linear programming approach is used to solved this problem. The optimal cost function is obtained respect to two major factors of degree of satisfaction and vagueness.

Keywords: Decision making, fuzzy systems, fuzzy linear programming, product-mix selection, optimization

1. INTRODUCTION

Due to limitations in resources for manufacturing a product and the need to satisfy certain conditions in manufacturing and demand, a problem of fuzziness occurs in industrial production planning. This problem occurs also in chocolate manufacturing when deciding a mixed selection of raw materials to produce varieties of products. This is referred here to as the Product- mix Selection Problem (Tabucanon, 1996). The data for this problem are taken from the data-bank of Chocoman Inc, USA (Tabucanon, 1996). Chocoman produces varieties of chocolate bars, candy and wafer using a number of raw materials and processes. The objective is to use the modified S-shaped membership function for obtaining a cost optimization procedure through FLP (Fuzzy Linear Programming).

The modified S-curve membership function is proved to be a flexible membership function through an analytical approach (Vasant, 2002). This membership function is to be used in FLP involving fuzzy objective coefficients, fuzzy technical coefficients and fuzzy resource variables. The modified S-curve membership function is flexible enough to describe vagueness in these fuzzy parameters.

The objective of the company is to minimize its cost, which is, alternatively, equivalent to minimizing the cost to produce eight products. That is to find the optimal product mix under uncertain constraints in the technical, raw material and market consideration. Furthermore, it is possible to show the relationship between the optimal cost and the corresponding membership values (Vasant et al., 2002a). According to this relationship, the decision maker can then obtain his optimal solution with a trade-off under a pre-determined allowable imprecision (Zimmermann, 1985) and (Vasant and Barsoum, 2005).

2. METHODOLOGY FOR FPS PROBLEM

The Fuzzy Product – mix Selection Problem (FPS) is stated as:
There are eight products to be manufactured by mixing eight raw materials with different proportion and by using nine varieties of processing. There are limitations in resources of raw materials. There are also ten constraints imposed by marketing department such as product – mix requirement, main product line requirement and lower and upper limit of demand for each product. All the above requirements and conditions are fuzzy. It is necessary to obtain optimal cost with certain degree of satisfaction by using fuzzy linear programming and modified S-curve membership function.

The first step is construction of S-curve membership function for the FPS problem. Then followed by formulation of FLP problem which represent FPS problem. This mathematical model of FLP problem will be solved by using LP toolbox in MATLAB®.

Here we only consider one problem of FPS in which the objective coefficients, technical coefficients and resource variables all are fuzzy. The FLP model for this problem is given in (1). The objective function is the cost for the FPS problem.

$$\begin{aligned} &\text{Minimize} \quad z = \sum_{j=1}^{8} \tilde{c}_j x_j \\ &\text{subject to} \quad \sum_{i=1}^{29} \tilde{a}_{ij} \le \tilde{b}_i \end{aligned} \tag{1}$$

where $\tilde{c}_j, \tilde{a}_{ij}$ and $\tilde{c}_j$ are fuzzy parameters.

The relation between objective coefficient of cost function and vagueness is given in the following equation.

$$\tilde{c}_j\Big|_{\mu=\mu_{C_J}} = c_j^a + \left(\frac{c_j^b - c_j^a}{\alpha}\right)\ln\frac{1}{C}\left(\frac{B}{\mu_{c_j}} - 1\right) \tag{2}$$

The following values are substituted in the equation (2) with $C = 0.001001001$, B = 1, $\alpha > 0$ and $\alpha = 13.8135$ (Vasant, 2002).

Equation(1) is solved by using parametric programming approach (Carlsson and Korhonen, 1986) and a modified S-curve membership function used as a methodology (Vasant et al., 2002b; Vasant 2004). The input data for c_j is the cost fuzzy values, a_{ij} technical coefficients and b_i is the resource variables for FPS problem. The fuzzy interval for objective coefficients c_j (cost) is calculated from the profit and revenue optimization (Tabucanon, 1996). The interval for cost is selected such a way that it's value less than profit. There are 29 constraints and 8 products and hence in (1), i = 1, 2, 3,...,29 and j = 1,2,3,...,8. Membership function and membership values for c_j's are constructed and valued. The FLP problem has been formulated and all the coefficients are parameterized. However, it will not be possible to use the linear parametric formulation to solve the FLP problem since the membership functions are non-linear (Watada, 1997). Then, it is needed to carry out a series of experiments for 21 membership values: $\mu_{aij} = \mu_{bi} = \mu_{cj} = \mu = 0.0010, 0.0509, 0.1008, ..., 0.9990$ with an interval of 0.0499. These experiments are carried out by using the Simplex Method in the Optimization Tool Box of MATLAB®.

First of all, construct the membership functions for the fuzzy parameters of $\tilde{A}, \tilde{c}_j$ and $\tilde{b}$. Here a non-linear membership function called as modified S-curve membership function is used for the convenience of selecting vague parameter α (Bells, 1999 and Kuzmin, 1981). The membership functions are represented by $\mu_{a_{ij}}$, and μ_{b_i} ,where a_{ij} are the technical coefficients of matrix A for i =1,…,29 and j =1,…,8 , b_i are the resource variables for i =1,…,29. The membership function for $\mu_{a_{ij}}$ and the fuzzy interval, a_{ij}^a to a_{ij}^b ,for $\tilde{a}_{ij}$ is given in Vasant (2004).

In similar way we can construct membership function for fuzzy resource variable and fuzzy objective coefficient and it's derivations (Vasant, 2002). Since the technical coefficients and resource variables are fuzzy therefore the outcome of the cost function will be fuzzy.

3. FUZZY SOLUTIONS AND THE OUTCOME

The FPS problem is solved by using MATLAB and its toolbox of Linear Programming (LP). The vagueness is given by α, and μ is the degree of satisfaction. The LP toolbox has two inputs namely α and μ in addition to the fuzzy parameters. There is one output z^*, the optimal cost function.

The given values of various parameters of Chocolate Manufacturing are fed to the toolbox. The solution can be tabulated and presented as 2 and 3 dimensional graphs.

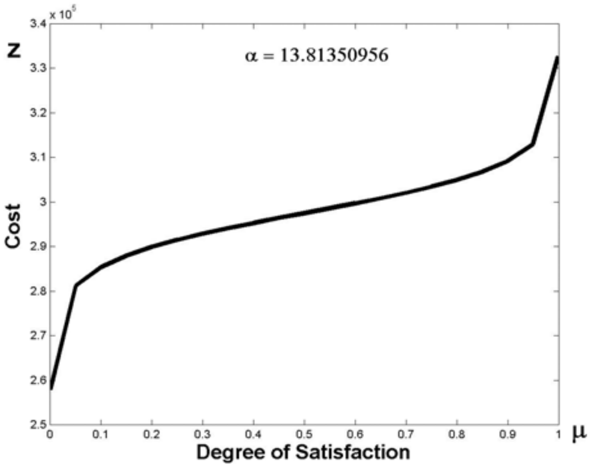

Figure II- Optimal Cost and Degree of Satisfaction
(α = 13..8135)

Table 1- Optimal Cost and Degree of Satisfaction (α = 13..8135)

No	Degree of Satisfaction (μ)	Optimal Cost (z^*)
1	0.0010	257920
2	0.0509	281240
3	0.1008	285410
4	0.1507	287980
5	0.2006	289910
6	0.2505	291500
7	0.3004	292880
8	0.3503	294130
9	0.4002	295300
10	0.4501	296410
11	0.5000	297500
12	0.5499	298580
13	0.5998	299690
14	0.6497	300830
15	0.6996	302050
16	0.7495	303390
17	0.7994	304910
18	0.8493	306740
19	0.8992	309150
20	0.9491	312980
21	0.9990	332660

From Figure II, we can see that the graph behaves as an increasing function. This shows that the objective values are increases as degree of satisfaction increases. The cost function (objective value) has a value 332660 at μ = 0.999. We define this as 99.9% degree of satisfaction. Accordingly a z^* of value 257920 has 0.1% degree of satisfaction. The possible realistic solution exists at μ = 0.5 (ie 50% degree of satisfaction) with a value of z^* as 297500. The non-fuzzy situation (i.e all the coefficients a_{ij}, c_j and b_i are precise) and the z^* value has been computed to be less than 257920 (Tabucanon, 1996). It is found that z^* becoming more than that of a totally non-fuzzy situation (Vasant, 2002;2003). The result obtained by using fuzzy optimization approach far better than the result obtained by deterministic approach (Tabucanon, 1996). The comparison for the profit function is available in Vasant (2002).

3.1 A. Objective Values for Various α

Figure II illustrates the variation of objective values z^* with respect to degree of satisfaction μ for one value of vagueness factor α = 13.8135. It will be useful for the decision maker to observe such variations for several values of α.

Figure III, shows the nature of variations of z^* with respect to μ when α varies from 1.5 to 39.5.

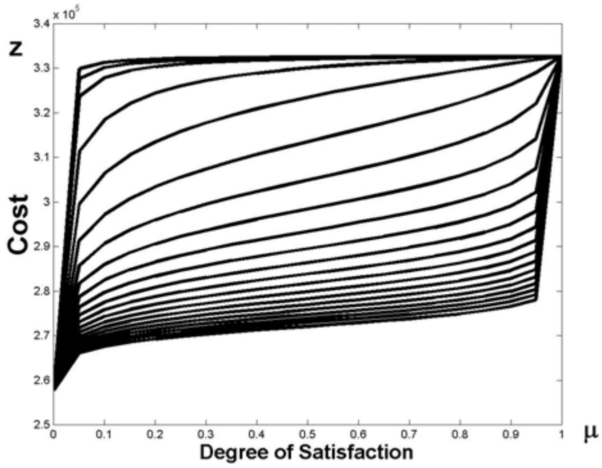

Figure III – Variation of Cost z^* in Terms of μ and α

The realistic solution with an uncertainties in fuzzy parameters of technical coefficients and resource variables exists at μ = 50%. Hence the result for 50% degree of satisfaction for 1.5 ≤ α ≤ 39.5 and the corresponding values for z^* are tabulated in Table 2.

Table 2- Vagueness α and Cost z^* for μ = 50%

Vagueness α	Cost z^*
1.5	332510
3.5	332070
5.5	330060
7.5	323470
9.5	313530
11.5	304890
13.5	298370
15.5	293400
17.5	289510
19.5	286390
21.5	283830
23.5	281690
25.5	279880
27.5	278320
29.5	276970
31.5	275790
33.5	274740
35.5	273800
37.5	272960
39.5	272090

The fuzzy outcome for the cost function decreases as vagueness α increases in the fuzzy parameters of technical coefficients, objective coefficients and resource variables. This is clearly shown in Table 2. Table 2 is very important to the decision maker in picking up the α so that the outcome will be at good enough satisfactory level.

The outcome in Table 3 shows that when the vagueness in increases results in less cost. Also it is found that the S-curve membership function with various values of α provides a possible solution with certain degree of satisfaction.

Furthermore the relationship between z^*, μ and α is given in Table 3. This Table is very useful for the decision maker to find the cost any given value of α with degree of satisfaction μ. From Table 3 it is observed that at any particular degree of satisfaction μ the cost of products z^* decreases as the vagueness α increases between 1.5 and 39.5. Similarly at any particular value of vagueness the cost of products are increases as the degree of satisfaction increases.

Table 3(a): Fuzzy Optimal Cost for $1.5 \leq \alpha \leq 7.5$

z^*	Vagueness α			
μ	1.5	3.5	5.5	7.5
0.0010	257920	257920	257920	257920
0.0509	329900	323710	311310	299520
0.1008	331330	327900	318470	306450
0.1507	331810	329530	322130	310560
0.2006	332060	330400	324480	313510
0.2505	332210	330940	326050	315840
0.3004	332310	331310	327250	317780
0.3503	332380	331580	328180	319450
0.4002	332240	331780	328930	320920
0.4501	332480	331940	329540	322250
0.5000	332510	332070	330060	323470
0.5499	332540	332180	330490	324600
0.5998	332560	332270	330870	325660
0.6497	332580	332340	331190	326650
0.6996	332600	332410	331480	327600
0.7495	332610	332460	331730	328510
0.7994	332630	332510	331960	329390
0.8493	332640	332560	332160	330240
0.8992	332650	332600	332350	331060
0.9491	332660	332630	332510	338700
0.9990	332660	332660	332660	332660

Table 3(b): Fuzzy Optimal Cost for $9.5 \leq \alpha \leq 11.5$

z^*	Vagueness α			
μ	9.5	11.5	13.5	15.5
0.0010	257920	257920	257920	257920
0.0509	291340	285760	281760	278770
0.1008	297180	290700	286020	282510
0.1507	300750	293740	288650	284880
0.2006	303410	296010	290620	286560
0.2505	305570	297870	292240	287990
0.3004	307440	299500	293650	289230
0.3503	309110	300960	294930	290360
0.4002	310660	302320	296120	291410
0.4501	312120	303620	297260	292480
0.5000	313530	304890	298370	293400
0.5499	314910	306150	299470	294380
0.5998	316280	307430	300600	295380
0.6497	317680	308750	301770	296410
0.6996	319130	310150	303010	297520
0.7495	320660	311670	304370	298730
0.7994	323310	313400	305920	300110
0.8493	324160	315440	307790	301780
0.8992	326300	318070	310230	303970
0.9491	328970	322030	314110	307480
0.9990	332660	332660	332660	332660

Table 3(c): Fuzzy Optimal Cost for $17.5 \leq \alpha \leq 23.5$

z^*	Vagueness α			
μ	17.5	19.5	21.5	23.5
0.0010	257920	257920	257920	257920
0.0509	276440	274580	273060	271790
0.1008	279780	277590	275800	274310
0.1507	281840	279450	277500	275870
0.2006	283390	280850	278770	277040
0.2505	284670	282010	279830	278010
0.3004	285780	283010	280750	278850
0.3503	286790	283930	281580	279620
0.4002	287730	284780	282360	280330
0.4501	288630	285590	283100	281020
0.5000	289510	286390	283830	281690
0.5499	290390	287190	284550	282360
0.5998	291280	288000	285300	283040
0.6497	292210	288840	286070	283750
0.6996	293210	289740	286890	284510
0.7495	294290	290720	287790	285340
0.7994	295540	291850	288820	286290
0.8493	297030	293210	290060	287430
0.8992	299010	295010	291710	288950
0.9491	302180	297890	294350	291390
0.9990	332660	332660	332660	332660

Table 3(d): Fuzzy Optimal Cost for $25.5 \leq \alpha \leq 31.5$

z^*	Vagueness α			
μ	25.5	27.5	29.5	31.5
0.0010	257920	257920	257920	257920
0.0509	270720	269800	269010	268310
0.1008	273050	271960	271030	270210
0.1507	274490	273310	272280	271380
0.2006	275570	274320	273220	272270
0.2505	276470	275150	274000	273000
0.3004	277250	275880	274680	273640
0.3503	277960	276530	275300	274220
0.4002	278620	277150	275880	274760
0.4501	279260	277740	276430	275280
0.5000	279880	278320	276970	275790
0.5499	280490	278900	277510	276290
0.5998	281130	279480	278060	276810
0.6497	281780	280100	278630	277350
0.6996	282480	280750	279240	277920
0.7495	283250	281470	279910	278550
0.7994	284140	282290	280680	279270
0.8493	285200	283280	281610	280150
0.8992	286600	284590	282840	283160
0.9491	288870	286700	284810	283160
0.9990	332660	332660	332660	332660

Table 3(e) : Fuzzy Optimal Cost for $33.5 \leq \alpha \leq 39.5$

z^*	Vagueness α			
μ	33.5	35.5	37.5	39.5
0.0010	257920	257910	257900	257780
0.0509	267700	267140	266640	266090
0.1008	269480	268830	268240	267610
0.1507	270590	269880	269240	268550
0.2006	271430	270670	269990	269260
0.2505	272120	271320	270600	269850
0.3004	272720	271890	271140	270360
0.3503	273260	272400	271630	270830
0.4002	273770	272890	272090	271260
0.4501	274260	273350	272530	271680
0.5000	274740	273800	272960	272090
0.5499	275220	274250	273390	272500
0.5998	275710	274710	274090	272910
0.6497	276210	275190	274280	273350
0.6996	276750	275710	274770	273810
0.7495	277350	276270	275300	274320
0.7994	278030	276910	275910	274900
0.8493	278850	277690	276650	275600
0.8992	279940	278730	277630	276540
0.9491	281700	280390	279210	278040
0.9990	332660	332660	332660	332660

μ = Degree of Satisfaction, z^*= Cost of Products and α = Vagueness.

The diagonal values in the Tables 3, show that the cost increases at lower value of μ (0.1% ≤ μ ≤ 5.09%). Then z^* value decreases for 5.09% < μ ≤ 89.92%. Lastly z^* value increases for 89.92% < μ ≤ 99.9%. This result shows that good decision (higher degree of satisfaction) does not guarantee minimum value in cost (objective value). This means one should satisfy with degree of satisfaction when come to making decision in a fuzzy environment. The result shows that, the outcome almost does not depend on the decision made at early stage of input level for fuzzy parameters of objective coefficients, technical coefficients and resource variables.

4. CONCLUSION

The S-curve membership function was used in generating fuzzy parameters towards solving an industrial production-planning problem. These parameters are defined in terms of the fuzzy linear programming problem and named as the fuzzy coefficients of the objective function, fuzzy technical coefficients and fuzzy resource variables. Membership values for this fuzzy parameters are created by using the S-curve membership function. This formulation is found to be suitable in applying the Simplex Method in Linear programming (LP) approach. This approach of solving industrial production planning problem can have feed back within the decision maker, the implementer and the analyst. It is to be noted that minimum cost need not lead to higher degree satisfaction. The decision maker has to satisfy with the cost, which was obtained through FLP process respect to degree of satisfaction. Since the emphasis is given to degree of satisfaction and vagueness in the fuzzy system so the problem of finding the well distributed z^* function becomes very important in this work. In order to obtain this, we need experience and expertise in selecting parameter α . The Tables and Figures are very useful for the decision maker and the implementer to make final decision for picking up the optimal cost.The input data for cost function was calculated from profit function and return. Since both profit and return are fuzzy input therefore the cost function also has to be fuzzy. The selection of fuzzy cost function for input data has made such a way that the value is less then the profit. The outcome of the cost function for this FPS problem almost equal to the profit function. It is possible to reduce the cost function by carrying out the solution procedure continuously in an interactive manner between decision maker and analyst. This will result in fuzzy system of industrial production system of interactive process. The best good enough outcome for minimum cost function can be achieved by designing self-organizing fuzzy system for the FPS problem.

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