ROBUST NONLINEAR GYROMOMENT CONTROL OF AGILE REMOTE SENSING SPACECRAFT [⋆]

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Abstract: Some problems on robust gyromoment motion control of agile spacecraft (SC) for remote sensing the Earth surface are considered. Elaborated methods for dynamic research of the SC programmed angular motion at principle modes under external, structural and parametric disturbances, partial discrete measurement of the state and digital control of the gyro moment cluster (GMC) by the minimum-excessive gyrodine scheme, are presented.

Keywords: agile spacecraft, nonlinear dynamics, robust control

1. INTRODUCTION

The dynamic requirements to the attitude control system (ACS) for the remote sensing SC are:

- guidance the telescope's line-of-sight to a predetermined part of the Earth surface with the scan in designated direction;
- stabilization of a image motion at the onboard optical telescope focal plane.

Moreover, for the remote sensing spacecraft these requirements are expressed by rapid angular manoeuvring and spatial compensative motion with a variable vector of angular rate, see Fig. 1. Increased requirements to such information satellites (lifetime up to 10 years, exactness of spatial rotation manoeuvers with effective damping the SC flexible construction oscillations, robustness, fault-tolerance as well as to reasonable mass, size and energy characteristics has motivated intensive development the gyro moment clusters (GMCs) based on excessive number of gyrodines (GDs) — single-gimbal control moment gyros.

Mathematical aspects of the SC *nonlinear* gyromoment control were represented in a number of research works (Junkins and Turner, 1986; Singh and Bossart, 1993; Hoelscher and Vadali, 1994; Schaub *et al.*, 1998) including authors' papers (Anshakov *et al.*, 1995; Somov, 1997; Matrosov *et al.*, 1997; Somov, 1998; Somov *et al.*, 1999*a*,*b*,*c*; Kozlov *et al.*, 1999; Somov, 2000, 2001, 2002; Somov *et al.*, 2002; Matrosov and Somov, 2003; Somov *et al.*, 2003*a*,*b*,*c*, 2004; Anshakov *et al.*, 2004) The paper suggests new results on *nonlinear* and *robust* gyromoment control of the agile SC.

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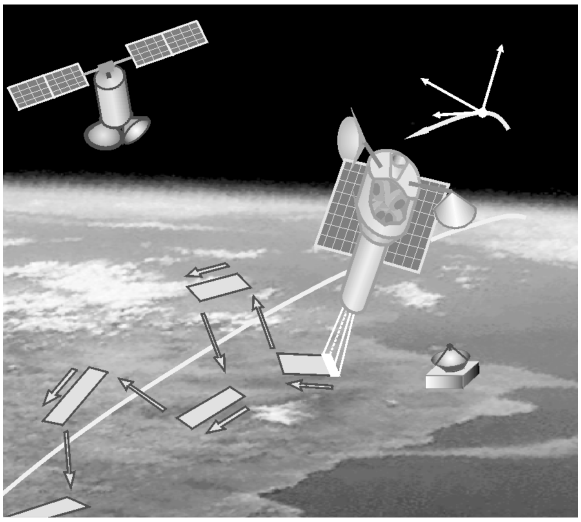


Fig. 1. The scanning pattern of given targets

2. THE GMC SCHEME

Primary problem consists in an estimating the domain $\mathbf{V}_k^{\rm p}$ of *nominal* variation of the SC body normed angular momentum (AM), starting from the SC purposes. The SC inertia tensor $\mathbf{J}$ usually is given in the BRF with any admittance, therefore guaranteeing calculation is carried out for the inertia tensor $\mathbf{J}^{\rm o}$, by which corresponds an inertia ellipsoid $\mathbf{J}^{\rm o} = {\rm diag}\{{\rm J}_x^{\rm o}, {\rm J}_y^{\rm o}, {\rm J}_z^{\rm o}\} \equiv \lceil {\rm J}_i^{\rm o} \rfloor$ with principle central axes on body reference frame (BRF) axes, and enveloping (i.e. including in itself) all possible the SC inertia ellipsoids. The domain of required variation by the SC vector *programmed* angular rate $\boldsymbol{\omega}^{\rm p}(t)$ conveniently is defined as ellipsoid $\mathbf{V}_\omega^{\rm p} = \{\boldsymbol{\omega}^{\rm p} : \sum(\omega_i^{\rm p}/\omega_i^{\rm m})^2 \le 1\}$, where $\omega_i^{\rm m}$ are given constants. Then the domain $\mathbf{V}_k^{\rm p}$ of nominal variation of the SC normed AM $\mathbf{k}^{\rm o}$ for its components $k_i^{\rm o}(t) = {\rm J}_i^{\rm o}\omega_i^{\rm p}(t)/h_g$ is obtained also in the form of ellipsoid $\mathbf{V}_k^{\rm p} = \{\mathbf{k}^{\rm o} : \sum(k_i^{\rm o}/k_i^{\rm m})^2 \le 1\}$, where constants $k_i^{\rm m} = {\rm J}_i^{\rm o}\omega_i^{\rm m}/h_g$ present its semi-axes. For information SC it is characteristics that the semi-axis dimensions of an ellipsoid $\mathbf{V}_k^{\rm p}$ have the essentially *different* values. For the agile remote sensing SC there is possible to stand out two classes of *programmed* angular rates of the SC movements — the course motions (CMs) and the rotational maneuvers (RMs):

- class $\boldsymbol{\mathcal{A}}$, when $\omega_i^{\rm m} = \omega^{\rm m}$ and $\|\boldsymbol{\omega}^{\rm p}(t)\| \le \omega^{\rm m}$;
- class $\boldsymbol{\mathcal{B}}$, when the ellipsoid's $\mathbf{V}_\omega^{\rm p}$ semi-axes are related among themselves as $a_1^\omega : a_2^\omega : a_3^\omega$ $(a_3^\omega \ge a_1^\omega >> a_2^\omega)$ at the fast movements with priority on the *pitch* axis Oz.

For typical remote sensing SC the values of inertia moments by its body ${\rm J}_x^{\rm o}$ and ${\rm J}_z^{\rm o}$ on the axes *roll* Ox and pitch Oz are compared among themselves, and no less twice exceeded the inertia moment by the SC body ${\rm J}_y^{\rm o}$ on the axis *yaw* Oy. Therefore the semi-axes $k_x^{\rm m}, k_y^{\rm m}$ $k_z^{\rm m}$ of ellipsoid $\mathbf{V}_k^{\rm p}$ are related among themselves approximately as 2:1:2 on the SC motions in class $\boldsymbol{\mathcal{A}}$ and as $2a_1^\omega : a_2^\omega : 2a_3^\omega$ in the basis $\mathbf{B}$ on the SC motions in class $\boldsymbol{\mathcal{B}}$. Domain $\mathbf{V}_k^{\rm m}$ of the SC body AM *required* variation must be given in the BRF with something to spare with respect to domain $\mathbf{V}_k^{\rm p}$. It is obviously that in class GD-systems the GMC mass will *minimal*, if at *least* number $m \ge 4$ applied GDs it ensures the domain $\mathbf{V}_h^{\rm m} = \mathbf{V}_\alpha^{\rm m}$ of the normed AM variation without *non-passed singular states* for condition $\mathbf{V}_k^{\rm m} \subseteq \mathbf{V}_h^{\rm m}$. In Crenshaw (1973) collinear pair of the stop-less GDs was named *Scissored Pair Ensemble* (*SPE*) and scheme based on two collinear pairs — as *2-SPE*. The angels $\gamma_{\rm s}^{\rm g}, 0 \le \gamma_{\rm s}^{\rm g} \le \pi/2, {\rm s} = 1, 2$ define the axis suspension positions for the GDs pairs. Topological analysis of *all* natural sets by singular GMC's states and results were presented in Somov *et al.* (2003*b*).

3. MATHEMATICAL MODELS

The spacecraft BRF attitude with respect to the inertial reference frame (IRF) is defined by quaternion $\boldsymbol{\Lambda} = (\lambda_0, \boldsymbol{\lambda}), \boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$. Assume that $\boldsymbol{\Lambda}^p(t)$ is a quaternion of the programmed SC body's motion in the IRF. The error quaternion is $\mathbf{E} = (e_0, \mathbf{e}) = \tilde{\boldsymbol{\Lambda}}^p(t) \circ \boldsymbol{\Lambda}$, the *Euler* parameters' vector is $\boldsymbol{\mathcal{E}} = \{e_0, \mathbf{e}\}$, and the attitude error's matrix is $\mathbf{C}_e \equiv \mathbf{C}(\boldsymbol{\mathcal{E}}) = \mathbf{I}_3 - 2[\mathbf{e}\times]\mathbf{Q}_e$, where $\mathbf{Q}_e \equiv \mathbf{Q}(\boldsymbol{\mathcal{E}}) = \mathbf{I}_3 e_0 + [\mathbf{e}\times]$ with $\det(\mathbf{Q}_e) = e_0$. Here and further the symbols $\langle\cdot,\cdot\rangle$, $\times$, $\{\cdot\}$, $[\cdot]$ for vectors and $[\mathbf{a}\times]$, $(\cdot)^{\rm t}$ for matrixes are conventional denotations.

For a fixed position of the SC *flexible* structures with some simplifying assumptions and $t \in {\rm T}_{t_0} = [t_0, +\infty)$ a SC angular motion model appears as:

$$\dot{\boldsymbol{\Lambda}} = \boldsymbol{\Lambda} \circ \boldsymbol{\omega}/2; \quad \mathbf{A}^o\{\dot{\boldsymbol{\omega}}, \ddot{\mathbf{q}}, \ddot{\boldsymbol{\beta}}\} = \{\mathbf{F}^\omega, \mathbf{F}^q, \mathbf{F}^\beta\}, \quad (1)$$

$$\mathbf{F}^\omega = \mathbf{M}^{\rm g} - \boldsymbol{\omega} \times \mathbf{G} + \mathbf{M}_d^o + \mathbf{Q}^o; \quad \mathbf{M}^{\rm g} = -\dot{\boldsymbol{\mathcal{H}}} = -\mathbf{A}_{\rm h}\dot{\boldsymbol{\beta}};$$

$$\mathbf{F}^q = \{-a_{jj}^q((\delta^q/\pi)\Omega_j^q \dot{q}_j + (\Omega_j^q)^2 q_j) + {\rm Q}_j^q(\boldsymbol{\omega}, \dot{q}_j, q_j)\};$$

$$\mathbf{F}^\beta = \mathbf{A}_{\rm h}^{\rm t}\boldsymbol{\omega} + \mathbf{M}^g + \mathbf{M}_d^g + \mathbf{M}_f^g + \mathbf{Q}^g; \ \mathbf{A}_{\rm h} = [\partial\boldsymbol{\mathcal{H}}(\boldsymbol{\beta})/\partial\boldsymbol{\beta}];$$

$$\mathbf{A}^o = \begin{bmatrix} \mathbf{J} & \mathbf{D}_q & \mathbf{D}_g \\ \mathbf{D}_q^{\rm t} & \mathbf{A}^q & \mathbf{0} \\ \mathbf{D}_g^{\rm t} & \mathbf{0} & \mathbf{A}^g \end{bmatrix}; \quad \begin{array}{l} \mathbf{G} = \mathbf{G}^o + \mathbf{D}_q\dot{\mathbf{q}} + \mathbf{D}_g\dot{\boldsymbol{\beta}}; \\ \mathbf{G}^o = \mathbf{J}\,\boldsymbol{\omega} + \boldsymbol{\mathcal{H}}(\boldsymbol{\beta}); \\ \boldsymbol{\omega} = \{\omega_i\}; \mathbf{q} = \{q_j\}; \boldsymbol{\beta} = \{\beta_p\}; \end{array}$$

the GMC's AM vector $\boldsymbol{\mathcal{H}}(\boldsymbol{\beta}) = {\rm h_g}\sum \mathbf{h}_p(\beta_p)$; a damping torque vector $\mathbf{M}_d^g$ is continuous function, and vector $\mathbf{M}_f^g$ of the friction torques in the GD's bearings is *discontinuous* function.

The components of the GMC control vector $\mathbf{M}^g$ are described as ${\rm M}_p^g = a_p^g\, u_p^{\rm c}(t)$, where the digital control voltages $u_p^{\rm c}(t) = {\rm Zh}[{\rm Sat}({\rm Qntr}(u_{pk}^{\rm c}, b_u^g), B_u^g), {\rm T}_u]$ and a_p^g are constants. Functions $u_{pk}^{\rm c}$ are the discrete outputs of nonlinear control law (NCL), and functions ${\rm Sat}(x, a)$ and ${\rm Qntr}(x, a)$ are general-usage ones, while the holder model with the period ${\rm T}_u$ is of the type: $y(t) = {\rm Zh}[x_k, {\rm T}_u] = x_k \quad \forall t \in [t_k, t_{k+1})$.

4. THE GMC ADJUSTMENT

Within *precession theory* of control moment gyros the the GMC torque vector $\mathbf{M}^{\rm g}$ is presented as

$$\mathbf{M}^{\rm g} = -\dot{\boldsymbol{\mathcal{H}}} = -h_g\mathbf{A}_\gamma\mathbf{A}_h(\boldsymbol{\beta})\,\mathbf{u}; \quad \dot{\boldsymbol{\beta}} = \mathbf{u}; \quad \dot{\mathbf{u}} = \mathbf{v}, \quad (2)$$

where matrix $\mathbf{A}_\gamma = \{[1, 0, 0], [0, s_\gamma, s_\gamma], [0, -c_\gamma, c_\gamma]\}$;

$$\mathbf{A}_h(\boldsymbol{\beta}) = \left[\frac{\partial\mathbf{h}(\boldsymbol{\beta})}{\partial\boldsymbol{\beta}}\right] = \begin{bmatrix} -{\rm y}_1 & -{\rm y}_2 & -{\rm z}_3 & -{\rm z}_4 \\ {\rm x}_1 & {\rm x}_2 & 0 & 0 \\ 0 & 0 & -{\rm x}_3 & -{\rm x}_4 \end{bmatrix};$$

$$\boldsymbol{\mathcal{H}}(\boldsymbol{\beta}) = h_g\mathbf{h}^{\rm c}; \mathbf{h}^{\rm c} = \{x_c, y_c, z_c\} = \mathbf{A}_\gamma\mathbf{h}; \gamma = \gamma_1^g = \gamma_2^g;$$

$$\mathbf{h}(\boldsymbol{\beta}) = \{{\rm x}, {\rm y}, {\rm z}\} = \sum\mathbf{h}_p(\beta_p); \ c_\alpha = \cos\alpha; \ s_\alpha = \sin\alpha;$$

$${\rm x} = {\rm x}_{12} + {\rm x}_{34}; \ {\rm y} = {\rm y}_1 + {\rm y}_2; \ {\rm z} = -({\rm z}_3 + {\rm z}_4); \ {\rm x}_p = c_{\beta_p};$$

$\mathsf{y}_p = s_{\beta_p}$; $\mathsf{z}_p = s_{\beta_p}$; $\mathsf{x}_{12} = \mathsf{x}_1 + \mathsf{x}_2$; $\mathsf{x}_{34} = \mathsf{x}_3 + \mathsf{x}_4$; $\mathbf{u}(t) \equiv \{\mathrm{u}_p(t), p = 1:4\}$; $\mathbf{v}(t) \equiv \{\mathrm{v}_p(t), p = 1:4\}$; $\mathrm{v}_p = (\mathrm{Sat}(\bar{\mathrm{v}}, \mathrm{u}_p^c(t)/T_u),\ if\ |\mathrm{u}_p| \le \bar{\mathrm{u}}) \vee (0,\ if\ |\mathrm{u}_p| > \bar{\mathrm{u}})$.

The distribution law (DL) of the normed AM $\mathbf{h}(\boldsymbol{\beta}) = \mathbf{A}_\gamma^{-1}\boldsymbol{\mathcal{H}}(\boldsymbol{\beta})/h_g$ between GD's pairs of GMC

$$f_\rho(\boldsymbol{\beta}) = (\tilde{\mathsf{x}}_1 - \tilde{\mathsf{x}}_2) + \rho\ (\tilde{\mathsf{x}}_1 \cdot \tilde{\mathsf{x}}_2 - 1) = 0, \qquad (3)$$

with $\tilde{\mathsf{x}}_1 = \mathsf{x}_{12}/\mathsf{q}_\mathsf{y}$; $\tilde{\mathsf{x}}_2 = \mathsf{x}_{34}/\mathsf{q}_\mathsf{z}$; $\mathsf{q}_\mathsf{s} = \sqrt{4 - \mathsf{s}^2}$, $\mathsf{s} = \mathsf{y}, \mathsf{z}$ and a fixed $\rho \in [\rho_0, \rho_1]$, where $\rho_0 = 2\sqrt{6}/5$, $\rho_1 = 1 - \rho_0$, differs from that proposed by J.W. Crenshaw (1973). For $\rho = \rho_0$ this DL ensures *global maximum* of the *Grame* determinant $\mathsf{G} = \det(\mathbf{A}_h \mathbf{A}_h^\mathrm{t}) = 64/27$ and the *maximum* module of the warranted control torque vector $\mathbf{M}^\mathbf{g}$ (2) in an arbitrary direction for the "park" state $\mathbf{h}(\boldsymbol{\beta}) = \mathbf{0}$, as well as large singularitiless central part inside of the GMC AM's variation domain

$$\mathbf{S} = \{\mathsf{x}^2 + \mathsf{y}^2 + \mathsf{z}^2 - 2\mathsf{q}_\mathsf{y}\mathsf{q}_\mathsf{z} < 8; |\mathsf{y}| < 2; |\mathsf{z}| < 2\}$$

and analytically described curves in the set of GMC's "passed" internal singularities $\mathbf{Q}_{\mathsf{yz}}(\boldsymbol{\beta}) = \mathbf{Q}_\mathsf{y}^p \cup \mathbf{Q}_\mathsf{z}^p$, $\mathbf{Q}_\mathsf{s}^p = \mathbf{Q}_\mathsf{s}^* \cap \mathbf{S}_\mathsf{s}^*$; $\mathbf{S}_\mathsf{s}^* = \{\mathsf{s} = 0; |\mathsf{s}_1| = |\mathsf{s}_2| = 1\}, \mathsf{s} = \mathsf{y}, \mathsf{z}$ with $\mathbf{Q}_\mathsf{y}^* = \{(\mathsf{x}_{34}/(2\rho))^2 + (\mathsf{z}/2)^2 = 1; \mathsf{x}_{34} < 0\}$ and $\mathbf{Q}_\mathsf{z}^* = \{(\mathsf{x}_{12}/(2\rho))^2 + (\mathsf{y}/2)^2 = 1; \mathsf{x}_{12} > 0\}$.

At "right-sided differential relay-hysteresis" tuning due to $D^+ f_\rho(\boldsymbol{\beta}) = \Phi_\rho(f_\rho(\boldsymbol{\beta}), \mathbf{h}(\boldsymbol{\beta}))$, where the function $\Phi_\rho(\cdot)$ was presented in Somov (2002), the DL (3) ensures the singular set $\mathbf{Q}_{\mathsf{yz}}(\boldsymbol{\beta})$ only at *separate* time moments (with *Lebesgue* zero measure) and *bijectively* connects the vector $\mathbf{M}^\mathbf{g}$ with vectors $\boldsymbol{\beta}$ and $\dot{\boldsymbol{\beta}}$.

Recently, the explicit *logic-dynamical* DL for the scheme *2-SPE* was elaborated (Somov, 2002; Somov and Butyrin, 2002*b*). This DL guarantees a GMC control torque vector for *any* time moment on arbitrary direction for all inner points of the domain $\mathbf{S}$. That law was synthesized by a logic flexible re-tuning the function $\rho(t)$ in (3) by its parametrization in a polynomial class. Let us be $t_{\nu i}$ and $t_{\nu f} = t_{\nu i} + T_\rho^g$ are the time moments by initial and finish of such ν recontruction with boundary conditions

$$\rho(t_{\nu i}) = \rho_{\nu i}; \quad \dot{\rho}(t_{\nu i}) = 0; \quad \ddot{\rho}(t_{\nu i}) = 0;$$
$$\rho(t_{\nu f}) = \rho_{\nu f}; \quad \dot{\rho}(t_{\nu f}) = 0; \quad \ddot{\rho}(t_{\nu f}) = 0,$$

where the values $\rho_{\nu i}, \rho_{\nu f} \in \{\rho_0, \rho_1\}$ for $\rho_{\nu i} \ne \rho_{\nu f}$. At notation $T_\rho = T_\rho^g/2$ and constant parameters $\sigma_\rho \equiv \rho_0 - \rho_1 = 2\rho_0 - 1$; $\omega_\rho \equiv \sigma_\rho/T_\rho$; $\varepsilon_\rho \equiv 6\omega_\rho/T_\rho$ there are defined the branch functions

$$\sigma_2(t) = \varepsilon_\rho \tau_1(1 - \tau_1), \sigma_1(t) = \omega_\rho \tau_1^2(3 - 2\tau_1),$$
$$\sigma(t) = \sigma_\rho \tau_1^3(2 - \tau_1)/2, \qquad t = t_{\nu i} + T_\rho \tau_1$$

and $\sigma_2(t) = -\varepsilon_\rho \tau_2(1 - \tau_2)$, $\sigma_1(t) = \omega_\rho[1 - \tau_2^2(3 - 2\tau_2]$, $\sigma(t) = \sigma_\rho[1/2 + \tau_2(1 - \tau_2^2(2 - \tau_2))/2], t = t_{\nu i} + T_\rho(1 + \tau_2)$, where the normed time parameters $\tau_1, \tau_2 \in [0, 1]$. For notation $S_{\rho\nu} = \mathrm{Sign}(\rho_{\nu i} - \rho_{\nu f})$ the function $\rho(t)$ and its the time derivatives are presented in *explicit* form

$$\rho(t) = \rho_{\nu i} + S_{\rho\nu} \cdot \sigma(t);\ \dot{\rho}(t) = S_{\rho\nu}\ \sigma_1(t);\ \ddot{\rho}(t) = S_{\rho\nu}\sigma_2(t)$$

with gratification of the boundary conditions.

5. PROGRAMMED CONTROL

The analytic matching solution have been obtained for problem of the SC programmed angular RM synthesis at given time interval $t \in \mathrm{T}_n \equiv [t_0^n, t_f^n]$, $t_f^n \equiv t_0^n + T_n$, when a space opto-electronic observation is carried out. This problem consists in determination of quaternion $\boldsymbol{\Lambda}(t)$ by the SC BRF $\mathbf{B}$ with respect to the IRF $\mathbf{I}$, angular rate vector $\boldsymbol{\omega}(t)$, vectors of angular acceleration $\boldsymbol{\varepsilon}(t) = \{\varepsilon_i(t)\} = \dot{\boldsymbol{\omega}}(t)$ and its derivative $\dot{\boldsymbol{\varepsilon}}(t) = \boldsymbol{\varepsilon}^*(t) + \boldsymbol{\omega}(t) \times \boldsymbol{\varepsilon}(t)$ in the form of *explicit* functions. Moreover values of vectors $\boldsymbol{\omega}_s \equiv \boldsymbol{\omega}(t_s)$ for the discrete time moments $t_s \in \mathrm{T}_n$ with period $T_q = t_{s+1} - t_s$, $s = 0, 1, 2 \ldots n_q \equiv 0 : n_q$, $n_q = T_n/T_q$ and initial value $\boldsymbol{\Lambda}(t_0^n) = \boldsymbol{\Lambda}_0$ are given.

Solution is based on a vector composition of all elemental motions in geodetic reference frame with regard to initial coordinates and the scan azimuth, the Sun zenith angle and a current observation perspective, proceed from principle requirement: optical image of the Earth given part must to move by desired way at focal plane of the telescope. The solution is obtained by *extrapolation* of the vector $\boldsymbol{\omega}_k \equiv \boldsymbol{\omega}(t_k)$ values which are defined in the time moments $t_k \in \mathrm{T}_n$ with step $T_a = t_{k+1} - t_k$, $k = 0 : n$, $n \equiv T_n/T_a$, for the period's ratio $k_q^a \equiv T_a/T_q = 2^{k_p}$ by degree $k_p \ge 2$. For such approximation the quaternion $\mathbf{M}(t)$ and vector $\mathbf{p}(t)$ have been appeared as explicit functions of a time $t \in \mathrm{T}_n$ which are neared to the quaternion $\boldsymbol{\Lambda}(t)$ and vector $\boldsymbol{\omega}(t)$ $\forall t \in T_n$ respectively, and in *explicit* form — the vector functions $\mathbf{q}(t) = \dot{\mathbf{p}}(t)$ and $\dot{\mathbf{q}}(t)$, corresponding to $\boldsymbol{\varepsilon}(t)$ and $\boldsymbol{\varepsilon}^*(t)$. Extrapolation of the discrete-assigned trajectory $\boldsymbol{\omega}_k$ by vector $\mathbf{p}(t)$ $\forall t \in \mathrm{T}_n$ at the conditions $\mathbf{p}(t_k) \equiv \mathbf{p}_k = \boldsymbol{\omega}_k$, $k = 0 : (n-1)$, $\mathbf{p}(t_n) = \mathbf{p}(t_f^n) = \boldsymbol{\omega}_n$ is obtained by set of n 3-degree *vector* splines $\mathbf{p}_k(\tau)$ at normed time $\tau = (t - t_k)/T_a \in [0, 1]$, with analytical obtaining a high-precise approximation both vector of programmed angular acceleration $\mathbf{q}(t) = \dot{\mathbf{p}}(t)$ with its local derivative $\dot{\mathbf{q}}(t)$ and quaternion $\mathbf{M}(t)$. These functions are applied at onboard computer for the time moments $t_s \in \mathrm{T}_n$.

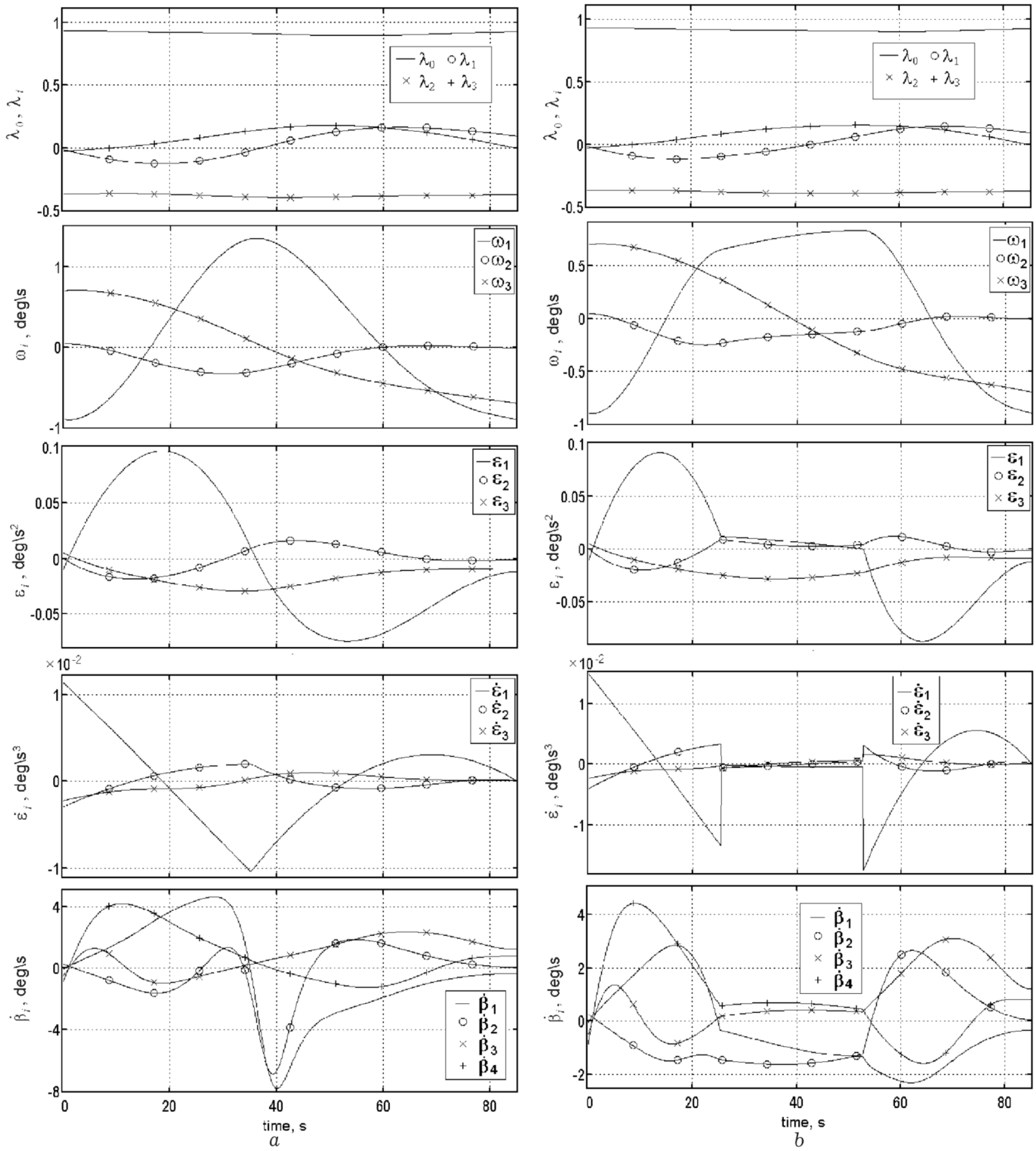
For control torque $\mathbf{M}^\mathbf{g}$ (2) and the SC model as a free rigid body the simplified controlled object is such:

$$\dot{\boldsymbol{\Lambda}} = \boldsymbol{\Lambda} \circ \boldsymbol{\omega}/2; \mathbf{J}\dot{\boldsymbol{\omega}} + [\boldsymbol{\omega}\times]\mathbf{G}^o = \mathbf{M}^\mathbf{g}; \dot{\boldsymbol{\beta}} = \mathbf{u}; \dot{\mathbf{u}} = \mathbf{v}. \quad (4)$$

The computer-aided algorithm for synthesis of the strict optimal control $\mathbf{v} = \mathbf{v}^p(t), t \in \mathrm{T_r} = [t_\mathrm{i}, t_\mathrm{f}]$ for the GMC AM's DL (3) during the SC rapid RMs with the *preassigned boundary conditions* on initial and final states $\boldsymbol{\Lambda}^\mathrm{s} \equiv \boldsymbol{\Lambda}(t_\mathrm{s})$; $\boldsymbol{\omega}^\mathrm{s} \equiv \boldsymbol{\omega}(t_\mathrm{s}), \mathrm{s} = \mathrm{i}, \mathrm{f}$; $\boldsymbol{\beta}^\mathrm{i} \equiv \boldsymbol{\beta}(t_\mathrm{i})$ with $\mathbf{h}(\boldsymbol{\beta}^\mathrm{i}) \subseteq \mathbf{S}^o \subset \mathbf{S}$ and $\dot{\boldsymbol{\beta}}^\mathrm{i} \equiv \dot{\boldsymbol{\beta}}(t_\mathrm{i}) = \mathbf{0}$ in the *optimization problem* of the energy index

$$\mathrm{I}^\circ = \frac{1}{2} \int_{t_\mathrm{i}}^{t_\mathrm{f}} (q\ \mathrm{Q}_1(\mathbf{u}(\tau)) + p\ \mathrm{Q}_2(\mathbf{v}^p(\tau))) d\tau \Rightarrow \min \qquad (5)$$

for $\boldsymbol{\Lambda}^\mathrm{i}$, $\boldsymbol{\omega}^\mathrm{i} \Longrightarrow \boldsymbol{\Lambda}^\mathrm{f}$, $\boldsymbol{\omega}^\mathrm{f}$, where $q, p, a_i = \mathrm{const}$ and $q \in [0, 1], p = 1 - q$; $\mathrm{Q}_i(\mathbf{x}) \equiv a_i^2 \langle \mathbf{x}, \mathbf{x} \rangle$ and a *preassigned* time $T_\mathrm{r} = t_\mathrm{f} - t_\mathrm{i}$ for $\mathbf{h}(\boldsymbol{\beta}(t)) \subseteq \mathbf{S}^o\ \forall t \in \mathrm{T_r} \subset \mathrm{T}_{t_0}$, has been created (Somov, 2000). Fast onboard algorithms with parametric optimization $\mathbf{v}^p(t)$ in the problem (5) for model (4) and restrictions to $\boldsymbol{\omega}(t), \dot{\boldsymbol{\omega}}(t), \ddot{\boldsymbol{\omega}}(t)$, corresponding restrictions to $\mathbf{h}(\boldsymbol{\beta}(t)), \dot{\boldsymbol{\beta}}(t) = \mathbf{u}(t), \ddot{\boldsymbol{\beta}}(t) = \mathbf{v}^p(t)$ in a class of the SC angular motions, were elaborated. At analytical synthesis of the SC RM



a *b*

Fig. 2. Coordinates of the SC and the GMC at the programmed rotation maneuver: *a* — without a limit of the SC angular rate in a *position transfer*; *b* — with such limit.

programme under given interval of a time $t \in \mathrm{T}_p \equiv [t_0^p, t_f^p]$, $t_f^p \equiv t_0^p + T_p$ a problem consists in determination the *explicit* time functions — quaternion $\mathbf{\Lambda}(t)$, vectors $\boldsymbol{\omega}(t)$, $\varepsilon(t)$ and $\dot{\boldsymbol{\varepsilon}}(t) = \boldsymbol{\varepsilon}^*(t) + \boldsymbol{\omega}(t) \times \boldsymbol{\varepsilon}(t)$ for the boundary conditions on left ($t = t_0^p$) and right ($t = t_f^p$) trajectory ends:

$$\mathbf{\Lambda}(t_0^p) = \mathbf{\Lambda}_0; \boldsymbol{\omega}(t_0^p) = \boldsymbol{\omega}_0 \equiv \omega_0\ \mathbf{e}_0^{\omega}; \boldsymbol{\varepsilon}(t_0^p) = \boldsymbol{\varepsilon}_0 \equiv \varepsilon_0 \mathbf{e}_0^{\varepsilon}; \quad (6)$$

$$\mathbf{\Lambda}(t_f^p) = \mathbf{\Lambda}_f; \boldsymbol{\omega}(t_f^p) \equiv \boldsymbol{\omega}_f \equiv \omega_f \mathbf{e}_f^{\omega}; \boldsymbol{\varepsilon}(t_f^p) = \boldsymbol{\varepsilon}_f \equiv \varepsilon_f \mathbf{e}_f^{\varepsilon}; \quad (7)$$
$$\dot{\boldsymbol{\varepsilon}}(t_f^p) = \dot{\boldsymbol{\varepsilon}}_f \equiv \varepsilon_f^*\ \mathbf{e}_f^{\varepsilon *} + \boldsymbol{\omega}_f \times \boldsymbol{\varepsilon}_f.$$

In (7) last condition presents requirements for a *smooth conjugation* of rotation maneuver with next the SC route motion. Developed approach to the problem is also based on *necessary* and *sufficient* condition for solvability of *Darboux* problem.

At general case the solution is presented as result of composition by six ($k = 1:6$) *simultaneously* derived elementary rotations of *embedded* bases $\mathbf{E}_k$ about units $\mathbf{e}_k$ of *Euler* axes, which position is defined from the boundary conditions (6) and (7) for initial spatial problem. For all 6 elementary rotations with respect to units $\mathbf{e}_k$ the boundary conditions are analytically assigned, moreover for first *five* elementary motions there are ensured zero value for local (own) derivative of acceleration on right end of trajectory. Into basis $\mathbf{I}$ the quaternion $\mathbf{\Lambda}(t)$ is defined by the production

$$\mathbf{\Lambda}(t) = \mathbf{\Lambda}_0 \circ \mathbf{\Lambda}_1(t) \circ \mathbf{\Lambda}_2(t) \circ \mathbf{\Lambda}_3(t) \circ \mathbf{\Lambda}_4(t) \circ \mathbf{\Lambda}_5(t) \circ \mathbf{\Lambda}_6(t), \quad (8)$$

where $\mathbf{\Lambda}_k(t) = (\cos(\varphi_k(t)/2), \sin(\varphi_k(t)/2)\ \mathbf{e}_k)$, $\mathbf{e}_k$ is unit of *Euler* axis by k's rotation, and functions $\varphi_k(t)$ present the elementary rotation angels in analytical form. These functions are selected in class of splines by relative degree. At initial denotations for vectors $\boldsymbol{\omega}^{(1)}(t) = \boldsymbol{\omega}_1(t)$, $\boldsymbol{\varepsilon}^{(1)}(t) = \boldsymbol{\varepsilon}_1(t)$ $\dot{\boldsymbol{\varepsilon}}^{(1)}(t) = \dot{\boldsymbol{\varepsilon}}_1(t)$ vector of angular rate $\boldsymbol{\omega}(t)$, vectors of angular acceleration

$\varepsilon(t)$ and its derivative $\dot{\varepsilon}(t)$ are *analytically* defined for each time moment $t \in \mathrm{T}_p$ by recurrent algorithm: for *upper* indexes $k = 2:6$ are *consequently* computed

$$\begin{aligned}
&\omega_q^k(t) =: \tilde{\mathbf{\Lambda}}_k(t) \odot \boldsymbol{\omega}^{(k-1)}(t) \odot \mathbf{\Lambda}_k(t), \\
&\qquad \boldsymbol{\omega}^{(k)}(t) = \boldsymbol{\omega}_k(t) + \boldsymbol{\omega}_q^k(t); \\
&\boldsymbol{\varepsilon}_q^k(t) =: \tilde{\mathbf{\Lambda}}_k(t) \odot \boldsymbol{\varepsilon}^{(k-1)}(t) \odot \mathbf{\Lambda}_k(t), \\
&\qquad \boldsymbol{\varepsilon}^{(k)}(t) = \boldsymbol{\varepsilon}_k(t) + \boldsymbol{\varepsilon}_q^k(t) + \boldsymbol{\omega}_q^k(t) \times \boldsymbol{\omega}_k(t); \qquad (9) \\
&\dot{\boldsymbol{\varepsilon}}_q^k(t) =: \tilde{\mathbf{\Lambda}}_k(t) \odot \dot{\boldsymbol{\varepsilon}}^{(k-1)}(t) \odot \mathbf{\Lambda}_k(t), \\
&\dot{\boldsymbol{\varepsilon}}^{(k)}(t) = \dot{\boldsymbol{\varepsilon}}_k(t) + \dot{\boldsymbol{\varepsilon}}_q^k(t) + (2\boldsymbol{\varepsilon}_q^k(t) \\
&\qquad + \boldsymbol{\omega}_q^k(t) \times \boldsymbol{\omega}_k(t)) \times \boldsymbol{\omega}_k(t) + \boldsymbol{\omega}_q^k(t) \times \boldsymbol{\varepsilon}_k(t),
\end{aligned}$$

and in result quested vectors are obtained in the form $\omega(t) = \omega^{(6)}(t)$, $\varepsilon(t) = \varepsilon^{(6)}(t)$, $\dot{\varepsilon}(t) = \dot{\varepsilon}^{(6)}(t)$. In (9) a module of a motion rate in a *position transfer* ($k=3$) may be limited (Somov and Butyrin, 2003). The technique is based on the generalized integral's properties for the AM of the mechanical system "SC+GMC" and allows to evaluate vectors $\boldsymbol{\beta}(t)$, $\dot{\boldsymbol{\beta}}(t)$, $\ddot{\boldsymbol{\beta}}(t)$ in the *analytical form* for an *arbitrary preassigned* SC motion $\mathbf{\Lambda}(t)$, $\boldsymbol{\omega}(t)$, $\dot{\boldsymbol{\omega}}(t)$, $\ddot{\boldsymbol{\omega}}(t)\ \forall t \in \mathrm{T_r}$. Let be $\mathbf{g}(t) = \mathbf{k}(t) + \mathbf{h^c}(t) = \tilde{\mathbf{\Lambda}}(t) \circ \mathbf{g}_{\mathrm{i}}^{\mathrm{I}} \circ \mathbf{\Lambda}(t)$, where $\mathbf{k}(t) = \mathbf{J}\,\boldsymbol{\omega}(t)/h_g$ and $\mathbf{g}_{\mathrm{i}}^{\mathrm{I}} = \mathbf{\Lambda}(t_{\mathrm{i}}) \circ \mathbf{g}(t_{\mathrm{i}}) \circ \tilde{\mathbf{\Lambda}}(t_{\mathrm{i}})$. By the DL (3) there is derived the priciple relation

$$\delta \;=\; d\;(1 - (1 - 2ac\rho - e\rho^2)^{1/2})/\rho,$$

where $a = \mathsf{x}/d$; $b = \mathsf{q_y q_z}/d^2$; $c = (\mathsf{q_y} - \mathsf{q_z})/d$; $d = \mathsf{q_y} + \mathsf{q_z}$; $e = 4b - a^2$. Then there are computed:

$$\begin{aligned}
\mathbf{h^c}(t) = \mathbf{g}(t) - \mathbf{k}(t) \implies \boldsymbol{\beta}(t);\ \mathbf{g}^a(t) = -\boldsymbol{\omega}(t) \times \mathbf{g}(t); \\
\mathbf{g}^b(t) = -\dot{\boldsymbol{\omega}}(t) \times \mathbf{g}(t) - \boldsymbol{\omega}(t) \times \mathbf{g}^a(t); \\
\dot{\mathbf{h}}^{\mathbf{c}}(t) \;=\; \mathbf{g}^a(t) - \dot{\mathbf{k}}(t) \implies \dot{\boldsymbol{\beta}}(t); \\
\ddot{\mathbf{h}}^{\mathbf{c}}(t) \;=\; \mathbf{g}^b(t) - \ddot{\mathbf{k}}(t) \implies \ddot{\boldsymbol{\beta}}(t).
\end{aligned}$$

These spline onboard algorithms ensure the desirable *profile smoothness* for the SC motion with small level of its flexible structure *oscillations*. Fig. 2 briefly presents the SC programmed rotation maneuver on time interval $t \in [0, 85]$ sec with boundary conditions, represented in Somov and Butyrin (2002*a*). At low part of this figure the precession angular rate $\dot{\beta}_p(t)$ values are presented for all four gyrodines.

6. NONLINEAR ROBUST CONTROL

If the error $\delta\boldsymbol{\omega} \equiv \tilde{\boldsymbol{\omega}}$ in the rate vector $\boldsymbol{\omega}$ is defined as $\tilde{\boldsymbol{\omega}} = \boldsymbol{\omega} - \mathbf{C}_e\boldsymbol{\omega}^p(t)$, and the GMC's required control torque vector $\mathbf{M^g}$ is formed as $\mathbf{M^g} = \boldsymbol{\omega} \times \mathbf{G}^o + \mathbf{J}(\mathbf{C}_e\dot{\boldsymbol{\omega}}^p(t) - [\boldsymbol{\omega}\times]\mathbf{C}_e\boldsymbol{\omega}^p(t) + \tilde{\mathbf{m}})$, then the simplest nonlinear model of the SC's attitude error is as follows:

$$\dot{e}_0 = -\langle \mathbf{e}, \tilde{\boldsymbol{\omega}} \rangle/2; \quad \dot{\mathbf{e}} = \mathbf{Q}_e\tilde{\boldsymbol{\omega}}/2; \quad \dot{\tilde{\boldsymbol{\omega}}} = \tilde{\mathbf{m}}. \qquad (10)$$

By the relations $\mathbf{Q}_e^{-1}\mathbf{Q}_e^{\mathrm{t}} = \mathbf{C}_e$; $\mathbf{Q}_e^{-1} = \mathbf{Q}_e^{\mathrm{t}} + \mathbf{e} \cdot \mathbf{e}^{\mathrm{t}}/e_0$; $\mathbf{Q}_e^{-1}\mathbf{e} = \mathbf{e}/e_0$; $\mathbf{I}_3 - e_0\mathbf{Q}_e^{-1} = \mathbf{Q}_e^{\mathrm{t}}[\mathbf{e}\times]$, which are used for $e_0 \neq 0$ (Somov, 1997), for model (10) a *nonlocal nonlinear* coordinate transformation is defined and used at analytical synthesis by the *exact feedback linearization.* This results in the NCL

$$\tilde{\mathbf{m}}(\boldsymbol{\mathcal{E}}, \tilde{\boldsymbol{\omega}}) = -\mathbf{A}_0 \cdot \mathbf{e} \cdot \mathrm{Sgn}(e_0) - \mathbf{A}_1 \cdot \tilde{\boldsymbol{\omega}}, \qquad (11)$$

where $\mathbf{A}_0 = ((2a_0^* - \tilde{\omega}^2/2)/e_0)\mathbf{I}_3$; $\mathbf{A}_1 = a_1^*\mathbf{I}_3 - \mathbf{R}_{e\omega}$, $\mathrm{Sgn}(e_0) = (1$, if $e_0 \geq 0) \vee (-1$, if $e_0 < 0)$, matrix $\mathbf{R}_{e\omega} = \langle \mathbf{e}, \tilde{\boldsymbol{\omega}} \rangle \mathbf{Q}_e^{\mathrm{t}}[\mathbf{e}\times]/(2e_0)$, and constants a_0^*, a_1^* are analytically calculated on spectrum $\mathrm{S}_{ci}^* = -\alpha_c \pm j\omega_c$.

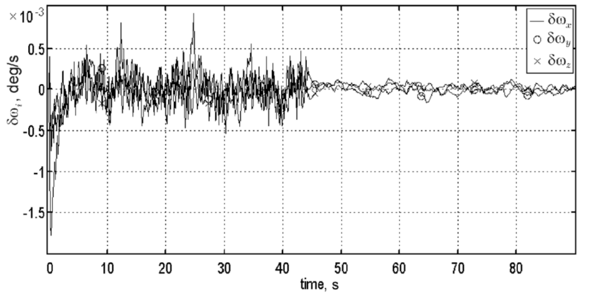


Fig. 3. The rate errors for consequence of the SC rotational maneuver and course motion

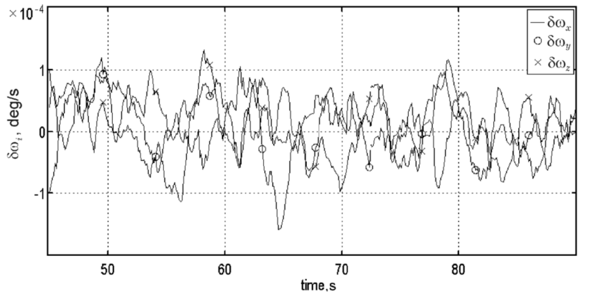


Fig. 4. The rate errors at the course motion

Simultaneously using the *Vandermonde* matrix the *vector Lyapunov function* (VFL) $\boldsymbol{\upsilon}(\boldsymbol{\mathcal{E}}, \tilde{\boldsymbol{\omega}})$ is analytically constructed for close-loop system (10) and (11). Taking into account restrictions on the GMC control, special nonlinear functions of the type *"division of variables with scaling"* were introduced in Somov (1997). In result the *nonlinear control law* was obtained for model (2) and (4), details see in Somov *et al.* (1999*b*). In stage 2, the problems of synthesising nonlinear control law were solved for model of the *flexible* spacecraft (1). Furthermore, the selection of parameters in the structure of the GMC *nonlinear robust* control law (which optimizes the main quality criterion for given restrictions) is fulfilled by a multistage numerical analysis and *parametric* optimization of the *comparison system* for the VLF. Thereto, the VLF has the structure derived above for the *error coordinates* $\boldsymbol{\mathcal{E}}, \tilde{\boldsymbol{\omega}}$ and the structure of other VLF components in the form of *sublinear norms* for vector variables $\mathbf{q}(t)$, $\dot{\mathbf{q}}(t)$, $\dot{\boldsymbol{\beta}}(t)$ using the vector $\boldsymbol{\beta}(t)$.

Applied onboard measuring system is based on the fiber-optic gyros corrected by the fine fixed-head star trackers. This system is intended for precise determination of the SC angular position and rate. Applied contemporary filtering & alignment calibration algorithms and a discrete *astatic* observer give finally a fine estimating the SC angular motion coordinates.

7. COMPUTER SIMULATION

Fig. 3 and Fig. 4 present some results on computer simulaton of a ACS for Russian remote sensing SC by the *Resource-DK* type. Here the rate errors are represented at consequence of the SC spatial rotational maneuver for time $t \in [0, 45)$ sec and the SC course motion for time $t \in [45, 90]$ sec with a nearly-constant vector of acceleration $\boldsymbol{\varepsilon}(t)$. Applied digital nonlinear control law is flexible switched at the time $t = 45$ sec on astatic ones with respect to the acceleration.

8. CONCLUSIONS

Contemporary approaches and some new results were presented for nonlinear robust ACSs applied at the agile remote sensing SC.

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