EXPERIMENTAL VERIFICATION OF STABILIZING SPLINE-BASED CONTINUOUS-TIME MODEL PREDICTIVE CONTROL SCHEME WITH ADAPTATION OF TERMINAL SET

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Abstract: In this paper a continuous-time Model-based Predictive Control (MPC) problem is approached in a sub-optimal way utilizing polynomial spline functions as the plant control input signals and B-splines for approximation of the continuous-time performance index. The optimization is performed with respect to a sequence of spline coefficients belonging to predicted control input profiles. The suggested solution enables us a continuous-time satisfaction of the plant signal constraints. Closed-loop stability is ensured through infinite horizon formulation by means of a terminal cost, a terminal constraint set, and a local controller. The presented scheme is applied on-line, supporting adaptation of the terminal set and resulting algorithm. The efficacy of the approach is illustrated through an experimental verification on a laboratory scale system.

Keywords: Predictive control, Constraints, Closed-loop stability, Spline functions

1. INTRODUCTION

The MPC algorithms have been widely developed mainly in a discrete-time context utilizing discrete-time models for continuous-time systems description and piecewise constant control signals. It is evident that in consequence of their discrete-time formulation, these algorithms are necessarily suboptimal with respect to the ideal continuous-time representations and performance indexes, for arguments see e.g. (Cannon, 1999), (Cannon and Kouvaritakis, 2000), (Blanchini *et al.*, 2002), (Magni and Scattolini, 2002) and the papers quoted there. The ideal MPC continuous-time formulation leads to a difficult optimization task assumed to be repeatedly solved at any continuous time instant. This is practically impossible, since any real implementation of MPC algorithm requires a non-negligible computational time. During the last years has been suggested several computationally tractable (sub-optimal) continuous-time MPC schemes, starting from the classical ones, see e.g. (Demircioğlu and Gawthrop, 1991), (Gawthrop *et al.*, 1998), (Ronco *et al.*, 1998), up to the guaranteed stability ones, see e.g. (Chen and Allgöwer, 1998), (Kouvaritakis *et al.*, 1999), (Cannon and Kouvaritakis, 2000), (Magni and Scattolini, 2004). The main objective of this paper is directed towards the design of a practically

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implementable, hybrid nature, continuous-time MPC scheme based on a simultaneous polynomial and B-spline parameterization of the continuous-time plant model, the used continuous-time performance index and the plant control and output signals. Considering a finite number of spline basis functions the parameterization enables us to design suitable discretization of all, in the MPC problem involved tasks, while satisfying their continuous-time nature on a desired level. The plant input/output signals and the applied constraints are treated in term of spline coefficients irrelevant to the used sampling time. Such treatment with the signals and their constraints allows us also to evaluate and guarantee their intersample behaviour, which in some practical cases could be significant. Taking the plant manipulated signals from a suitable chosen class of polynomial spline functions, the overall MPC optimization procedure leads to a finite dimensional problem with a finite number of decision variables, which are the spline coefficients of the predicted input profiles over the given prediction horizon. Following the receding horizon control philosophy only the first polynomial segment of the optimal input profile is applied to the manipulated variables with an arbitrary small sampling frequency, while the whole optimization procedure can be performed in parallel, with a smaller sampling frequency. The plant closed loop stability is ensured by means of a terminal cost, a terminal constraint set and a local controller acting after some finite horizon, following the quasi-infinite horizon approach of (Chen and Allgöwer, 1998) and solutions in (Mayne *et al.*, 2000). For simplicity only the plant input constraints are considered. Executing the above ideas two distinct sampling time intervals are introduced in this work:

(1) the *implementation* sampling time T_g - vanishingly small. Using zero order hold, it serves only for calculation of values of the spline input polynomial segment applied to the plant control during a time interval between the spline knot points.
(2) the *control* sampling time T - the time interval between the uniformly located spline input signal knots ($T = ng.T_g$). The size of T can be small, it depends only on the MPC procedure computational time.

2. PROBLEM STATEMENT AND PRELIMINARY RESULTS

Consider the continuous-time plant described near a given reference operating point by following matrix input-output convolution relation written in the Laplace domain as

$$\mathcal{C}^{-1}(s)\mathcal{A}(s)\mathbf{y}(s) = \mathcal{C}^{-1}(s)\mathcal{B}(s)\mathbf{u}(s) + \mathbf{e}(s) \qquad (1)$$

where $\mathbf{y}(s)$, $\mathbf{u}(s)$ and $\mathbf{e}(s)$ are $(p \times 1)$ output, $(m \times 1)$ input and $(p \times 1)$ disturbance vectors, respectively. $\mathcal{A}(s)$ and $\mathcal{C}(s)$ are $(p{\times}p)$ and $\mathcal{B}(s)$, $(p{\times}m)$ polynomial matrices. Without too much loss of generality, the matrices $\mathcal{A}(s)$ and $\mathcal{C}(s)$ are assumed to be diagonal. Vector $\mathbf{e}(s)$ represents a transform of disturbance signals, which are considered of finite intensity and uncorrelated with inputs and which become white discrete processes in the case of fast sampling.

Consider that the plant manipulated signals $\mathbf{u}(t)$ are chosen from a linear space $P_{r,\xi,\alpha}$ of polynomial spline functions of order r with knot sequence ξ where α counts the number of continuity conditions required at the knot points,

$$\mathbf{u}(t) \ \in \ P_{r,\xi,\alpha} \qquad (2)$$

Then using a technique of *spline filtration* elaborated in (Kárný *et al.*, 1990), (Rohaľ-Ilkiv, 2003) the relation (1) can by effectively identified from the filtered data and written in standard regression form, for the ith plant output as

$$y_i(t) \ = \theta_i^T \ \varphi_i(t) + \epsilon_i(t) \qquad (3)$$

where θ_i^T is the vector of filtered plant parameters and $\varphi_i(t)$ is the vector of real data observed within data windows of appropriate chosen length.

An important by-product of the spline filtering technique is knowledge of all (spline) derivatives of the plant input/output signals thereby enabling us to create a simple phase variable state-space realization of the plant (Gopal, 1985)

$$\dot{\mathbf{x}}(t) = \mathbf{A}\ \mathbf{x}(t) + \mathbf{B}\ \mathbf{u}(t) \qquad (4)$$

$$\mathbf{y}(t) = \mathbf{C}\ \mathbf{x}(t) \qquad (5)$$

where the entries of state vector $\mathbf{x}(t)$ are accessible to straight calculation from the derivatives, which considerably support the practical calculation of the guaranteed stability "ingredients".

Ideally, the objective of the continuous-time MPC design considered in the work is to minimize an infinite horizon quadratic performance index

$$J_{inf} = \int_t^\infty [\|\mathbf{y}(t+\tau|t)\|^2_{\mathbf{Q}} + \|\mathbf{u}(t+\tau|t)\|^2_{\mathbf{R}}]d\tau \quad (6)$$

subject to (1) and input constraints

$$-\mathbf{u}_{min} \le \mathbf{u}(t+\tau|t) \le \mathbf{u}_{max} \quad \tau \ge t$$

$$-\dot{\mathbf{u}}_{min} \le \dot{\mathbf{u}}(t+\tau|t) \le \dot{\mathbf{u}}_{max} \quad \tau \ge t$$

In (6) $\mathbf{Q}$ and $\mathbf{R}$ are positive definite weighting matrices and all signals are written as differences from their reference values. $\mathbf{u}_{min}, \mathbf{u}_{max}, \dot{\mathbf{u}}_{min}, \dot{\mathbf{u}}_{max}$ are positive bounds imposed on control input signals. Applying the quasi-infinite horizon approach of (Chen and Allgöwer, 1998) to guarantee the closed-loop stability the infinite horizon index (6) can be formulated as

$$J(\mathbf{x}(t), \mathbf{u}(.), T_h) = \int_t^{t+T_h} [\|\mathbf{y}(t+\tau|t)\|^2_{\mathbf{Q}} + \|\mathbf{u}(t+\tau|t)\|^2_{\mathbf{R}}]d\tau + \|\mathbf{x}(t+T_h|t)\|^2_{\mathbf{Q}_h} \qquad (7)$$

subject to

$$-\mathbf{u}_{min} \le \mathbf{u}(t+\tau|t) \le \mathbf{u}_{max} \quad \tau \ge t \tag{8}$$

$$-\dot{\mathbf{u}}_{min} \le \dot{\mathbf{u}}(t+\tau|t) \le \dot{\mathbf{u}}_{max} \quad \tau \ge t \tag{9}$$

$$\mathbf{x}(t_k+T_h|t_k) \in \Omega \tag{10}$$

where

$$\|\mathbf{x}(t+T_h|t)\|^2_{\mathbf{Q}_h} \ge \int_{t+T_h}^{\infty} [\|\mathbf{x}(\tau|t)\|^2_{\mathbf{Q}_x} + \|\mathbf{u}(\tau|t)\|^2_{\mathbf{R}}]d\tau$$

$$\mathbf{u} = \mathbf{F}\mathbf{x}, \quad \forall \mathbf{x}(\tau|t) \in \Omega \tag{11}$$

$\mathbf{Q}_h$ is the terminal state penalty matrix, $\mathbf{Q}_x = \mathbf{C}^T\mathbf{Q}\mathbf{C}$ and Ω is the terminal constraint set. The set have to be chosen as invariant with respect to a local linear state feedback $\mathbf{u} = \mathbf{F}\mathbf{x}$, virtually acting for $\tau \in [t+T_h, \infty]$, and feasible with (8), (9).

3. SPLINE APPROXIMATION OF THE FINITE HORIZON PERFORMANCE INDEX

The task is to find at some decision time instant $t_k(t_{k+1} = t_k + T)$ optimal continuous-time control input signals, $\mathbf{u}(t_k+\tau|t_k), \tau \in [t_k, t_k+T_h]$, meeting constraints (8),(9) and (10) such that the performance index (7) reaches its minimal value. Let us simplify the optimization problem using the spline approximation to the finite horizon performance index considering the condition (2). Then applying the integration in (7) over chosen spline bases functions and over polynomial segments of all spline control signals the approximation gives us following version of the performance index

$$J(\mathbf{x}(t_k), \mathbf{p}_{h_k}, T_h) = \|\mathbf{c}_h(k)\|^2_{\mathbf{Q}_1} + \|\mathbf{p}_{h_k}\|^2_{\mathbf{Q}_u} + \\ + \|\mathbf{x}(t_k+T_h|t_k)\|^2_{\mathbf{Q}_h} \tag{12}$$

where vector $\mathbf{p}_{h_k}$ collects the coefficients of all polynomial segments which belong to the projected input profiles calculated over the prediction horizon, $[t_k, t_k+T_h]$ and vector $\mathbf{c}_h(k)$, collects all (B-spline) coefficients of the plant output signals. The new penalty matrices $\mathbf{Q}_1$, $\mathbf{Q}_u$ are calculated performing integration over mutual products of the individual spline basis functions chosen for the approximation.

Applying well-known formulas for spline interpolation it is possible to exchange the vector of B-spline coefficients $\mathbf{c}_h(k)$ in (12) with a virtual vector $\hat{\mathbf{y}}_k$ of predicted plant outputs sampled with the sampling period T, then

$$\mathbf{c}_h(k) = \mathbf{M}_h^{-1}\hat{\mathbf{y}}_k \tag{13}$$

$$\mathbf{M}_h = \begin{bmatrix} \mathbf{M}_h(t_{k+1}) \\ \mathbf{M}_h(t_{k+2}) \\ \vdots \\ \mathbf{M}_h(t_{k+nh}) \end{bmatrix} \qquad \hat{\mathbf{y}}_k = \begin{bmatrix} \hat{\mathbf{y}}(t_{k+1}) \\ \hat{\mathbf{y}}(t_{k+2}) \\ \vdots \\ \hat{\mathbf{y}}(t_{k+nh}) \end{bmatrix}$$

where $\mathbf{M}_h(t)$ is a matrix consisting from the chosen spline basis functions evaluated for the time instant t. Considering the above relation in (12) we can arrive at the next final version of the approximated finite horizon performance index

$$J(\mathbf{x}(t_k), \mathbf{p}_{h_k}, T_h) = \|\hat{\mathbf{y}}_k\|^2_{\mathbf{Q}_y} + \|\mathbf{p}_{h_k}\|^2_{\mathbf{Q}_u} + \\ + \|\mathbf{x}(t_k+T_h|t_k)\|^2_{\mathbf{Q}_h} \tag{14}$$

where

$$\mathbf{Q}_y = [\mathbf{M}_h^{-1}]^T\mathbf{Q}_1\mathbf{M}_h^{-1} \tag{15}$$

For practical calculations of the terminal state penalty matrix $\mathbf{Q}_h$ and the terminal set Ω the state-space description (4) and the state feedback gain $\mathbf{F}$ of the local controller have to be applied.

4. TERMINAL CONSTRAINT SETS DEFINITION

The quasi-infinite horizon prediction strategies, the terminal state penalty and terminal constraint sets are imperative tools for guarantee of the closed-loop stability in constrained MPC schemes. The existence, characterization and practical calculation of the sets for dynamical systems is therefore a basic issue for successful implementation of MPC algorithms.

This paper utilizes following *low-complexity* polytopic invariant sets as the terminal sets for continuous-time linear systems explained in more detail in (Rohal̆-Ilkiv, 2003)

$$\mathscr{P}(\tilde{\mathbf{W}}, \hat{\mathbf{w}}_1, \hat{\mathbf{w}}_2) = \{\mathbf{x} \in \mathbb{R}^n; -\hat{\mathbf{w}}_1 \le \tilde{\mathbf{W}}\mathbf{x} \le \hat{\mathbf{w}}_2\} \tag{16}$$

Starting from a suitable chosen stabilizing local controller $\mathbf{u} = \mathbf{F}\mathbf{x}$ one effective linear programming based algorithm for calculation of the polytope *definition* matrix $\tilde{\mathbf{W}}$ and *boundary* vectors $\hat{\mathbf{w}}_1, \hat{\mathbf{w}}_2$ in the case of amplitude and rate input constraints (8), (9) can be found in (Rohal̆-Ilkiv, 2004). The algorithm will be further used for the adaptation of the terminal set $\Omega \equiv \mathscr{P}(\tilde{\mathbf{W}}, \hat{\mathbf{w}}_1, \hat{\mathbf{w}}_2)$.

The necessary condition for the successful determination of the terminal set Ω and the terminal state penalty matrix $\mathbf{Q}_h$ requires the suitable choice of the local controller gain $\mathbf{F}$ in such a way that the closed-loop poles must be placed within a sector region prescribed in the left-half complex plane, Figure 1. For proof see the literature cited above. An efficient LMI based technique of LQ/ $\mathcal{H}_2$ optimal sector pole placement, exploiting further in the practical calculations, has been adopted from (Rusko, 2004).

5. PREDICTION EQUATIONS

The equations for prediction of the plant outputs $\hat{\mathbf{y}}_k$ can be easily derived simulating the model (3) from time instant t_k forward in time over the nh

virtual samples situated in the prediction interval $[t_k, t_k + T_h]$, as

$$\hat{\mathbf{y}}_k = \mathbf{G}\mathbf{p}_{h_k} + \mathbf{f}_k \quad (17)$$

where matrix $\mathbf{G}$ and vector $\mathbf{f}_k$ are determined using appropriate Toeplitz and Hankel matrices.

The further task is the prediction of the plant terminal state, $\mathbf{x}(t_k + T_h|t_k)$. Making use of next relation for prediction of the vector of spline coefficients belonging to predicted plant outputs

$$\mathbf{c}_h(k) = \mathbf{M}_h^{-1}[\mathbf{G}\mathbf{p}_{h_k} + \mathbf{f}_k] \quad (18)$$

we can obtain following relation for evaluation of the predicted plant terminal state

$$\mathbf{x}(t_k + T_h|t_k) = \mathbf{D}_1\mathbf{p}_{h_k} + \mathbf{D}_2\mathbf{f}_k \quad (19)$$

where matrices $\mathbf{D}_1$ and $\mathbf{D}_2$ moreover depend on the chosen spline basis functions and their derivatives, for details see (Rohaľ-Ilkiv, 2003).

6. MPC ALGORITHM AND CONSTRAINTS SATISFACTION

Employing obtained predictions (17) and (19) the performance index (14) can arrive at the following quadratic relation

$$J(\mathbf{x}(t_k), \mathbf{p}_{h_k}, T_h) = \frac{1}{2}\mathbf{p}_{h_k}^T\mathbf{H}\mathbf{p}_{h_k} + \mathbf{p}_{h_k}^T\nabla J(0) \quad (20)$$

with $\mathbf{H}$ and $\nabla J(0)$ calculated as

$$\mathbf{H} = 2[\mathbf{G}^T\mathbf{Q}_y\mathbf{G} + \mathbf{Q}_u + \mathbf{D}_1^T\mathbf{Q}_h\mathbf{D}_1]$$
$$\nabla J(0) = 2[\mathbf{G}^T\mathbf{Q}_y\mathbf{f}_k + \mathbf{D}_1^T\mathbf{Q}_h\mathbf{D}_2\mathbf{f}_k]$$

Thus the problem of minimization of the performance index from the point of view of vector $\mathbf{p}_{h_k}$, subject to constraints (8) $\sim$ (10), reduces to a typical quadratic programming problem. In the submitted experiments an efficient QP technique of (Powell, 1985) has been employed.

Because standard QP techniques minimize a quadratic performance index with *general* linear equality-inequality constraints written usually in a matrix form as

$$\mathcal{A}^T\mathbf{p}_{h_k} \geq \mathbf{d}(k) \quad (21)$$

all constraints (8) $\sim$ (10) consequently have to be *re-parameterized* into this form through a proper design of the matrix $\mathcal{A}$ columns and the vector $\mathbf{d}(k)$ elements. *Subsets* of properly designed *elementary* constraints of type

$$\mathbf{a}_j^T\mathbf{p}_{h_k} \geq d_j(k) \qquad j = 1, \dots \quad (22)$$

acting on the optimized vector $\mathbf{p}_{h_k}$, where $\mathbf{a}_j$ and $d_j(k)$ denote the jth column of $\mathcal{A}$ and the jth element of $\mathbf{d}(k)$, respectively, then represent all those constraints we want to have applied during the design procedure of the predictive control algorithm. This re-parameterization of the original continuous-time constraints to constraints acting on the vector of spline coefficients or a vector of spline control points is a key moment in the problem of their intersample satisfaction. Some examples of this re-parameterization for spline function of low orders (2,3,4) are presented in (Rohaľ-Ilkiv, 2003). Constraining properly the coefficients of individual spline polynomial segments of the control signal, at the sampling time instants t_k corresponding with the spline knots, enables us to govern the segment intersample behaviour and to fulfill the constraints (8), (9) between the control sampling time instants.

The relevant continuous-time MPC algorithm then consists of performing the following steps at each time instant t_k.

Spline based stabilizing MPC algorithm with adaptation of terminal set:

(1) for current plant data $\mathbf{y}_k$, $\mathbf{p}_k$ and reference values, using a suitable recursive algorithm (e.g. alternative covariance matrix (Kulhavý and Kraus, 1996)), update models (1), (4) and adapt the calculation of the terminal set $\Omega \equiv \mathscr{P}(\tilde{\mathbf{W}}, \hat{\mathbf{w}}_1, \hat{\mathbf{w}}_2)$
(2) then *update* $\mathbf{H}$, $\nabla J(0)$ and $\mathbf{d}(k)$
(3) and using a QP routine minimize

$$J(\mathbf{x}(t_k), \mathbf{p}_{h_k}, T_h) = \frac{1}{2}\mathbf{p}_{h_k}^T\mathbf{H}\ \mathbf{p}_{h_k} + \mathbf{p}_{h_k}^T\nabla J(0)$$

subject to

$$\mathcal{A}^T\mathbf{p}_{h_k} \geq \mathbf{d}(k)$$

(4) finally apply with the possibly shortest implementation period $T_g, T_g \ll T$, first polynomial segments $\mathbf{p}_h(k+1)$ of $\mathbf{p}_{h_k}$ to the plant control during the time interval $[t_k, t_k + T]$, that means calculate control signals $s_{u_j}(t_k + \tau)$, for a relative time variable $\tau = iT_g,\ i = 1, \dots ng,\ T = ng.T_g$, as the spline polynomial segment of order r_u and implement them on the plant control through zero order hold
(5) repeat the procedure from (1.) for next starting time $t_{k+1} = t_k + T$.

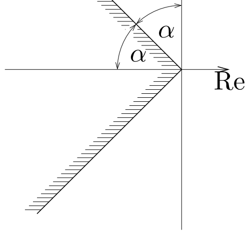

Fig. 1. The admissible sector for closed-loop poles

Naturally, the outlined algorithm is open to other process limitations, which can be included to the procedure of creating the terminal set and can be adapted to the system of linear constraints (21).

7. EXPERIMENTAL VERIFICATION

The efficiency of the algorithm can be judged from real-time experiments performed on a laboratory setup consisting from a 3-phase AC motor, powered by a frequency converter, and coupled with a DC generator representing a load of the system. Two types of experiments were examined - reference value tracking and disturbance rejection. A special starting procedure based on the system proper exciting was designed for the models (3), (4) parameters inital tuning after which the complete MPC algorithm was activated. Through experiments, the following setting of some interesting algorithm design parameters was used:

$T_g = 0.05s$	$T = 0.5s$	(sampling times)
$r_y = 4$	$r_u = 3$	(spline orders)
$T_h = 12T$		(tracking problem)
$T_h = 9T$		(disturbance rejection)

For spline filtering of the output signal - speed of the motor-generator set - the smoothing spline was mounted. In all experiments the penalty matrix of control signals was set to zero, $\mathbf{Q}_u = 0$. The amplitude and gradient constraints of the control signal were set as

$$\left[\, 3.5 \,\right] \leq \mathbf{u}(t) \leq \left[\, 9.0 \,\right] \left[\, -0.5 \,\right] \leq \dot{\mathbf{u}}(t) \leq \left[\, 0.5 \,\right]$$

All signals of the laboratory setup (input, output and load) were scaled in the Volts.

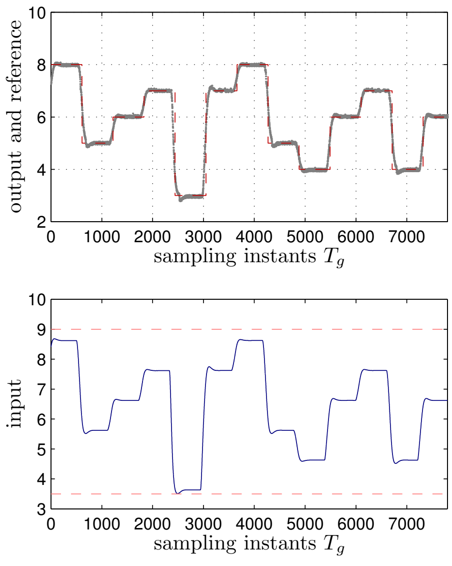

Fig. 2. Input, output and reference signals

After a preliminary phase of the experiments, dealt with identification of the models, following results were obtained.
Figure 2 illustrates the tracking abilities of the tested MPC algorithm. On Figure 3 the time responses of the individual entries ($n = 3$ - the state-space order) of the vectors $\hat{\mathbf{w}}_2, \hat{\mathbf{w}}_1$ belonging to the adapted terminal set $\Omega = \mathscr{P}(\tilde{\mathbf{W}}, \hat{\mathbf{w}}_2, \hat{\mathbf{w}}_1)$ are depicted - $\hat{\mathbf{w}}_2$ are the upper curves, $\hat{\mathbf{w}}_1$ are the bottom curves. The disturbance (the DC generator load) rejection experiment is depicted on Figure 4 and the adaptation of the terminal set is on Figure 5. The load of the motor-generator set was realized with varying the actuating voltage of the generator.
The computation was performed on a PC with a Pentium IV procesor with speed 2.4 GHz using the software LMI Control Toolbox in the Matlab.

The obtained results supports the applicability of the proposed MPC scheme.

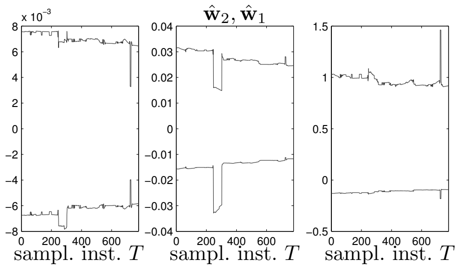

Fig. 3. Adaptation of the terminal set

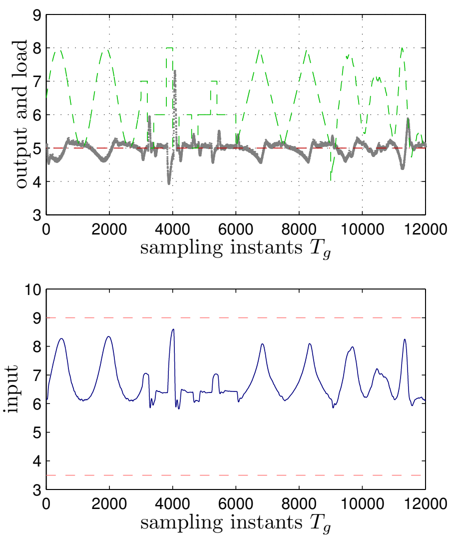

Fig. 4. Input, output and load signals

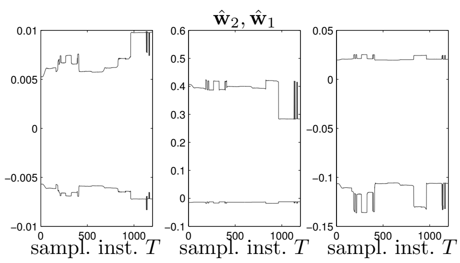

Fig. 5. Adaptation of the terminal set

8. CONCLUSION

The domain of attraction of the spline based algorithm depends on the size of the calculated terminal set Ω and the selected prediction horizon T_h. Increasing both of these may yield a bigger domain of attraction. Usually in the most known MPC algorithms an increase in prediction horizon automatically leads to a greater number of decision variables - degrees of freedom - and consequently to a greater computational effort. The submitted spline-based MPC algorithm do not suffer from this - in such extent - since the extension of the horizon here does not lead immediately to the larger number of decision variables, i.e. elements of the vector $\mathbf{p}_{h_k}$. Using a larger distance between knots of the projected spline control signal it is possible to extent the prediction horizon T_h satisfying the same length of the vector and hence the same number of decision variables and the same computational load. Accordingly, by increasing the prediction horizon, the domain of attraction of the MPC algorithm can be enlarged without expense of a greater computational burden.

Of course many problems related to the proposed approximative continuous-time MPC scheme are still open and will be further elaborated from the practical and theoretical point of view.

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