A HYBRID AUTOMATA APPROACH FOR ROBOTIC PERCEPTIVE CONTROL AND PLANNING

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Abstract: This paper presents a hybrid automata approach for modeling and analyzing the robotic motion planning and control. Robotic systems acquire data from perceptive sensors and respond to the perceptions through decision and control process. The hybrid perceptive framework is built based on continuous and discrete perceptive references. The robot can plan and modifying the original path through switching the continuous controller at discrete level. The hybrid system is stable in switching. The evolution of the hybrid references can be guaranteed during the unexpected events that can block the continuous or discrete references. *Copyright*[©] 2005 IFAC.

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1. INTRODUCTION

As autonomous systems, robots process perception, the environmental information obtained from onboard sensors, and respond to the perception by changing the original path planning and control schemes. A perceptive frame can be built for robotic systems modeling and analysis. The perceptive framework introduces a concept of a perceptive reference s, a parameter that is directly relevant to measured sensory outputs and the task. The s can be, for example, the distance the robot system traveled. Thus, instead of time, the control input is parameterized by the new reference, which is a function of the real time measurement. The planner is triggered by the new reference and generates the desired values of the system, according to the on-line computed reference s. The perceptive frame has been applied to the path planning and control for a single manipulator and multiple manipulators coordination with a continuous perceptive reference (Xi et al., 1996) A perceptive scheme was developed for integration of task scheduling, action planning and control of robotic manufacturing systems in (Song et al., 2000). The perceptive approach guarantees the stability of the robot system in the presence of unexpected events. However, unexpected events become serious problem, as they can block the evolution of the reference, for example, an obstacle. When the reference is blocked by an unexpected obstacle, the robot will stop until the blocked reference is released and then resume evolution. It is crucial for the reference to keep evolving. A hybrid perceptive framework is considered to solve the break in evolution of a single reference.

Hybrid systems have been attracting attention from control engineers recently and extensively investigated in (Lygeros *et al.*, 2003), (Braniky, 1995), (Pettersson and Lennartson, 1993) and a variety of hybrid system models and frameworks have been proposed (Grossman *et al.*, 1993) (Antsaklis *et al.*, 1993) (Braniky, 1995). Linguistic

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Fig. 1. Perceptive Framework for Robotic Systems

method was proposed for motion planing and control of mobile robots by Brockett. In his research, a motion description language was developed for kinetic state machines, which are the continuous analog of finite automata (Brockett, 1993). (Manikonda *et al.*, 1998) defined atoms in the alphabet that describes motion behaviors. The functions of the language are improved to deal with multiple interruptions. In Egerstedt's research, a complexity problem of a multiple obstacle avoidance is analyzed by using motion description language of Brockett's hybrid model (Egerstedt, 2001). However, both the continuous part and the discrete part of the system use time reference.

In this paper, using a hybrid framework, the hybrid perceptive references enable both the continuous and discrete part of the system to deal with unexpected events. The discrete part of the system governs the system based on perceptive information such that the system can keep the reference evolving through modifying original plans.

2. HYBRID PERCEPTIVE FRAMEWORK

Based on (Song *et al.*, 2000), a perceptive framework represented as a hybrid system model, as shown in Fig.1, is developed for robot system to process both continuous and discrete information. The perceptive framework is composed of two major parts, a hierarchical command execution and a hybrid perceptive reference. All the units of the model are modeled by hybrid automata. in this framework, the discrete level references trigger the tasks and actions to execute. The information, carrying commands and perceptive references, is expressed using hybrid languages.

2.1 Hybrid Perceptive Automata and languages

As described in (Sun *et al.*, 2003), the task set T, action set A, corresponding reference sets R_{Task} and R_{Action} , and continuous perceptive reference s, are defined in perceptive frame.

The formal languages L_A, L_T, L_{RA} and L_{RT} can be built by the alphabets $\Sigma_A, \Sigma_T, \Sigma_{RA}$, and Σ_{RT} that are generated by sets $A, T, R_{Action}, R_{Task}$, respectively.

A Hybrid Language (HL) is generated from an original alphabet Σ , consisting of discrete atoms, and numbers $N \in \mathbb{R}^n$. The alphabet of the HL is an extended alphabet of a formal language. An HL is defined as the set of all the strings over the extended alphabet Σ^H . a string is concatenation of the elements lying in the product set of the original alphabet and the Euclidean spaces. An HL derived from the extended alphabet Σ^H is denoted L^H . A word in a hybrid language L_T^H can be " $T_1(x, y)$ " meaning "going to point(x,y)". The alphabet gives the qualitative description, the numeric part is the quantization of the linguistic expressions.

A perceptive automaton can be defined as a tuple

$$M_e = (Q, \Sigma, \Sigma_R, \delta, Q_0, \eta, O).$$
(1)

where Q is the set of the discrete states of the automaton and Σ is the discrete control input set.

The following properties describe the state transitions and the output action triggered by the control command input and the reference inputs.

$$q_j = \delta(q_i, \sigma_j), O_k = \eta(q_i, \sigma_k) \tag{2}$$

where $q_i, q_j \in Q, O_k \in O, \sigma_j \in \Sigma, \sigma_k \in \Sigma_R$.

Corresponding different inputs the automaton can have different responses including state switching and command issuing. A hybrid automaton is capable of bridging the two kinds of variables. Based on (Lygeros *et al.*, 2003) it can be considered to be a tuple

$$M^{H} = (Q, X, \Sigma, \delta, Q_0, \eta, 0), \qquad (3)$$

where X is the set of continuous variables. The automaton can be described as

$$q_i = \delta(q_i, \sigma_i, X), \sigma_i \in \Sigma \tag{4}$$

$$O_k = \eta(q_i, \sigma_i, X) \tag{5}$$

A hierarchical automaton consists of several nodes, some of which are other automata. The first automaton is an upper automaton, the other automata denoting the nodes of the upper automaton are called embedded automata. This architecture can be described as a tuple, $M_h = (Q_U, Q_{EM}, \Sigma_U, \Sigma_{EM}, \delta_U, \delta_{EM}, Q_{U0}, Q_{EM0}, O)$,

 Σ_U is the input set for the upper automaton, Σ_{EM} is the input set of embedded automaton, Q_U is a set of the states(the nodes of the upper automaton), Q_{EM} is a set of the states of the automata embedded into nodes of Q_U , Q_{U0} is the set of the starting states of Q_U , Q_{EM0} is the set of the starting states of Q_{EM} . δ_U and δ_{EM} describe the transition functions of Q_U and Q_{EM} , respectively.

Perceptive, hybrid and hierarchical automata refer to three basic prototypes of automata in the framework.

2.2 Hierarchical Task Execution Architecture

As shown in Fig.1, The task execution includes three parts: task scheduler, action and motion planner. which can be modeled using automata.

The task scheduler sequentially generates tasks according to the commands and discrete task level references. It is a perceptive automaton hybrid variables.

 $M_{TaskSch} = (Q, \Sigma_T^H, \Sigma_{RT}^H, X, \delta, Q_0, \eta, O_{TaskSch}),$ where the unexpected event input and task level reference input are in language L_{RT}^H , and L_T^H , respectively. The inputs from the Task Level Reference automaton trigger the task output in hybrid language L_T^H . Logically, it chooses the higher priority task from the inputs, e.g. obstacle avoidance.

The function of the planner is to generate a sequence of actions according to the task input from the task level. According to the discussion above, the action planner is modeled as a hybrid perceptive automaton which can be described as a tuple, $M_{ActPlan} = (Q, \Sigma_T^H, \Sigma_{RA}^H, X, \delta, Q_0, \eta, O_{ActPlan}).$ It has perceptive reference inputs in hybrid language L_{RA}^H and task inputs in hybrid language L_T^H denoted as:

$$I_{ActPlan1} = L_T^H \tag{6}$$

$$I_{ActPlan2} = L_{RA}^H \tag{7}$$

The output $O_{ActPlan}$ is in the language L_A^H .

The continuous variables in set X carried out by hybrid language inputs give the automaton continuous evolutions and can also be used to generate the outputs in the hybrid language.

The transition function and the output functions of the Action Planner are as follows

$$q_{ActPlanj} = \delta(q_{ActPlani}, \sigma_j), \sigma_j \in \Sigma_T^H \quad (8)$$

$$O_{ActPlank} = \eta(q_{ActPlani}, \sigma_k), \sigma_k \in \Sigma_{RA}^H \quad (9)$$

The task inputs cause the state transition of the automaton. The perceptive reference input triggers the output, The output can be a vector comprising of several commands.

The motion planner in Fig. 1 can generate a continuous trajectory for the desired motion, based on the actions from the action planner. The motion planner is a hybrid automaton triggered by the perceptive reference. We describe it as a tuple: $M_{MotPlan} = (Q, \Sigma_A^H, s, X, \delta, Q_0, \eta, O_{MotPlan})$ The output is parameterized by reference s. Another input of the automaton is in hybrid language L_A^H from the alphabet Σ_A^H . The discrete part of the input σ in L_A^H make the automaton switch to the appropriate node. Each node of the automaton is a dynamic system (or a controller) which can issue the planned continuous trajectory.

Therefore, it can be formulated as:

$$q_{MotPlanj} = \delta(q_{MotPlani}, \sigma_j), \sigma_j \in \Sigma_A \quad (10)$$

$$O_{MotPlank} = \eta(q_{MotPlani}, s) \quad (11)$$

It can be seen that the state transition and output are triggered separately by two different inputs. For motion actions, the output to the robot controller can be a vector, $O_{MotPlank} = [X(s), Y(s), \dot{X}(s), \dot{Y}(s)]^T$, for a complete motion trajectory.

2.3 Reference Generation

The system has configuration sensors and environmental sensors. The environmental sensory measurements are used by the higher levels for discrete perceptive reference generation, and the motion reference can be issued based on the configuration sensor measurements. Reference generation can be described as automata. Task and action level references are mechanism to trigger the next task or action based on the environmental sensor measurements and unexpected events.

The detailed description of reference generation can be found in (Sun *et al.*, 2003).

3. HYBRID PERCEPTIVE CONTROL FOR MANIPULATION AND TELE-OPERATION

A typical task is to pick up an object and transport it to a desired destination with a mobile manipulator. In the route of transporting the object, an unexpected obstacle occurs. In order to complete the task, the mobile manipulator will execute an obstacle avoidance task. The mobile manipulator will then resume its original task and reach the destination.

To build the hybrid languages from task set and action set, some continuous variables can be involved to the atoms. In the extended task set, the T1(m1,n1)(m2,n2) denotes "transporting an object from point(m1,n1) to point(m2,n2)." $T2(m,n,l_O)$ denotes "to avoid an obstacle at(m, n) at the distance l_O ." In the extended action set, A1(m,n,l) denotes "following a straight line with directional cosine (m,n) and length l." A2(m,n,r) denotes "go through an arc centered at (m, n) with length r." A3 and A4 denote "close the gripper" and "open the gripper", respectively.

In manipulation, the units behaves as follows: Task Scheduler: The output of the task scheduler is T1(m1, n1)(m2, n2) until the obstacle has been detected. Action Planner: Based on T1(m1, n1)(m2, n2), the transition and the outputs are $q_{AP1} = (q_0, T1)$ and $o_{AP1} =$ $\eta(q_{AP1}, reset) = A1$. When the planner has the reference input A1 meaning that the A1 action is finished, the output action triggered by A1 is a vector $o_{AP1} = \eta(q_{AP1}, A1) = [A2, A3]^T$. The output triggered by action reference A11 is another action $o_{AP1} = \eta(q_{AP2}, A11) = A4$. The directional cosines and the length in A1 and A11 can be found with the parameters in T1. Motion Planner: The inputs A1(m1, n1, l1), $A11(m11, n11, l11) \in \Sigma_A^H$ cause the states transition of the motion planner. $q_{MotPlan1} = \delta(q_0, A1),$ $q_{MotPlan11} = \delta(q_0, A11)$. The states denote different vector fields, i.e., trajectories, namely, straight line path and arc path.

4. ANALYSIS ON SWITCHING SYSTEM

During switching between the tasks, which can be described in subspaces, stability is a very crucial issue. The values of the vector in each subspace represent the parameter disturbance of the low level controllers.

4.1 Mapping from high level task to low level controllers and regions

Task space can be defined as a N-dimensional linear vector space.

$$T^{s} = [T_{1}T_{2}...T_{n}]^{T}$$
(12)

$$T_i = diag[0...I_i...0] \tag{13}$$

$$T^{s} = [0...x_{i1}, x_{i2}...x_{ip}...0]^{T}$$

Where I_i is a diagonal matrix with order p. Therefore, for each task, we have a multidimensional vector space. For several independent tasks, the combined task space can be generated and, each task is defined in a subspace. For example, the T_i is a task in p dimensional space. The dimension of the subspace depends on the specific task. Similarly, the action space can be defined as:

$$A^{s} = [A_{1}A_{2}...A_{m}]^{T}$$
(14)

$$A_{j} = diag[0...D_{j}...0]A^{s}$$
(15)
$$A^{s} = [0...x_{j1}, x_{j2}...x_{jq}...0]^{T}$$

Where D_j is a diagonal matrix with order q. The transform would result in mapping the task from a task subspace into an action subspace. The mapping $T^s - > A^s$, therefore, can be described as a linear mapping.

According to the control model of action planning,

$$O_k^A = L(q_i, \sigma_i, \sigma_k) = L'(q_i, \sigma_k, T^s)$$
(16)

Where D_i is a diagonal matrix. The switch between the task level inputs is equivelent to the switch of the Linear mappings. Given a different task level input, there exist a linear mapping to generate the the vectors in Action Space.

$$O^{M} = L_{M}(\eta(q_{i}^{M}, \sigma_{i}, s)) = L''(q_{i}, s, A^{s}) \quad (17)$$

Where the D_j is a diagonal matrix. L' and L'' are linear mappings.

Fig. 1 describes the model with hybrid perceptive references and states, A hybrid perceptive trajectory is a finite or infinite sequence of reference interval $e = \{S_{TRi}S_{ARj}S_K\}$, where $S_K \in [S_k, s'_k], S_{TRi} \in \Sigma_{RT}^H, S_{ARj} \in \Sigma_{RA}^H$. Furthermore, An execution of a hybrid perceptive automaton describes a collection $\chi = \{e, q, x\}$, where e is a hybrid perceptive reference trajectory. q is a map from $S_{TRi}S_{ARj}$ to q, x maps the continuous reference s to the continuous state space.

4.2 switching conditions for low level controllers

For the system dx/ds = f(x), we say that V(s) is a candidate Lyapunov function if V(s) is a continuous positive definite function (about the origin) with continuous partial derivatives. Note this assumes V(0) = 0.

Considering switched hybrid systems, the systems operate in a hybrid metric space. The stability property of such systems can be described in a hybrid state space of the perceptive frame. Therefore, multiple Lyapunov function can be used to discuss the stability issue.

Given a dynamical system in the perceptive frame, if there exist Lyapunov functions $V_1(s^h), V_2(s^h)...$ with hybrid perceptive references, corresponding to the hybrid perceptive reference trajectory and different segments of executions, for a given strictly increasing sequence of $S_{TRi}S_{ARj}$ (discrete reference values), denoted by $I = S_{TRi}S_{ARj}$ in perceptive reference, we say that V is a Lyapunovlike function for function f and execution $\chi = \{e, q, x\}$, if 1. Given $s \in (s_i, s_{i+1})$ where $dV(x(s^h))/ds \leq 0$ 2. V is monotonically non-increasing on ordered set I.

Theorem 1 (Stability): Suppose we have candidate Lyapunov functions $V_{i,i} = 1...N$, and vector field $dx/ds = f_i(x)$ with $f_i(0) = 0$, Let E be the set of all switching sequences associated with the system. If for each $s^h \in E$ we have for all i, V_i is Lyapunov-like for f_i . Then the system is stable in the sense of Lyapunov.

Theorem 2(Exponential Stability): Suppose we have candidate Lyapunov functions $V_{i,i} =$ 1...N, and vector field $dx/ds = f_i(x)$ with $f_i(0) =$ 0, Let E be the set of all switching sequences associated with the system. In the perceptive frame, if for each $s^h \in E$ we have for all i, V_i is Lyapunovlike for f_i . Then the system is exponentially stable in the sense of Lyapunov. If There exist constants $\alpha_g, \beta_g, \gamma_g, g = 1, ..., l$, such that:

$$--\forall x \in \Omega, \alpha_g(||x||) \le V_g(x) \le \beta_g(||x||) (18)$$

$$--\forall x \in \Omega,$$

$$--\forall x \in E_{gr}, V_r(x) \le V_g(x).$$
(19)

for switching from q to r.

4.3 continuity of input-output mapping

The offset on the continuous variables of the task level linguistic inputs. For the tasks, which have the same discrete task but different continuous variables, i.e., two task controls in the same task subspace, if the continuous parts approximate sufficiently, then the states of the system are sufficiently close to each other.

It can be formally described by the followings:

For a given task subspace T_i , the mapping Lis referred to as a continuous mapping, if there always exists an ϵ , for given δ such that $\|\Delta A\| < \epsilon$, when $\|\Delta T\| < \delta$. The intuitive meaning is that both the task and the action will evolve continuously. The norm $\|\Delta A\|$ is referred to as the sum of all the action offset. I.e., $\|\Delta A\| = \Sigma \|\Delta A_i\|$. Thus, the input and output of the task controller in the linear space T^s and A^s are continuously dependent.

4.4 stability conditions for switched systems

The stability conditions include two parts. First, the switching between the task subspace with different dimension is stable under certain conditions of Lyapunov functions. Second, the continuous variable of the task input do not spoil the stability, if for a given value, the system is stable. Assumption: The input and the output of the task controllers in linear space are continuous dependent.

Claim 3: If the vector fields corresponding to the switching task T1 and T2 in Ω_1 and Ω_2 are exponentially stable. Then given any continuous part T1 and T2, the switching tasks will be stable.

5. MODELING OF UNEXPECTED EVENT PROCESSING

The ability to deal with unexpected events is crucial for a perceptive control system. Q represents the set of the vector fields, $(q_1, ..., q_n)$, for a mdimensional space, at least it has m m-dimensional orthogonal vector fileds. $f_1...f_m$. Therefore, for any 2 points x1 and x2, the system can start from x1, x2 within finite time of switches over the vector fields. The continuous reference s should satisfies: $L_f S(q, x) > 0$, it guarantees that the continuous reference is increasing over time t.

5.1 Unexpected Event(UE) Processing

Unexpected events can be described as follows: For a given $e = \{S_{TRi}S_{ARj}S_K\}$, where $S_K \in [S_k, s'_k], S_{TRi} \in \Sigma^H_{RT}, S_{ARj} \in \Sigma^H_{RA}$. and an execution of a hybrid perceptive automaton $\chi = \{e, q, x\}$, A unexpected event happends, when the following two condition hold:

$$ds/dt \mid_{s_{u}^{h}} = 0, s_{u}^{h} \in (s_{0}^{h}, s_{f}^{h}),$$
(20)

$$s_u^h \neq S_{TRi}, S_{ARj}.$$
 (21)

Furthermore, an unexpected event represents a convex region U_A that is an open set of states, in which the above conditions hold, the boundary is a scalar function $\omega(x) = 0$, and $\omega(x) > 0$ for $x \in U_A$, when UE happens, using "Lie Derivative", we have $L_f \omega(x) > 0, \omega(x) = 0$.

It is a local blocking event of Task T if we can find a subset of the trajectory $\chi_1 = \{s_t, q, x\}$, where $s_t = (s_u^h, s_f^h) \notin U_A$.

For hybrid automata, the unexpected events can be described as a disturbance for a system. The system is desired to be able to return to the designed trajectory after the disturbance. Or, the unexpected events can be treated as a task executed before finishing the current task. The new task is to switch out of the blocking state then return to and resume the previous task. For the given model, the unexpected event will affect the states, causes a discrete state transition. A new vector field will be applied on the continuous states of the system, which is an orthogonal vector field to the old one, $\langle f_i(x) \cdot f_{i+1}(x) \rangle = 0$, then it can satisfy the following conditions: $L_{f_{i+1}}\omega(x) < 0$, $L_{f_{i+1}}S(q,x) > 0$, the former makes the system leave domain U_A from the boundary $\omega(x) = 0$, the latter guarantees the evolution of the continuous reference.

Theorem 4 shows the existence of the solution at unexpected events in the perceptive frame.

Theorem 4: if the unexpected event e_u is a local blocking event, there exists another finite automaton and with the same alphabet, corresponding to the new automaton, the automaton can go back to the original task trajectory $\chi = \{e, q, x\}$.

The detailed proofs of Theorem 1-4 can be found in (Sun, 2004).

5.2 UE Processing in Mobile Manipulation

After an UE that blocks the evolution of continuous reference is detected, the task reference generates the UE with the obstacle info $T2(m_o, n_o, l_o)$, the output of the task scheduler is $T2(m_o, n_o, l_o)$. It causes a discrete switch over discrete variables, an obstacle avoidance on the task execution to switch and resume the evolution of the continuous reference, which leads the robot to the previous task and reach the destination. The perceptive reference is never blocked.

The transition function and the output function are $q_{AP2} = \delta(q_{AP1}, T2(m_o, n_o))$ and $o_{AP2} = \eta(q_{AP2}, reset) = A2(m_{a2}, n_{a2}, r_{a2})$, respectively.

The motion planner receives the action A2(.) from the action planner to generate the trajectory. This automaton has a state transition on the upper automaton due to the unexpected event and returns to the previous state after processing the unexpected event. The triggered procedure is the same as in the task execution process.

In teleoperation, The time delay of the communication between the operator and controller will significantly affect the control performance. it is crucial to synchronize the command from the operator to the arm controller. If the controller can not receive the next position or velocity command before it finishes executing the current one. It has to stop or just keep going on the current velocity, the former reduces the smoothness of execution while the later causes control errors. In the hybrid model, the delayed command is thought of as an unexpected event on action level, that can prevent the discrete reference from evolving.

The routing for the UE is to switch to another action which decelerates the motion to make the trajectory smooth until the next control command is received then switch back to the previous state and the path plan is modified. As the result, the hybrid perceptive reference can not be blocked.

6. CONCLUSION

This paper presents a hybrid model for the perceptive robotic systems. The model integrates the continuous perceptive reference and discrete references for planning and control using hybrid automata. Compare to continuous reference, the hybrid perceptive reference keeps evolving despite the occurrence of an unexpected event, which could block the continuous perceptive reference or the discrete perceptive reference. It can be shown that the system is stable during the switches.

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