# SECOND ORDER SLIDING MODE ADAPTIVE NEUROCONTROL FOR ROBOT ARMS WITH FINITE TIME CONVERGENCE

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Abstract: In this paper we present a low dimensional adaptive neural network controller for robot manipulators with fast convergence of tracking error. Its novelty lies in the low dimensional network, smooth control input and very fast convergence that reduce the computational cost that face the problem of over parameterization. The control strategy is based on a second order sliding surface which drives the controller and the online computation of weights with a chattering-free control output. Furthermore, a time base generator induces well-posed finite time convergence of tracking errors for any initial condition. We validate our approach including experimental results obtained in a planar 2 dgf manipulator. *Copyright @ 2005 IFAC*

Keywords: Neural Network Control, Robot Manipulators, Second Order Sliding Mode

## 1. INTRODUCTION

Approaches based on neuro-adaptive or neuro-sliding mode control (Ertugrul and Kaynak, 2000; Lewis and Abdallaah, 1994; Ge and Harris, 1994; Karakasoglu and Sundareshan, 1995) approximate the unknown dynamics of a plant using an ANN, however, to achieve an exact approximation it is required a large number of nodes, even for simple applications (Cotter, 1990). Relevant results constraint the network to a bounded number of nodes that means reconstruction errors which don't guarantee the convergence of tracking error (Yu, 2003), (M. Yamakita, 1999), (Cotter, 1990), (F.L.Lewis, 1998), (Ge and Harris, 1994), (F.C Sun, 1999), (G. Kulawski, 2002), (E. N. Sanchez, 2003). Some authors introduced an additional high frequency input in a first order sliding mode to assure the convergence however this design is usually impossible to implement (O. Barambones, 2002), (Ertugrul and Kaynak, 2000), (Karakasoglu and Sundareshan, 1995), (C.H. Lin, 2001).

In this paper we present a controller where the regressor of the system is approximated by a low dimensional single layer neural network, and a second order sliding mode compensates the reconstruction error. Exponential convergence arises and if a time varying gain is introduced, finite time convergence of the tracking errors is guaranteed. The close loop system renders a TBG sliding mode for all time whose solution converges in finite time; hence a perfect tracking is obtained. Furthermore, the second order sliding mode drives synergisti-

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cally the neural network dynamics. Experimental results on a robot manipulator verify the closed loop stability properties.

## 2. REGRESSOR IN AN ADAPTIVE CONTROLLER

The dynamics of a rigid serial $n$-link robot manipulator is described by

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \qquad (1)$$

with $(q,\dot{q},\ddot{q}) \in \Re^{3n}$, generalized joint coordinates, $H(q) \in \Re^{nxn}$, symmetric positive definite inertial matrix, $C(q,\dot{q}) \in \Re^{nxn}$, Coriolis and centripetal forces, $g(q) \in \Re^{n}$, gravitational torques, and $\tau \in \Re^{n}$ the torque input.

Since (1) can be parameterized linearly in terms of a nominal reference $(\dot{q}_r, \ddot{q}_r) \in \Re^{2n}$ (Lewis and Abdallaah, 1994), consider

$$H(q)\ddot{q}_r + C(q,\dot{q})\dot{q}_r + G(q) = Y_r\Theta_r \qquad (2)$$

where the regressor $Y_r = Y_r(q,\dot{q},\dot{q}_r,\ddot{q}_r) \in \Re^{nxp}$ is composed of know nonlinear functions, and $\Theta \in \Re^{p}$ assumed to represent unknown but constant parameters with $(\dot{q}_r, \ddot{q}_r)$ to be defined. Subtracting equation (2) into (1), the open loop error equation is given by

$$H(q)\dot{S}_r + C(q,\dot{q})S_r = \tau - Y_r\Theta, \qquad (3)$$

where $S_r = \dot{q} - \dot{q}_r$, the extended error, carries out a change of coordinates through $(\dot{q}_r, \ddot{q}_r)$. If the regressor $Y_r$ is known, it suffices to design an adaptive control

$$\tau = -K_d S_r + Y_r\Theta \qquad (4)$$

$$\dot{\hat{\Theta}} = -\Gamma Y_r^T S_r \qquad (5)$$

where $K_d \in \Re_+^{nxn}$ and $\Gamma \in \Re_+^{nxn}$, to produce an overparametrized asymptotically stable closed-loop system. If the regressor is unknown, then (4)-(5) cannot be implemented and we have to solve the problem of finding a way to get an approximated regressor. Then the problem statement is *to design a continuous control $\tau$ based on low dimensional neural control such that*

$$\Delta q(t) \doteq q(t) - q_d(t) = 0 \qquad \forall\, t \geq t_g > 0 \quad (6)$$

*for $0 < t_g < \infty$, with measurable state $(q,\dot{q})^T \in R^{2n}$ assuming that $Y_r$ is not available, for any know desired trajectory $q_d \in C^2$.*

The challenge lies in how to obtain fast convergence of tracking errors $\Delta q(t)$ when the regressor $Y_r$ is unknown, under the constraint of a smooth controller $\tau$.

## 3. ERROR: MANIFOLDS AND DYNAMICS

Let us now design a convenient open loop error dynamic system. The nominal reference $\dot{q}_r$ is

$$\dot{q}_r = \dot{q}_d - \alpha(t)\Delta q + S_d - K_i\sigma \qquad (7)$$
$$\dot{\sigma} = sign(S_q)$$

where

$$S_q = S - S_d \qquad (8)$$

$$S = \Delta\dot{q} + \alpha(t)\Delta q, \quad S_d = S(t_0)exp^{-kt}. \qquad (9)$$

with $K_i = K_i^T \in \Re_+^{nxn} > 0$, $\alpha(t)$ time-varying feedback gain, $k > 0$; the *sign(x)* is the discontinuous *signum*$(x)$ function of vector $x \in \Re^n$; $S_d \in C^1$ converges monotonously to zero with initial conditions $S_d(t_0) = S(t_0)$ at time $t = t_0$, then $S_d(t_0) = 0$.

The derivative of (7) becomes

$$\ddot{q}_r = \ddot{q}_d + \dot{S}_d - \dot{\alpha}(t)\Delta q - \alpha(t)\Delta\dot{q} - K_i sign(S_q)$$
$$= \ddot{q}_{cont} + K_i Z \qquad (10)$$

where

$$\ddot{q}_{cont} = \ddot{q}_d + \dot{S}_d - \dot{\alpha}(t)\Delta q - \alpha(t)\Delta\dot{q} - K_i \tanh(\lambda S_q)$$
$$Z = \tanh(\lambda S_q) - sign(S_q), \qquad (11)$$

for tanh(.) the continuous hyperbolic tangent and $\lambda = \lambda^T \in \Re^{nxn} > 0$. Notice that $\ddot{q}_{cont}$ is continuous which will be very useful in the next section. The equation (11) has the following properties: $Z \geq -1$, $Z \leq 1$, $Z_{S_q \to 0^-} = -1$, $Z_{S_q \to 0^+} = +1$ and $Z_{S_q \to \pm\infty} = 0$.

Now, using equation (7) and (10), the parametrization of (1) becomes

$$H(q)\ddot{q}_r + C(q,\dot{q})\dot{q}_r + G(q) = Y_{cont}\Theta - \tau_d \quad (12)$$

where the regressor $Y_{cont} = Y_r(q,\dot{q},\dot{q}_r,\ddot{q}_{cont})$ is continuous due to $(\dot{q}_r, \ddot{q}_{cont}) \in C^1$, and $\tau_d = H(q)K_i Z$ stands for a bounded endogenous high frequency signal. Adding and subtracting (12) into (1) yields the following open-loop error dynamics

$$H(q)\dot{S}_r = -C(q,\dot{q})S_r + \tau - Y_{cont}\Theta - \tau_d \quad (13)$$

where $\tau_d$ is a bounded endogenous high frequency term and $S_r$ has the form

$$S_r = \underbrace{\Delta\dot{q} + \alpha(t)\Delta q}_{S} - S_d + K_i \int sign(S_q)$$
$$= S_q + K_i \int sign(S_q). \qquad (14)$$

which is obtained, substituting (7) in the extended error $S_r$.

## 4. NEURAL NETWORK ESTIMATOR

Based on the Stone-Weierstrass theorem, any smooth function $f(x) \in C^m(\mathbf{S})$, where $\mathbf{S}$ is a compact set simply connected set of $R^n$, can be approximated using with a low dimensional neural network

$$f(x) = \phi(\mathbf{W}_2^T X) + \epsilon(x) \tag{15}$$

where $\phi$ is a linear function, $\mathbf{W}_2^T$ is a subset of $\mathbf{W}_1^T$ the optimal bounded weight vector, $X$ belongs to a compact set $\mathbf{K} \subset \Re^{2n}$, that is $\mathbf{S} := \{x : \|x\| \leq \mathbf{S}\}$ such that $f(x) = \phi(\mathbf{W}_1^T X)$ (Cotter, 1990), and $\epsilon(x)$, a bounded functional reconstruction error, $\|\epsilon(x)\| \leq \epsilon_N$ with $\epsilon_N > 0$. Consider now that the unknown nonlinear function $f(x)$ is parameterized by static Adaline neural network with output $\hat{f}(x, \mathbf{W}_{n_2}) = \mathbf{W}_{n_2}^T X$ where $\mathbf{W}_{n_2} \in \Re^{n_2}$ is the matrix of adjustable weights and $n_2$ denotes a low number of weights, and $n_2 \ll n_1$. This type of neural network provides easily an approximation of $\hat{f}$ without concerning about its accuracy. Besides this, the size $n_2$ of the network can be obtained roughly by checking carefully the dynamics of a general n-link rigid arm.

*Without lack of generality, in the rest of the paper we refer $\boldsymbol{W}_{n_2}$ as $\boldsymbol{W}$, omitting its subindex.* Now, let

$$\hat{f}(x) = Y_{cont}\Theta \equiv \hat{\mathbf{W}}^T X + \epsilon(x) \tag{16}$$

where $Y_{cont} \in R^{nxp}$, $\Theta \in R^{px1}$ stand for the function to be approximated

$$X = [q, \dot{q}, \dot{q}_r, \ddot{q}_{cont}]. \tag{17}$$

In this way, using low dimensional neural network, the estimation of $f(x)$ is $\hat{f}(x)$, where $\hat{f}(x)$ stands for the online estimation of $Y_{cont}\hat{\Theta}$. Notice that for a 2 degrees of freedom robot, it is required a total of 2 neurons, with 4 online adaptive weights each.

**Remark** *In this paper, our approach is similar to (F.L.Lewis, 1998), (Ge and Harris, 1994) a neural network approximation of $f(x) \doteq Y_{cont}\Theta$, while $\tau_d$ will be treated as an endogenous discontinuous disturbance function. The difference is the use of a low dimensional neural network, based on linear associator, where we are able to prove convergence, in contrast to those references that guarantee only bounded tracking.*

## 5. CONTROL DESIGN

### 5.1 Exponential Convergence

Substituting (16) in (13) we obtain

$$H(q)\dot{S}_r = -C(q, \dot{q})S_r + \tau - \mathbf{W}^T - \epsilon(x) - \tau_d \tag{18}$$

Now consider the following adaptive control law

$$\tau = -K_d S_r + \hat{\mathbf{W}}^T X \tag{19}$$

$$\dot{\hat{\mathbf{W}}} = -\Gamma \mathbf{X}^T S_r \tag{20}$$

where $K_d = K_d^T \in R_+^{nxn}$, $\hat{\mathbf{W}} \in R^{nxp}$ the adaptive neural network weights, $\mathbf{X} \in R^p$ input to the network and $\Gamma = \Gamma^T \in R_+^{pxp}$.

Now, substituting (19)-(20) into (18) gives rise to the following closed-loop error dynamics

$$H(q)\dot{S}_r = -C(q, \dot{q})S_r - K_D S_r + \Delta W^T X + \epsilon(x) - \tau_d \tag{21}$$

$$\Delta \mathbf{W} = \Gamma \mathbf{X}^T S_r \tag{22}$$

for $\Delta \mathbf{W} = \mathbf{W} - \hat{\mathbf{W}}$. Finally, we have the following result.

**Theorem 1** *Exponential Stability.* Consider the closed-loop error dynamics (21)-(22). Then, exponential convergence of tracking errors arises if $K_i > \varepsilon_4$ and $\alpha(t) = \alpha$ a constant. Furthermore, a sliding mode is enforced for all time with a low dimensional neural network and smooth control effort.

**Proof:** See Appendix 1.

### 5.2 Finite Time Convergence, Faster that Exponential Convergence

Terminal attractor have been proposed using fractional power (X. Yu and Man, 1998), however those are ill-defined for at least some initial period of time. In (Parra-Vega, 2001) a well-posed time base generator (TBG) is proposed to render finite time convergence. The following result can be stated now.

**Theorem 2** *Finite Time Stability.* Consider the closed-loop error dynamics (21). Then, arbitrarily finite time convergence of tracking errors if $K_i > \varepsilon_4$ and if feedback gain $\alpha(t)$ is tuned as follows

$$\alpha(t) = \alpha_0 \frac{\dot{\xi}}{1 - \xi + \delta}, \tag{23}$$

where $\alpha_0 = 1 + \epsilon$ with $0 < \epsilon \ll 1$ and $0 < \delta \ll 1$. The time base generator $\xi = \xi(t) \in C^2$ must be provided by the user so as to $\xi$ goes smoothly from 0 to 1 in finite time $t = t_b$ and $\dot{\xi} = \dot{\xi}(t)$ is a bell shaped derivative of $\xi$ such that $\dot{\xi}(t_0) = \dot{\xi}(t_b) \equiv 0$. Furthermore, a sliding mode is enforced for all time with a low dimensional neural network and smooth control effort.

**Proof:** See Appendix 2.

## 6. EXPERIMENTAL RESULTS

In order to demonstrate usefulness of our controller, we present some experimental results obtained on a high performance robot showed in figure 1. Real time implementation uses only two nodes for each degree of freedom and four weight to approximate the inverse dynamics in a given error coordinate system. The objective is the tracking of a desired trajectory by final effector in a finite time. In this case, the task is a circle of $0.1m$ radius in $2.5s$ centered at $X = (0.5, 0)m$ in the Cartesian workspace under different initial conditions. The initial neural network weights were zero, zero initial velocity and 100% of parametric uncertain, this means, the neural network compensate the regressor matrix only based in the states of this function (extend error). Figs. (3) and (4) show exponential convergence of errors and (Fig.2) show a chattering free control input. Figs. (5,6,7) shows the control input, tracking errors applying Theorem 2, where the convergence of errors is driven by a time base generator (TBG). Each run has an average running of 12 s for 1 ms sampling period on Pentium 4, running at $1.5Ghz$ under Windows 2000.

## 7. CONCLUSION

A low dimensional neuroadaptive controller that uses a simple continuous second order change of coordinates is proposed to guarantee convergence of tracking errors. Two theorems show formal convergence results for exponential and finite time stability. The second order sliding mode does not exhibit high frequency commutation, typical of standard first order sliding mode. The experiments results allow to visualize the stability properties.

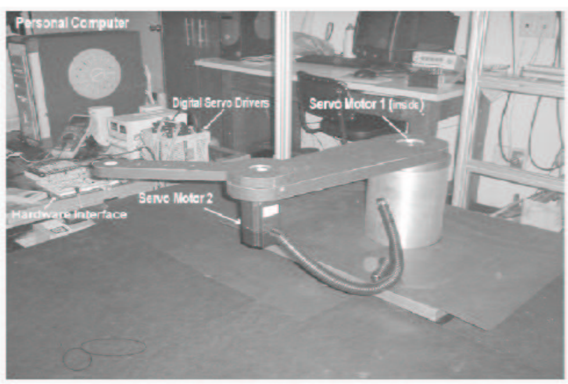


Fig. 1. Robot planar at Division of Mechatronics

Table 1. Parameters of robot arm.

| Parameter | $m_1$ | $m_2$ | $l_1$ | $l_2$ | $l_{c1}$ |
|---|---|---|---|---|---|
| Value | 8 | 5 | 0.5 | 0.35 | 0.19 |
| Parameter | $I_1$ | $I_2$ | $B_1$ | $B_2$ | $l_{c2}$ |
| Value | 0.02 | 0.16 | 5 | 5 | 0.12 |

This novel method solves two traditional drawbacks in neural network control for robots: *(i.)* very fast tracking error converges, and *(ii.)* few nodes and weights are required. Besides these, other important characteristic are: *(iii.)* smooth control, and *(iv.)* the regressor is not required .

## APPENDIX 1: PROOF OF THEOREM 1

The prove is divided in three parts: firstly, we prove that above equation shows boundedness of all system trajectories; secondly, we show the conditions to induce sliding modes, and thirdly, conditions of exponential convergence of tracking errors are shown.

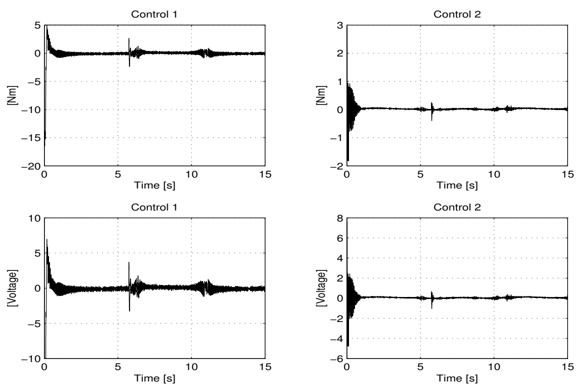


Fig. 2. Theorem 1: Control input for both joints.

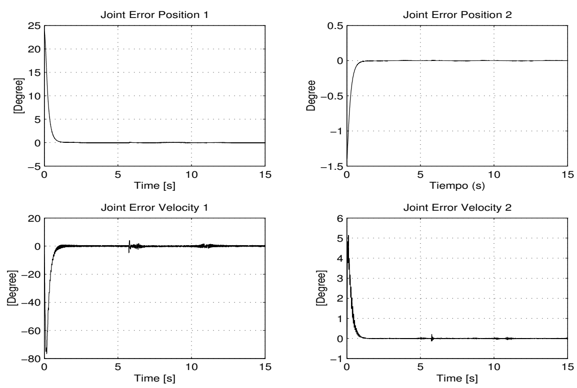


Fig. 3. Theorem 1: Position and velocity tracking errors

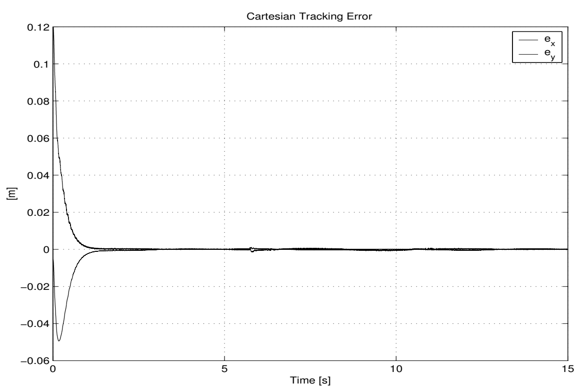


Fig. 4. Theorem 1: Cartesian tracking error.

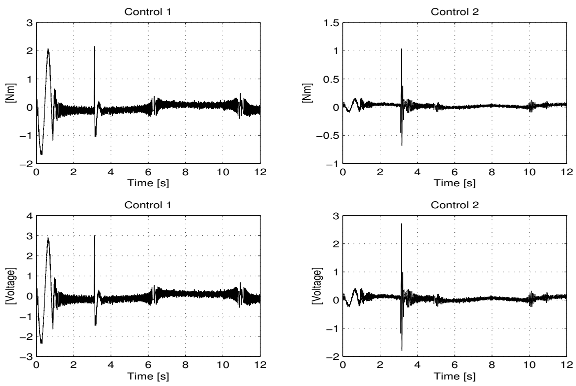


Fig. 5. Theorem 2: Control input for both joints for $t_b = 1.5s$

**Part 1** *Boundedness of Closed-loop Trajectories.* Consider the following Lyapunov function

$$V = \frac{1}{2} S_r^T H S_r + \frac{1}{2} \Delta \mathbf{W}^T \Gamma^{-1} \Delta \mathbf{W} \qquad (24)$$

whose total derivative along its solution is as follows

$$\begin{aligned} \dot{V} &= S_r^T H \dot{S}_r + \frac{1}{2} S_r^T \dot{H} S_r + \Delta \mathbf{W}^T \Gamma^{-1} \Delta \dot{\mathbf{W}} \\ &= -S_r^T K_d S_r + S_r^T \epsilon(x) - S_r^T \tau_d \\ &\leq -S_r^T K_d S_r + S_r^T \epsilon_N - S_r^T \tau_d \qquad (25) \end{aligned}$$

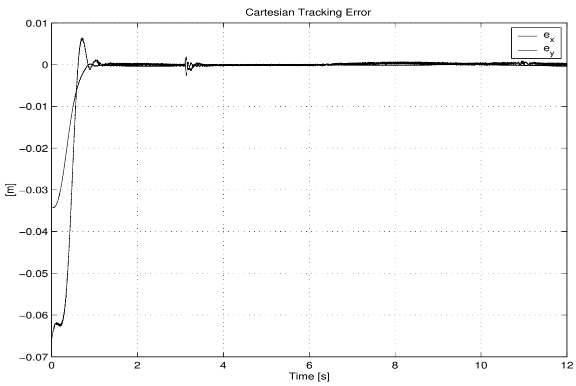


Fig. 6. Theorem 2: Cartesian tracking error for $t_b = 1.5s$

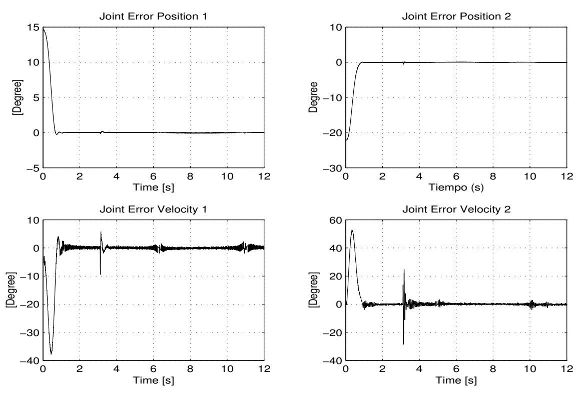


Fig. 7. Theorem 2: Position and velocity tracking error for $t_b = 1.5s$

Note that the term $\tau_d$ is radially unbounded only when $S_r \to \infty$ and for bounded signals it is zero only at $S_r = 0$. This arguments implies that $\|S_r^T \tau_d\| \leq \eta \|S_r\|$ where $\eta = \|H(q)\| \, \|K_i\|$, with $\|X\| = \sqrt{\lambda_m X^T X}$ and $\lambda_m$ stands for the maximum eigenvalue of matrix X. Then, equation (25) becomes

$$\dot{V} \leq -S_r^T K_D S_r + \eta \|S_r\| + \|S_r\| \|\epsilon\| \qquad (26)$$

If $K_d$ is large enough, and initial errors are sufficiently small, we conclude the seminegative definiteness of (26) outside a hyperball $\epsilon_0 = \{S_r | \dot{V}_r \leq 0\}$ centered in the origin such that the following properties of the state of the closed loop system arise

$$S_r \in L_\infty \Longrightarrow \|S_r\| < \varepsilon_1 \qquad (27)$$

where $\varepsilon_1 > 0$ is a bounded scalar. Then $(S_q, \sigma) \in L_\infty$ and since desired trajectories are $C^2$ and feedback gains are bounded, we have that $(\dot{q}_r, \ddot{q}_{cont}) \in L_\infty$, which implies that $Y_{cont} \in L_\infty$. In this way, from equation (26) render $\|S_r\| \|\epsilon\| \leq \varepsilon_2$ with $\varepsilon_2 > 0$ is bounded and $\hat{\mathbf{W}} \in L_\infty$. The right hand side of (18) is therefore bounded and aimed at the boundedness of the inertial, coriolis and gravitational matrices, then $\dot{S}_r \in L_\infty$ and therefore there exists a bounded scalar $\varepsilon_4 > 0$ such that

$$\|\dot{S}_r\| < \varepsilon_4 \qquad (28)$$

So far, we conclude the boundedness of all closed-loop error signals.

**Part 2** *Sliding Mode.* Now, we show that a sliding mode at $S_q = 0$ arises for all time. If we derivative (14), and multiply by $S_q^T$, rearranging we obtain the sliding mode condition

$$\begin{aligned} S_q^T \dot{S}_q &= S_q^T (\dot{S}_r - K_i sign(S_q)) \\ &\leq \varepsilon_4 |S_q| - K_i |S_q| \qquad (29) \\ &\leq -\mu |S_q| \qquad (30) \end{aligned}$$

with $\mu = K_i - \varepsilon_4$. Thus, we can always choose $K_i > \varepsilon_4$, in such a way that $\mu > 0$, guarantees

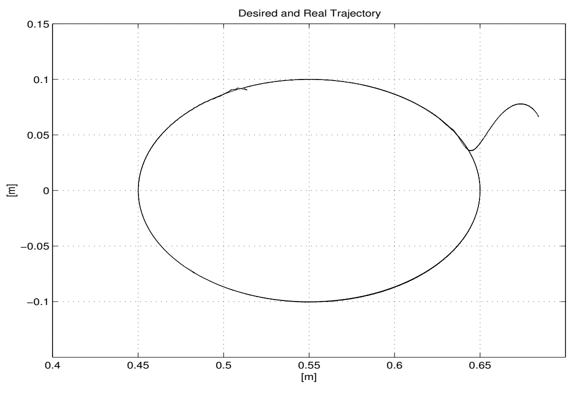


Fig. 8. Theorem 2: Phase plane for $t_b = 1.5s$

the existence of a sliding mode at $S_q = 0$ at time $t_q \leq \frac{|S_q(t_0)|}{\mu}$. However, notice that for any initial condition $S_q(t_0) = 0$, and hence $t_q \equiv 0$ implies that a sliding mode in $S_q(t) = 0$ is enforced for all time without reaching phase and then (8) renders $S = S_d \; \forall \; t$.

**Part 3** *Exponential Convergence.* If $k$ in (9) is tuned large enough such that $S_d \approx 0$ for some small time $0 < t_d \ll 1$ then (8) yields

$$S = 0 \; \forall \; t \geq t_d > 0. \tag{31}$$

guaranteeing exponential stability of tracking errors since the solution of $S = 0$ goes to zero exponentially. **QED**

## APPENDIX 2: PROOF OF THEOREM 2

Parts 1 and 2 are similar to theorem 1, which guarantee the existence of a sliding mode for all time. Part 3 is as follows

**Part 3: Convergence in Finite Time** Substituting (23) in (9), and eliminating the independent variable $t$ we obtain

$$\frac{d}{d\xi}\Delta q = -\alpha_0 \frac{\Delta q}{(1-\xi)+\delta} \tag{32}$$

which attains the solution

$$\Delta q(t) = \Delta q(t_0)[1-\xi(t)+\delta]^{\alpha_0} \tag{33}$$

Then, tracking errors converge in finite time to an arbitrary small vicinity of the equilibrium. Afterwards, and since by assumption $\xi(t) = 1$ at time $t = t_b > 0$, then (33) becomes

$$\Delta q(t_g) = \Delta q(t_0)\delta^{1+\epsilon}. \tag{34}$$

Considering that $\delta$ and $\epsilon$ are very small, then at $t = t_b$, tracking errors belong to a very small vicinity $\varepsilon$ of the origin $[\Delta q, \Delta\dot{q}]^T = [0,0]^T$, which in practice may stand for the required precision or zero error. Note that at $t > t_b$ the time-varying feedback gain $\alpha(t)$ becomes a positive constant near zero. Thus $\alpha(t)$ must be reset to a desired constant $\alpha_c > 0$ at time $t = t_b$. Now considering that a sliding mode is enforced for all time and that $v_q > 0$ and (30) guarantee the finite time monotonic decreasing behavior of $\|S(t)\|(\equiv \|\Delta\dot{q}(t) + \alpha_c\Delta q(t)\|)$, thus for $t > t_b$ we have that $\Delta q(t) \in \epsilon$ and furthermore $\Delta q(t)$ converges exponentially since $\Delta\dot{q}(t) = -\alpha_c\Delta q(t) \; \forall t > t_b$. **QED**

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