DEADTIME BLOCK COMPENSATION SLIDING MODE CONTROL OF LINEAR SYSTEM WITH DELAY

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Abstract: This paper applies the block control method to form a decomposed control law suitabal for multivariable linear time-delay systems. A block controllable form is introduced and a non-singular transformation that reduces the system to this form, is proposed. A block deadtime compensation algorithm wich gives a sliding manifold, is derived. An example of the application of the proposed control strategy is illustrated.

Keywords: linear system, time delay, sliding mode.

1. INTRODUCTION

The feedback stabilization of time-delay systems remains is one of most interest problems in control theory because many industrial processes modelled by delay differential equations. It is well-known that they can dramatically limit the performance and sometimes destabilize the closed loop system. This problem has been extensively studied and several controllers and stability criteria based on optimal control method (see Zavarei (1987), Shen (1991), Feron (1992)), including H_∞ and LMI approaches (Lee (1994), Lehman (1997), Li (1996), Li (1997)), or averaging theory (Leyman (1992)), have been proposed. The common feature of the referred papers is that their derivations are based on analysis of full order system. To decompose the design procedure and introduce a robustness property the sliding mode technique have been applied (Gouaisbaut, Dambrine and Richard (2002), Gouasbaut (1999), Fridman, Strygin and Polyakov (2004)).

In this paper, to stabilize linear time invariant systems with delayed state and input, we use the block control principle which is fruitful and relatively simple, especially when dealing with multivariable systems because the control problem is decomposed into a number of simpler sub-problems of lower dimensions. In order to achieve this, a special state representation must be used which will be referred to as the *Block Controllable form* (or BC-form), consisting of a set of controlled blocks. This approach has successfully been employed to stabilize linear systems (Dodds (1997)) including time delayed systems (Loukianov (2003)). Here, the possibility of applying the same method to design a predictor based sliding mode control, is investigated. Note that a continuos feedback controller using Smith compensator was investigated, for example, by Palmor (1980) and Furutani and Araki (1998), a sliding mode predictor controller was designed by Roh and Oh (1999) for linear systems with delay in the control input only. We consider a linear system with delay in both the state and control variables.

The paper consists of the following parts. In Section 2 the Block Controllable form for time-delay systems is introduced, and the existence conditions, and the transformation of the original system to the BC-form are derived. In Section 3 the block deadtime compensation control strategy is designed, and stability conditions of the closed-loop system are given. In Section 4 an example of the application of proposed decomposition strategy is illustrated.

2. BLOCK CONTROLLABLE FORM FOR SYSTEMS WITH DELAY

Consider a linear time-delay system described by the following state equation

$$\dot{x}(t) = Ax(t) + Cx(t-\tau) + Bu(t-\tau) \qquad (1)$$

where $x \in R^n$, $u \in R^m$ and A, C, B and D are matrices of appropriate dimensions, and $x(t) = \varphi(t)$, $\forall t \in [t_0 - \tau,\, t_0]$, $t_0 \geq 0$, $\varphi(t)$ is a continuous vector-valued initial function.

The essential feature of the proposed method is the conversation of the system (1) to the following introduced BC-form consisting of r blocks

$$\begin{aligned} \dot{x}_r(t) &= A_{rr}x_r(t) + B_r x_{r-1}(t-\tau) \\ \dot{x}_i(t) &= \sum_{j=i}^{r} A_{ij}x_j(t) + B_i x_{i-1}(t-\tau), \qquad (2) \\ &\quad i = 2, \ldots r-1 \\ \dot{x}_1(t) &= \sum_{j=1}^{r} A_{ij}x_j(t) + B_1 u(t-\tau) \end{aligned}$$

where $\bar{x}(t) = [x_1(t), \ldots, x_r(t)]^T$ and $\text{rank}B_i = \dim(x_i) = n_i$, $i = 1, \ldots, r$ and $\sum_{i=1}^{r} n_i = n$. In this paper we consider the case $n_i = m$, $i = 1, \ldots, r$, or $n = r \times m$.

The initial system is brought to the form (2) though the iterative transformation procedure that consists of $(r-1)$ steps.

Step 1. We introduce the following assumption which will be carried for each step of the procedure:

A11. $\text{rank}B = n_1 = m$.

Using this assumption, vector $x(t)$ and matrix B can be partitioned as

$$x(t) = \begin{bmatrix} x_{12}(t) \\ x_1(t) \end{bmatrix}, \; B = \begin{bmatrix} B_{12} \\ B_1 \end{bmatrix}$$

where $\text{rank}B_1 = n_1$. Performing the nonsingular orthogonal transformation $x''(t) = M_1 x(t)$,

$$M_1 = \begin{bmatrix} I_{n-n_1} & -B_{12}B_1^{-1} \\ 0 & I_{n_1} \end{bmatrix}, \; M_1 \begin{bmatrix} B_{12} \\ B_1 \end{bmatrix} = \begin{bmatrix} 0 \\ B_1 \end{bmatrix} \qquad (3)$$

we assume that

A12. The system (1) has a structure such that

$$M_1 A M_1^{-1} = \begin{bmatrix} A'_{22} & 0 \\ A_{12} & A_{11} \end{bmatrix},$$

$$M_1 C M_1^{-1} = \begin{bmatrix} C'_{22} & B'_2 \\ 0 & 0 \end{bmatrix}.$$

Then the initial system (1) is represented as

$$\begin{aligned} \dot{x}'_2(t) &= A'_{22}x'_2(t) + C'_{22}x'_2(t-\tau) \qquad (4) \\ &\quad + B'_2 x_1(t-\tau) \\ \dot{x}_1(t) &= A_{12}x'_2(t) + A_{11}x_1(t) + B_1 u(t-\tau) \quad (5) \end{aligned}$$

where $x''(t) = [x_1(t), x'_2(t)]^T$, $x'_2(t) \in R^{n-n_1}$, $x_1(t) \in R^{n_1}$.

Step k. Consider the system obtained at $(k-1)^{th}$ step $(k < r-1)$

$$\begin{aligned} \dot{x}'_k(t) &= A'_k x'_k(t) + C'_{kk}x'_k(t-\tau) \qquad (6) \\ &\quad + B'_k x_{k-1}(t-\tau) \\ \dot{x}_i(t) &= \sum_{j=i}^{k} A_{ij}x_j(t) + B_i x_{i-1}(t-\tau), \qquad (7) \\ &\quad i = 2, \ldots k-1 \\ \dot{x}_1(t) &= \sum_{j=1}^{k} A_{ij}x_j(t) + B_1 u(t-\tau) \qquad (8) \end{aligned}$$

where rank $B_i = n_i = m$, $i = 1, \ldots, k-1$. For this step, we generalize assumptions A11 and A12 as follows:

Ak1. $\text{rank}B'_k = n_k = m$.

Based on this assumption the subsystem (6) is partitioned as

$$x'_k(t) = \begin{bmatrix} x_{k,2}(t) \\ x_k(t) \end{bmatrix}, \; B'_k = \begin{bmatrix} B_{k,2} \\ B_k \end{bmatrix}$$

where rank $B_k = n_k$, , and $x_k(t)$ and $x_{k,2}(t)$ are $n_k \times 1$ and $(n - \sum_{j=1}^{k-1} n_j - n_k) \times 1$ vectors, respectively.

Proceeding as in the first step, under the previous assumption, we use transformation for subsystem (6) similar to (3)

$$\begin{aligned} x''_k(t) &= M_k x'_k(t), \\ M_k &= \begin{bmatrix} I_{n-n_1-\cdots-n_k} & -B_{k,2}B_k^{-1} \\ 0 & I_{n_k} \end{bmatrix}, \qquad (9) \\ M_k \begin{bmatrix} B_{k,2} \\ B_k \end{bmatrix} &= \begin{bmatrix} 0 \\ B_k \end{bmatrix}. \end{aligned}$$

Ak2. The subsystem (6) has a structure such that

$$M_k A'_k M_k^{-1} = \begin{bmatrix} A'_{k+1,k+1} & 0 \\ A_{k,k+1} & A_{k,k} \end{bmatrix},$$

$$M_k C'_k M_k^{-1} = \begin{bmatrix} C'_{k+1,k+1} & B'_{k+1} \\ 0 & 0 \end{bmatrix}.$$

Then the system (6) - (8) is represented as

$$\begin{aligned} \dot{x}'_{k+1}(t) &= A'_{k+1,k+1}x'_{k+1}(t) + C'_{k+1,k+1}x'_{k+1}(t-\tau) \\ &\quad + B'_{k+1}x_k(t-\tau) \\ \dot{x}_k(t) &= A_{k,k+1}x'_{k+1}(t) + A_{k,k}x_k(t) \\ &\quad + B_k x_{k-1}(t-\tau) \\ \dot{x}_i(t) &= \sum_{j=i}^{k} A_{ij}x_j(t) + B_i x_{i-1}(t-\tau) \\ \dot{x}_1(t) &= \sum_{j=1}^{k} A_{ij}x_j(t) + B_1 u(t-\tau), \; i = 2, ..k-1 \end{aligned}$$

with $x''_k(t) = [x_k(t), x'_{k+1}(t)]^T$, and rank $B_i = n_i$, $i = 1, ..., k$.

Step (r-1). At this last step the system (1) is presented similar to (6) - (8) with $k = r-1$. Then we assume that

A(r-1)1. rank$B'_{r-1} = n_r = m$.

A(r-1)2. The subsystem (6) with $k = r-1$ has a structure such that

$$M_{r-1}A'_{r-1}M_{r-1}^{-1} = \begin{bmatrix} A_{r,r} & 0 \\ A_{r-1,r} & A_{r-1,r} \end{bmatrix} \text{ and}$$

$$M_{r-1}C_{r-1}M_{r-1}^{-1} = \begin{bmatrix} 0 & B_r \\ 0 & 0 \end{bmatrix}$$

Then the system (1) is presented in the BC-form (2).

From the previous algorithm, we may state the following result:

Theorem 1. Assume that the system (1) with the structure $n = r \times m$ is controllable and at each step of the BC-form algorithm assumptions Ak1 and Ak2 hold. Then, there exists an integer $r \leq n$ such that the system (1) takes the form (2).

Remark. The assumption Ak1 is not necessary since this condition implies that the system (1) has the structure

S1: $m = n_1 = n_2 = \cdots = n_r$ *or* $n = r \times m$

If at some k^{th} step of the BC-form algorithm we have rank$B'_k = n_k < m$, then the system (1) has the structure

S2: $m \geq n_1 \geq n_2 \geq \cdots \geq n_r$

In the last case the inverse matrix B_k^{-1} in the transformation (9) can be replaced by the pseudoinverse matrix B_k^+, $B_k^+ = B_k^T(B_k B_k^T)^{-1}$.

3. BLOCK CONTROL DESIGN

In this section, a state feedback control law is developed for transformed system (2) with structure S1. It is more conveniently to renumber the state variables of (2) as

$$\dot{x}_1(t) = A_{11}x_1(t) + B_1 x_2(t-\tau) \quad (10)$$
$$\dot{x}_2(t) = A_{21}x_1(t) + A_{22}x_2(t) + B_2 x_3(t-\tau) \quad (11)$$
$$\dot{x}_i(t) = \sum_{j=1}^{i} A_{ij}x_j(t) + B_i x_{i+1}(t-\tau), \quad (12)$$
$$\dot{x}_r(t) = \sum_{j=1}^{r} A_{ij}x_j(t) + B_r u(t-\tau), \quad (13)$$
$$i = 3, ..., r-1$$

A control strategy for (10)-(13) can be developed considering x_{i+1} as a fictitious control vector in the i^{th} block and designing a predictor. This procedure is outlined in the following.

Step 1. Define $z_1(t) = x_1(t+\tau)$. A predictor for the block (10) with fictitious control x_2 can be designed as

$$z_1(t) = e^{A_{11}\tau}x_1(t) + \int_{-\tau}^{0} e^{-A_{11}\theta}B_1 x_2(t+\theta)d\theta. \quad (14)$$

Taking the derivative of $z_1(t)$ (14),

$$\begin{aligned} \dot{z}_1(t) &= e^{A_{11}\tau}\dot{x}_1(t) + A_{11}\int_{-\tau}^{0} e^{-A_{11}\theta}B_1 x_2(t+\theta)d\theta \\ &\quad + B_1 x_2(t) - e^{A_{11}\tau}B_1 x_2(t-\tau) \end{aligned}$$

and using (10) yields

$$\dot{z}_1(t) = A_{11}z_1(t) + B_1 x_2(t). \quad (15)$$

The fictitious control $x_2(t)$ in (15) is chosen as

$$x_2(t) = x_2^c(t) + B_1^{-1}[K_1 z_1(t) + z_2(t)] \quad (16)$$

where $z_2(t)$ is a new variables vector, K_1 is a design matrix, and $x_2^c(t)$ is calculated from the equation $\dot{z}_1(t) = 0$ as

$$x_2^c(t) = -B_1^{-1}A_{11}z_1(t) \quad (17)$$

The transformed 1^{st} block (15) with input (16) and (17) has the desired form without delay

$$\dot{z}_1(t) = K_1 z_1(t) + z_2(t). \quad (18)$$

The algorithm (16) and (17) defines the transformation for $z_2(t)$

$$\begin{aligned} z_2(t) &= B_1 x_2(t) + x_2^c(t) + K_1 z_1(t) \quad (19) \\ &:= B_1 x_2(t) + R_{21}z_1(t), \\ R_{21} &= [A_{11} - K_1]. \end{aligned}$$

Step 2. Taking the derivative of (19) with respect to time gives

$\dot{z}_2(t) = \bar{A}^1_{21} z_1(t) + \bar{A}_{22} z_2(t) + D_{21} x_1(t) + \bar{B}_2 x_3(t-\tau)$

where $\bar{A}_{21} = [K_1 R_{21} - B_1 A_{22} B_1^{-1} R_{21}]$, $\bar{A}_{22} = [R_{21} + B_1 A_{22} B_1^{-1}]$, $D_{21} = B_1 A_{21}$, $\bar{B}_2 = B_1 B_2$. Then defining $\varphi_3(t-\tau) = \bar{A}_{21} z_1 + D_{21} x_1(t) + \bar{B}_2 x_3(t-\tau)$ yields

$$\dot{z}_2(t) = \bar{A}_{22} z_2(t) + \varphi_3(t-\tau). \tag{20}$$

As on the first step, the predictor for $\bar{z}_2(t) = z_2(t+\tau)$ can be designed similar to (14) as

$$\bar{z}_2(t) = e^{\bar{A}_{22}\tau} z_2(t) + \int_{-\tau}^{0} e^{-\bar{A}_{22}\theta} \varphi_3(t+\theta) d\theta.$$

with $\varphi_3(t-\tau) = \dot{z}_2(t) - \bar{A}_{22} z_2(t)$. Then

$$\dot{\bar{z}}_2(t) = \bar{A}_{22} \bar{z}_2(t) + \varphi_3(t). \tag{21}$$

Now the fictitious input vector $\varphi_3(t)$ in (21) is chosen similar to (16) and (17):

$$\varphi_3(t) = \varphi_3^c(t) + K_2 \bar{z}_2(t) + z_3(t) \tag{22}$$

where $z_3(t)$ is a new variables vector, K_2 is a design matrix, and again $\varphi_3^c(t)$ is found from the equation $\dot{\bar{z}}_2(t) = 0$ as

$$\varphi_3^c(t) = -\bar{A}_{22} \bar{z}_2(t). \tag{23}$$

Thus, equation (21) with (22) and (23) takes the same form of equation (18), namely

$$\dot{\bar{z}}_2(t) = K_2 \bar{z}_2(t) + z_3(t).$$

This procedure may be performed iteratively obtaining on the i^{th} step $i = 3, ..., r$

$$\begin{aligned}
\varphi_{i+1}(t-\tau) &= \dot{z}_i(t) - \bar{A}_{ii} z_i(t) \\
\bar{z}_i(t) &= e^{\bar{A}_{ii}\tau} z_i(t) + \int_{-\tau}^{0} e^{-\bar{A}_{ii}\theta} \varphi_{i+1}(t+\theta) d\theta \\
\varphi_{i+1}(t) &= \varphi_{i+1}^c(t) + K_i z_i(t) + z_{i+1}(t) \qquad (24) \\
\varphi_{i+1}^c(t) &= -\bar{A}_{ii} \bar{z}_i(t)
\end{aligned}$$

where K_i is a design matrix. The variables, obtained from this procedure form a transformation given by

$$\begin{aligned}
z_1(t) &= x_1(t+\tau) \\
z_2(t) &= B_1 x_2(t) + x_2^c(t) + K_1 z_1(t) \\
\varphi_3(t-\tau) &= \dot{z}_2(t) - \bar{A}_{22} z_2(t) \\
\bar{z}_2(t) &= e^{\bar{A}_{22}\tau} z_2(t) + \int_{-\tau}^{0} e^{-\bar{A}_{22}\theta} \varphi_3(t+\theta) d\theta. \\
z_i(t) &= \bar{B}_{i-1} x_i(t) + x_i^c(t) + K_{i-1} z_{i-1}(t) \qquad (25) \\
\bar{z}_i(t) &= e^{\bar{A}_{ii}\tau} z_i(t) + \int_{-\tau}^{0} e^{-\bar{A}_{ii}\theta} \varphi_{i+1}(t+\theta) d\theta \\
\bar{z}_i(t) &= z_i(t+\tau), \quad i = 2, ..., r,
\end{aligned}$$

where $x_i^c(t)$ is calculated from $\dot{\bar{z}}_i(t) = 0$. On the last step, the system (10)-(13) can be presented in the new variables $z_1, \bar{z}_2, ..., \bar{z}_r$, of the form

$$\begin{aligned}
\dot{z}_1(t) &= K_1 z_1(t) + z_2(t) \\
\dot{\bar{z}}_i(t) &= K_i \bar{z}_i(t) + z_{i+1}(t), \ i = 2, ..., r-1 \\
\dot{\bar{z}}_r(t) &= \bar{A}_{r,r} z_r(t) + \sum_{i=0}^{i=r-2} A^1_{r,i} z_1(t+i\tau) + \\
&\sum_{i=0}^{i=r-2} A^2_{r,i} z_2(t+i\tau) + \sum_{i=0}^{i=r-3} A^3_{r,i} z_3(t+i\tau) \\
&+ \cdots + \sum_{i=0}^{i=1} A^{r-1}_{r,i} z_{r-1}(t+i\tau) \qquad (26) \\
&+ D_{r,r+1} x_1(t) + B_r u(t).
\end{aligned}$$

To generate a sliding mode in system (26), a natural choice for a switching function $\sigma(t)$ is taking

$$\begin{aligned}
\sigma(t) &= \bar{z}_r(t) \\
\bar{z}_r(t) &= e^{\bar{A}_{r,r}\tau} z_r(t) + \int_{-\tau}^{0} e^{-\bar{A}_{r,r}\theta} \varphi_{r+1}(t+\theta) d\theta.
\end{aligned}$$

Then the following combined control law is proposed

$$u = u^c - k_r B_r^{-1} sign(\sigma(t)), \ k_r > 0 \tag{27}$$

where the control component u^c coincides with the equivalent control calculated as the solution of equation $\dot{\sigma}(t) = 0$ of the form

$$\begin{aligned}
u_{eq} = -B_r^{-1}[&\bar{A}_{r,r} z_r(t) + \sum_{i=0}^{i=r-2} A^1_{r,i} z_1(t+i\tau) + \\
&\sum_{i=0}^{i=r-2} A^2_{r,i} z_2(t+i\tau) + \sum_{i=0}^{i=r-3} A^3_{r,i} z_3(t+i\tau) \\
&+ \cdots + \sum_{i=0}^{i=1} A^{r-1}_{r,i} z_{r-1}(t+i\tau) \qquad (28) \\
&+ D_{r,r+1} x_1(t)].
\end{aligned}$$

Substitution (27) and (28) into (26) yields

$$\dot{\sigma}(t) = -k_r sign(\sigma(t)).$$

The state vector reaches the manifold $\sigma(t) = 0$ in a finite time, and then the sliding mode motion on this manifold is described by the reduced order system

$$\begin{aligned}
\dot{z}_1(t) &= K_1 z_1(t) + \bar{z}_2(t-\tau) \\
\dot{\bar{z}}_i(t) &= K_i \bar{z}_i(t) + \bar{z}_{i+1}(t-\tau), \qquad (29) \\
&i = 2, ..., r-2 \qquad (30) \\
\dot{\bar{z}}_{r-1}(t) &= K_{r-1} \bar{z}_{r-1}(t).
\end{aligned}$$

Theorem 2. If the matrices $K_i,\ i = 1, ..., r-1$, are Hurwitz then the sliding mode equation (1) is asymptotically stable.

4. AN APPLICATION EXAMPLE

In this section, the proposed control method is applied to control a high-speed clossed-air wind tunnel. The the main objective of the control is to provide a fast response so to reduce the cost of liquid nitrogen losses during the transient regimes. A linearized model of the wind tunnel is given by (Manitius and Tran (1986), Manitius (1984))

$$\begin{aligned}\dot{x}_1(t) &= -ax_1(t) + akx_2(t-\tau)\\ \dot{x}_2(t) &= x_3(t)\\ \dot{x}_3(t) &= -\omega^2 x_2(t) - 2\zeta\omega x_3(t) + \omega^2 u(t)\end{aligned}$$

where the state variable x_1, x_2 and x_3 present the Mach number, actuator position guide vane angle in a driving fan and actuator rate, respectively, $a = \frac{1}{1.964}$; $k = -0.117$; $\omega = 6$; $\zeta = 8$; $\tau = 0.33s$. The delay τ represents the time of the transport between the fan and the test section.

Rename the system as

$$\begin{aligned}\dot{x}_1(t) &= a_{11}x_1(t) + b_1 x_2(t-\tau)\\ \dot{x}_2(t) &= x_3(t)\\ \dot{x}_3(t) &= a_{31}x_2(t) + a_{32}x_3(t) + b_3 u(t) \qquad (31)\end{aligned}$$

for $a_{11} = -a,\ b_1 = ak,\ a_{31} = -\omega^2,\ a_{32} = -2\zeta\omega$, $b_3 = \omega^2$.

The predictor is designed similar to (14) as

$$z_1(t) = e^{a_{11}\tau}x_1(t) + \int_{-\tau}^{0} e^{-a_{11}\theta} b_1 x_2(t+\theta)d\theta.$$

Then

$$\begin{aligned}\dot{z}_1(t) = e^{a_{11}\tau}\dot{x}_1(t) + a_{11}\int_{-\tau}^{0} e^{-a_{11}\theta} b_1 x_2(t+\theta)d\theta\\ +b_1 x_2(t) - e^{a_{11}\tau}b_1 x_2(t-\tau)\end{aligned}$$

or

$$\dot{z}_1(t) = a_{11}z_1(t) + b_1 x_2(t).$$

Introducing the desired dynamics as $-k_1 z_1$, $k_1 > 0$, the transformation (19) is now defined of the form

$$z_2 = \bar{a}_{21}z_1(t) + b_1 x_2(t)$$

where $\bar{a}_{21} = (a_1 + k_1)$.

Then the first block of (31) is represented in the new variables z_1 and z_2 as

$$\dot{z}_1(t) = -k_1 z_1(t) + z_2(t).$$

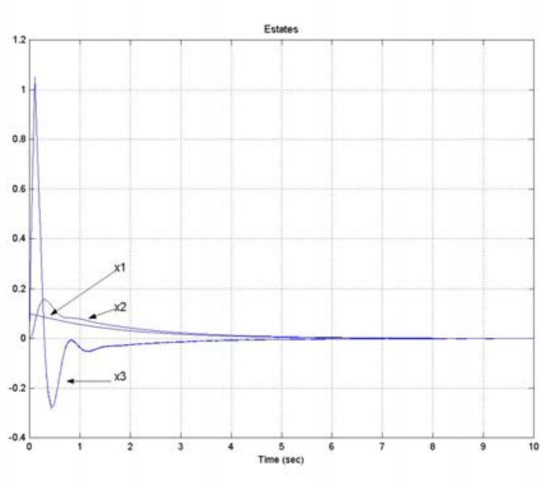

Fig. 1. The response of the state variables x_1, x_2 and x_3.

At the second step, taking the derivative of z_2,

$$\dot{z}_2(t) = \bar{a}_{21}(a_{11}z_1(t) + b_1 x_2(t)) + b_1 x_3(t)$$

and choosing the fictitious control $x_3(t)$ similar to (16) yields

$$\dot{z}_2(t) = -k_2 z_2(t) + z_3(t).$$

Then the transformation for z_3 is of the form

$$z_3(t) = \tilde{a}_{31}z_1(t) + \tilde{a}_{32}z_2(t) + b_1 x_3(t)$$

where $\tilde{a}_{31} = -\bar{a}_{21}k_1$ and $\tilde{a}_{32} = (a_{21} + k_2)$.

The original system (31) is now represented in the new variables z_1, z_2 and z_3 of the form

$$\begin{aligned}\dot{z}_1(t) &= -k_1 z_1(t) + z_2(t)\\ \dot{z}_2(t) &= -k_2 z_2(t) + z_3(t)\\ \dot{z}_3(t) &= \bar{a}_{31}z_1(t) + \bar{a}_{32}z_2(t) + \bar{a}_{33}z_3(t) +\\ &\quad b_r u(t) \qquad (32)\end{aligned}$$

with $\bar{a}_{31} = (a_{31}(\bar{a}_{21} - 1) - \tilde{a}_{31}k_1)$, $\bar{a}_{32} = (\tilde{a}_{31} - \tilde{a}_{32}k_2 - a_{31} + a_{32}(\bar{a}_{21} + k_2))$, $\bar{a}_{33} = \tilde{a}_{32} - a_{32}$ and $b_r = b_1 b_3$.

To generate a sliding mode in the system (32), we choose a switching function $\sigma(t)$ as

$$\sigma(t) = z_3(t) = \tilde{a}_{31}z_1(t) + \tilde{a}_{32}z_2(t) + b_1 x_3(t).$$

The control law is selected of the form

$$u(t) = u_{eq} - k_r sign(\sigma),\ k_r > 0$$

where the equivalent control u_{eq} is calculated from $\dot{\sigma} = 0$ as

$$u_{eq} = -(b_r)^{-1}(\bar{a}_{31}z_1(t) + \bar{a}_{32}z_2(t) + \bar{a}_{33}z_3(t)).$$

For the simulation, the values of the control parameters k_1, k_2 and k_r are adjusted to 0.75, 1 and 0.5, respectively. The responses of the original state and control variables are shown in the fig.1 and fig.2 respectively.

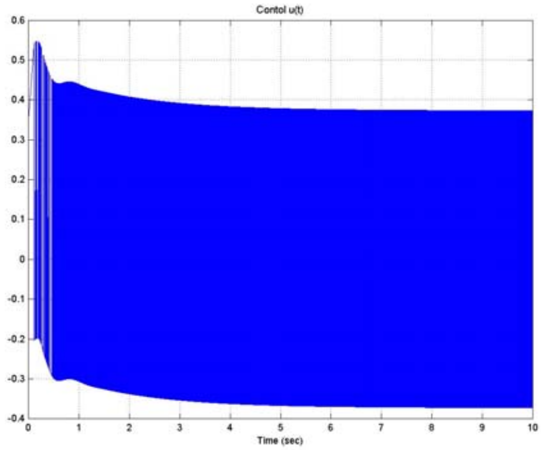

Fig. 2. The response of control input variable u.

5. CONCLUSIONS

The decomposition block deadtime compensation sliding mode control method has been formulated for control of linear time-delay systems which can be transformed into BC-form. The proposed transformation and control design procedures have step-by step character that simplifies the solution of the problem. This method enables to solve one of the classical problem design of pole placement state feedback for linear systems with delayed state and control input.

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