

FAULT-TOLERANT SYSTEM BASED ON OUTPUT FEEDBACK \mathcal{H}_∞ MARKOVIAN CONTROL FOR MANIPULATOR ROBOTS

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Abstract: In this paper, it is presented a fault-tolerant system based on output feedback \mathcal{H}_∞ Markovian control for manipulator robots. Firstly, it is shown that with a control strategy based on deterministic nonlinear \mathcal{H}_∞ approach the stability is guaranteed only if the robot stops completely after the fault detection, and then restarts the movement from zero velocity in an underactuated configuration. Secondly, it is shown that with the fault-tolerant system proposed in this paper, the stability is guaranteed, after the fault be detected, even with the robot still moving. Experimental results are presented. *Copyright © 2005 IFAC*

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1. INTRODUCTION

Parametric uncertainties and exogenous disturbances increase the difficulty of reference tracking control for robotic manipulators. Additionally, free torque failures, where the torque supply in the motor breaks down suddenly, can turn the system uncontrolled with the possibility of damage for the manipulator components. Furthermore, the movement must be completed according to the manipulator fault configuration if the robot is working in hazardous or unstructured environment. When a free torque failure occurs the fully actuated manipulator changes to an underactuated configuration. Nonlinear \mathcal{H}_∞ and Markovian controllers are used in (Siqueira and Terra, 2004b) to generate a fault-tolerant robotic system, where the control strategies are based on state feedback control. However, the velocity signal generally is not available and it is obtained indirectly

from the measured position. This procedure can present noises and delays. An output feedback controller can be used in order to avoid these problems. In this paper, the output feedback gain-scheduling controller proposed in (Apkarian and Adams, 1998), named *Projected Characterization*, is applied to an actual robot manipulator. The robot manipulator is represented as a quasi linear parameter-varying (Quasi-LPV) system in which the parameters depend on the state. When the manipulator changes, after a fault occurrence, from a totally actuated to an underactuated configuration, the system stability is not guaranteed with the output feedback controllers proposed in (Apkarian and Adams, 1998). To use this kind of controller (defined in a deterministic approach) in a fault-tolerant robot system, it is necessary to stop completely the movement of all joints after the fault detection, restarting it from zero

velocity. To avoid the necessity of stopping the robot when a fault occurs, the Markov theory is used to design a procedure to incorporate abrupt changes in the manipulator configuration, and Markovian controllers are used to guarantee stability. The Markov theory is based on stochastic linear systems subject to abrupt variations, namely, Markovian jump linear systems (MJLS's) (Ji *et al.*, 1991). The nonlinear system, which represents the dynamics of the manipulator, is linearized around operation points, and a Markovian model is developed regarding the changes of the operation points and the probability of a fault (Siqueira and Terra, 2004a). The fault-tolerant control system for manipulators developed in this paper utilizes the output feedback \mathcal{H}_∞ Markovian control proposed in (de Farias *et al.*, 2000). This paper is organized as follows: in Section 2, the Quasi-LPV representation of a manipulator robot is presented; experimental results obtained with the actual UArm II robot, using the deterministic output feedback \mathcal{H}_∞ LPV controller, are presented in Section 3; the alternative output feedback control, using Markov theory, and experimental results obtained via Markovian control are addressed in Section 4.

2. QUASI-LPV REPRESENTATION OF THE MANIPULATOR

2.1 Totally actuated manipulator

The dynamic equations of a robot manipulator can be found by the Lagrange theory as

$$\tau = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + F\dot{q} + G(q), \quad (1)$$

where $q \in \mathfrak{R}^n$ is the joint position vector, $M(q) \in \mathfrak{R}^{n \times n}$ is the symmetric positive definite inertia matrix, $C(q, \dot{q}) \in \mathfrak{R}^{n \times n}$ is the Coriolis and centripetal matrix, $F \in \mathfrak{R}^n$ is the diagonal matrix of frictional torque coefficients, $G(q) \in \mathfrak{R}^n$ is the gravitational torque vector and $\tau \in \mathfrak{R}^n$ is the applied torque vector. A parametric uncertainty can be introduced dividing the parameter matrices $M(q)$, $C(q, \dot{q})$, F and $G(q)$ into a nominal and a perturbed part, where, $M_0(q)$, $C_0(q, \dot{q})$, F_0 and $G_0(q)$ are the nominal matrices, and $\Delta M(q)$, $\Delta C(q, \dot{q})$, ΔF and $\Delta G(q)$ are the parametric uncertainties. A finite energy exogenous disturbance, τ_d , can also be introduced. After these considerations (1) becomes

$$\tau + \delta(q, \dot{q}, \ddot{q}, \tau_d) = M_0(q)\ddot{q} + C_0(q, \dot{q})\dot{q} + F_0\dot{q} + G_0(q), \quad (2)$$

with

$$\delta(\cdot) = -(\Delta M(q)\ddot{q} + \Delta C(q, \dot{q})\dot{q} + \Delta F\dot{q} + \Delta G(q) - \tau_d).$$

The state is defined as $x = [\dot{q}^T \ q^T]^T$, where q and \dot{q} are the positions and the velocities of

the manipulator joints, respectively. The state dynamic equation is given by

$$\dot{x} = A(q, \dot{q})x + B(q)u + B(q)\delta(q, \dot{q}, \ddot{q}, \tau_d), \quad (3)$$

where

$$A(q, \dot{q}) = \begin{bmatrix} -M_0^{-1}(q)(C_0(q, \dot{q}) + F_0) & 0 \\ I_n & 0 \end{bmatrix},$$

$$B(q) = \begin{bmatrix} M_0^{-1}(q) \\ 0 \end{bmatrix}, \quad u = \tau - G_0(q).$$

2.2 Underactuated manipulator

Underactuated robot manipulators are mechanical systems with less actuators than degrees of freedom. For this reason, the control of passive joints is made considering the dynamic coupling between them and the active joints. Here, the manipulator is considered with n joints, in which n_p are passive and n_a are active joints. From (Arai and Tachi, 1991), no more than n_a joints of the manipulator can be controlled at every instant when breaks are used in the passive joints. Let n_u be the number of passive joints that have not already reached their set-point in a given instant. If $n_u \geq n_a$, n_a passive joints are controlled and grouped in the vector $q_u \in \mathfrak{R}^{n_a}$, the remaining passive joints, if any, are kept locked, and the active joints are grouped in the vector $q_a \in \mathfrak{R}^{n_a}$. If $n_u < n_a$, the n_u passive joints are controlled applying torques in n_u active joints. The remaining active joints are kept locked. In this case, $q_u \in \mathfrak{R}^{n_u}$ and $q_a \in \mathfrak{R}^{n_u}$. The strategy is to control all passive joints until they reach the desired position, considering the two possibilities above, and then turn on the brakes. After that, all the active joints are controlled by themselves as a totally actuated manipulator. The dynamic equation (2) can be partitioned as

$$\begin{bmatrix} \tau_a \\ 0 \end{bmatrix} + \begin{bmatrix} \delta_a \\ \delta_u \end{bmatrix} = \begin{bmatrix} M_{aa} & M_{au} \\ M_{ua} & M_{uu} \end{bmatrix} \begin{bmatrix} \dot{q}_a \\ \dot{q}_u \end{bmatrix} + \begin{bmatrix} C_{aa} & C_{au} \\ C_{ua} & C_{uu} \end{bmatrix} \begin{bmatrix} \dot{q}_a \\ \dot{q}_u \end{bmatrix} + \begin{bmatrix} F_{aa} & 0 \\ 0 & F_{uu} \end{bmatrix} \begin{bmatrix} \dot{q}_a \\ \dot{q}_u \end{bmatrix} + \begin{bmatrix} G_a \\ G_u \end{bmatrix}, \quad (4)$$

where the indices a and u represent the active and free (breaks not actioned) passive joints, respectively. Factoring out the vector \dot{q}_a in the second line of (4) and substituting in the first one, one has

$$\tau_a + \bar{\delta}(q, \dot{q}, \ddot{q}, \tau_d) = \bar{M}_0(q)\ddot{q}_u + \bar{C}_0(q, \dot{q})\dot{q}_u + \bar{F}_0(q)\dot{q}_u + \bar{D}_0(q, \dot{q})\dot{q}_a + \bar{G}_0(q), \quad (5)$$

with

$$\begin{aligned}
\overline{M}_0(q) &= M_{au} - M_{aa}M_{ua}^{-1}M_{uu}, \\
\overline{C}_0(q, \dot{q}) &= C_{au} - M_{aa}M_{ua}^{-1}C_{uu}, \\
\overline{D}_0(q, \dot{q}) &= C_{aa} - M_{aa}M_{ua}^{-1}C_{ua} + F_{aa}, \\
\overline{F}_0(\dot{q}) &= -M_{aa}M_{ua}^{-1}F_{uu}, \\
\overline{G}_0(q) &= G_a - M_{aa}M_{ua}^{-1}G_u, \\
\overline{\delta}(q, \dot{q}, \ddot{q}, \tau_d) &= \delta_a - M_{aa}M_{ua}^{-1}\delta_u.
\end{aligned}$$

The state is defined as $x_u = [\dot{q}_u^T q_u^T]^T$. Hence, a Quasi-LPV representation of the underactuated manipulator can be defined as follows

$$\dot{x}_u = A(q, \dot{q})x_u + B(q)u + B(q)\overline{\delta}(q, \dot{q}, \ddot{q}, \tau_d), \quad (6)$$

with

$$\begin{aligned}
A(q, \dot{q}) &= \begin{bmatrix} -\overline{M}_0^{-1}(q) (\overline{C}_0(q, \dot{q}) + \overline{F}_0(q)) & 0 \\ I & 0 \end{bmatrix}, \\
B(q) &= \begin{bmatrix} \overline{M}_0^{-1}(q) \\ 0 \end{bmatrix}, \\
u &= \tau_a - \overline{D}_0(q, \dot{q})(\dot{q}_a - \overline{G}_0(q)).
\end{aligned}$$

To apply the control technique presented as *Projected Characterization* in (Apkarian and Adams, 1998), the robot manipulator needs to be represented according to the equation

$$\begin{aligned}
\dot{x} &= A(\rho(t))x + B_1(\rho(t))w + B_2(\rho(t))u, \\
z &= C_1(\rho(t))x + D_{11}(\rho(t))w + D_{12}(\rho(t))u, \\
y &= C_2(\rho(t))x + D_{21}(\rho(t))w.
\end{aligned} \quad (7)$$

Consider as system disturbances, the desired position, q^d , and the combined torque disturbance, δ , that is: $w = [\delta^T (q^d)^T]^T$. The system outputs, z , are the position error, $q^d - q$, and the control input, u . The control output is the position error, $y = q^d - q$, since we only have the position measured directly. Note that for the underactuated case, instead of using (q, \dot{q}) as the state, one must use (q_u, \dot{q}_u) . Hence, the robot system can be described by (7) with

$$\begin{aligned}
A(\rho) &= A(q, \dot{q}), \\
B_1(\rho) &= [B(q) \ 0], \quad B_2(\rho) = B(q), \\
C_1(\rho) &= \begin{bmatrix} 0 & -I \\ 0 & 0 \end{bmatrix}, \quad C_2(\rho) = [0 \ -I], \\
D_{11}(\rho) &= \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, \quad D_{12}(\rho) = [0 \ I]^T, \\
D_{21}(\rho) &= [0 \ I], \quad D_{22}(\rho) = 0,
\end{aligned}$$

where the matrices $A(q, \dot{q})$ and $B(q)$ are obtained from (3), for the totally actuated case, and (6), for the underactuated case.

3. EXPERIMENTAL RESULTS

The output feedback \mathcal{H}_∞ LPV controller mentioned in Section 2.2 was applied to the experimental underactuated manipulator UArm II (Underactuated Arm II), designed and built by H. Ben Brown, Jr. of Pittsburgh, PA, USA. This 3-link manipulator has special-purpose joints containing

each an actuator and a brake, so that they can act as active or passive joints. The robot parameters can be found in (Siqueira *et al.*, 2003).

3.1 Totally actuated case

For the experiment, the selected parameters, which are part of the state vector are $\rho(x) = [q_2 \ q_3]^T$. The parameter space, P , is defined as $\rho \in [40 \ -40]^\circ \times [40 \ -40]^\circ$. The parameter variation rate is bounded by $|\dot{\rho}| \leq [90 \ 90]^\circ/s$. $X(\rho)$ and $Y(\rho)$ were defined as follows

$$\begin{aligned}
X(\rho) &:= X_0 f_1(\rho), \\
Y(\rho) &:= Y_0 f_1(\rho) + Y_1 f_2(\rho) + Y_2 f_3(\rho),
\end{aligned} \quad (8)$$

where $f_1(\rho) = 1$, $f_2(\rho) = \sin(q_2) + \cos(q_2)$, $f_3(\rho) = \sin(q_3) + \cos(q_3)$. The parameter space was divided in $L = 5$ for each parameter. The best level of attenuation found was $\gamma = 2.49$.

3.2 Underactuated case

The underactuated configuration used to validate the proposed methodology was the APA configuration, i.e., the joint 2 is passive and the joints 1 and 3 are actives. For this configuration, two control phases are necessary to control all joints to the set-point. In the first control phase, the passive joint 2 is controlled by the dynamic coupling with the active joint 1, that is, $q_u = q_2$, $q_a = q_1$, and joint 3 is kept locked; in the second one, the active joints 1 and 3 are controlled. For the first control phase of the experiment, the parameter ρ selected is the state representing the position of joint 2, $\rho(x) = [q_2]$. The parameter space, P , is defined as $\rho \in [30, -30]^\circ$. The parameter variation rate is bounded by $|\dot{\rho}| \leq [90^\circ/s]$. $X(\rho)$ and $Y(\rho)$ are defined like in (8) with, $f_1(\rho) = 1$, $f_2(\rho) = \sin(q_2)$, $f_3(\rho) = \cos(q_2)$. The parameter space was divided in $L = 5$. The best level of attenuation found was $\gamma = 1.87$. For the second control phase, the selected parameters, that are part of the state vector, were $\rho(x) = [q_1 \ q_3]^T$. The parameter space, P , is defined as $\rho \in [40, -40]^\circ \times [40, -40]^\circ$. The parameter variation rate is bounded by $|\dot{\rho}| \leq [90^\circ/s \ 90^\circ/s]$. $X(\rho)$ and $Y(\rho)$ are the same as in (8) with, $f_1(\rho) = 1$, $f_2(\rho) = \sin(q_1) + \cos(q_1)$, $f_3(\rho) = \sin(q_3) + \cos(q_3)$. The parameter space was divided in $L = 5$ for each parameter. The best level of attenuation found was $\gamma = 2.36$.

3.3 Fault Occurrence

The controllers designed in Sections 3.1 and 3.2 do not guarantee that the joints will reach the set-point if a free torque fault occurs in the second joint, changing suddenly the configuration from AAA to APA. To verify this behavior, one experiment was performed considering initially

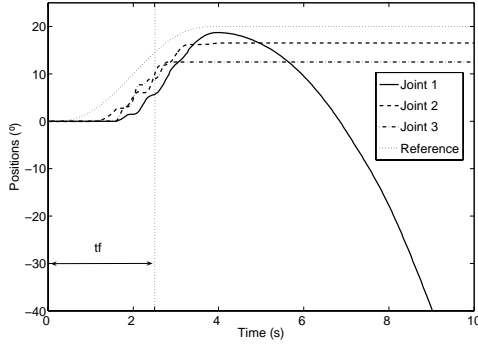


Figure 1. Joint position, Quasi-LPV control not using brakes in the reconfiguration.

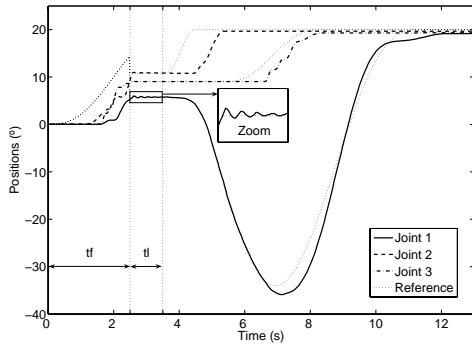


Figure 2. Joint position, Quasi-LPV control using brakes in the reconfiguration.

the manipulator in the totally actuated configuration AAA, with the initial position $q(0) = [0^\circ \ 0^\circ \ 0^\circ]^T$ and the desired final position $q(T) = [20^\circ \ 20^\circ \ 20^\circ]^T$, where $T = [4.0 \ 4.0 \ 4.0] \text{ s}$. When the joint positions reached approximately 15° for all joints, at $t_f = 2.5 \text{ s}$, a free torque fault was introduced in the second joint. Here, we assume that a fault detection system (Terra and Tinós, 2001) indicates the fault as soon as it occurs. Hence, the controller changes from the fully actuated configuration to the underactuated one maintaining the manipulators movement. As can be seen in Fig. 1, the LPV controllers were not able to react immediately to the fault occurrence. An alternative procedure is to use brakes during the control reconfiguration. In this case, all joints are locked for t_l seconds between the fault detection and the beginning of the APA configuration control phase. One disadvantage of this procedure is the necessity of turning on the brakes with the robot moving which can damage some components, mainly when it is performing high velocities movements. For the UArm II, when the brakes are turned on, there are some oscillations in the joint positions which take at least 1 second to vanish, see Fig. 2. Hence, it is chosen $t_l = 1.0 \text{ s}$. The results of this experiment can be seen in Fig. 2. The next objective is to design a controller that eliminates

the necessity of stopping the joints between the fully actuated and underactuated control phases.

4. OUTPUT FEEDBACK MARKOVIAN CONTROL

In this section, Markov theory is used to solve the free-joint fault problem described above. The dynamic model of an underactuated manipulator (5) can be represented as

$$\tau_a = \overline{M}_0(q)\ddot{q}_u + \overline{b}_0(q, \dot{q}) + \overline{\delta}(q, \dot{q}, \ddot{q}), \quad (9)$$

with $\overline{b}_0(q, \dot{q}) = \overline{C}_0(q, \dot{q})\dot{q}_u + \overline{F}_0(q)\dot{q}_u + \overline{D}_0(q, \dot{q})\dot{q}_a + \overline{G}_0(q)$. The totally actuated manipulator (2) can be represented by (9), with $q_u = q$, $\overline{M}_0(q) = M_0(q)$, $\overline{b}_0(q, \dot{q}) = b_0(q, \dot{q}) = C_0(q, \dot{q})\dot{q} + F_0\dot{q} + G_0(q)$, and $\overline{\delta}(q, \dot{q}, \ddot{q}) = \delta(q, \dot{q}, \ddot{q})$. A preliminary PD controller can be introduced in the form $\tau = [K_P \ K_D]x + u$, in order to pre-compensate model imprecisions. The linearization of (9) around an operation point with position q_0 and velocity \dot{q}_0 , is given by

$$\begin{aligned} \dot{x} &= Ax + Bu, \\ z &= C_1x + D_1u, \\ y &= C_2x, \end{aligned} \quad (10)$$

with

$$\begin{aligned} A &= \begin{bmatrix} 0 \\ -\frac{\partial}{\partial q} \left(\overline{M}_0^{-1}(q)\overline{b}_0(q, \dot{q}) \right) + \overline{M}_0^{-1}(q)K_P \\ -\overline{M}_0^{-1}(q) \left[\frac{\partial}{\partial \dot{q}} \left(\overline{b}_0(q, \dot{q}) \right) - K_D \right] \end{bmatrix} \Bigg|_{(q_0, \dot{q}_0)}, \\ B &= \overline{B}, \quad C_1 = \begin{bmatrix} \alpha I & 0 \\ 0 & 0 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0 \\ \beta I \end{bmatrix}, \\ C_2 &= \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \quad x = \begin{bmatrix} q^d - q \\ \dot{q}^d - \dot{q} \end{bmatrix}, \end{aligned}$$

where q^d is the desired trajectory, α and β are constants defined by the designer, and used to adjust the Markovian controllers.

4.1 Markovian Model

The Markovian model presented in this section describes the changes between the linearization points of the plant (10), and the probability of a fault occurrence for the 3-link manipulator UArm II (Siqueira and Terra, 2004a). In this paper, the workspace of each joint is divided in two sectors of 10° each (the set-point is defined as 20° for all joints, with initial position 0°). For each sector a linearization point is defined, 5° for the first sector and 15° for the second one. All the possible combinations to position the three joints, q_1 , q_2 , q_3 , in these two points are used to map the manipulator workspace. Then, eight linearization points, with the velocities set to zero, are found. These points are shown in Table 1. For a 3-link manipulator robot, seven possible fault configurations

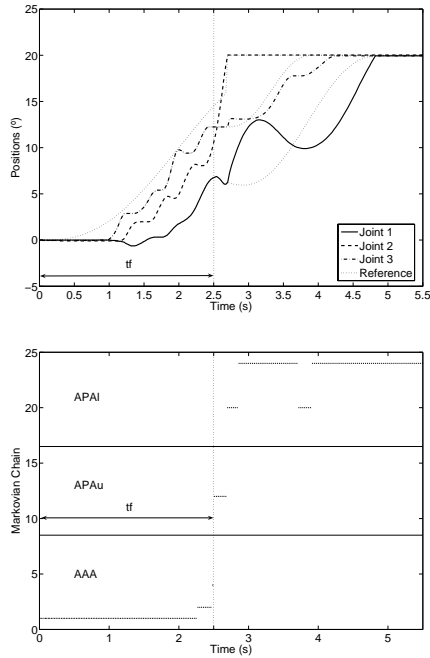


Figure 4. Joint position and Markovian chain states.

duced to verify the robustness of the controllers. For these experiments, the preliminary PD controllers were selected heuristically as

$$K_{P_{AAA}} = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.15 & 0 \\ 0 & 0 & 0.12 \end{bmatrix}, K_{D_{AAA}} = \begin{bmatrix} 0.02 & 0 & 0 \\ 0 & 0.02 & 0 \\ 0 & 0 & 0.02 \end{bmatrix},$$

$$K_{P_{APA_u}} = -2.8, \quad K_{D_{APA_u}} = -0.07,$$

$$K_{P_{APA_l}} = \begin{bmatrix} 1.25 & 0.20 \\ 0.06 & 1.30 \end{bmatrix}, \quad K_{D_{APA_l}} = \begin{bmatrix} 0.27 & 0.02 \\ 0.01 & 0.01 \end{bmatrix}.$$

The Markovian controllers were computed considering $\alpha = 50$ and $\beta = 100$ for all configurations, see (10). The best value of γ found was $\gamma = 1.5$. The transition rate matrix Λ is selected as (11) with

$$\Lambda_{AAA(ij)} = 0.09, \quad i \neq j, \quad \Lambda_{APA_u(ij)} = 0.08, \quad i \neq j,$$

$$\Lambda_{APA_l(ij)} = 0.08, \quad i \neq j, \quad \Lambda_f = 0.1I_8, \quad \Lambda_s = 2\Lambda_f,$$

and $\Lambda_{AAA(ii)} = -0.73$, $\Lambda_{APA_u(ii)} = -0.76$, and $\Lambda_{APA_l(ii)} = -0.76$. Differently from the transition probability matrix P , which has the sum of each row equal to one, the transition rate matrix Λ has the equivalent sum equal to zero. See (Ji and Chizeck, 1990) for more details. The experimental results, joint positions and Markovian chains for the output feedback \mathcal{H}_∞ Markovian controller are shown in Fig. 4. Even after the fault, the system kept the stability, with the manipulator in movement during the control reconfiguration.

5. CONCLUSION

The fault-tolerant control strategy developed in this paper, based on output feedback Markovian

control, was applied in a robot manipulator with one fault. Figure 4 shows the effectiveness of this approach. The stability is also guaranteed, even if several faults occur.

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