SYNTHESIS OF FIXED STRUCTURE CONTROLLERS FOR DISCRETE TIME SYSTEMS

Waqar A. Malik * **Swaroop Darbha** *
S.P. Bhattacharyya **

* *Department of Mechanical Engineering, Texas A&M University, College Station, TX 77843*
** *Department of Electrical Engineering, Texas A&M University, College Station, TX 77843*

Abstract: In this paper, we develop a linear programming approach to the synthesis of stabilizing fixed structure controllers for a class of linear time invariant discrete-time systems. The stabilization of this class of systems requires the determination of a real controller parameter vector (or simply, a controller), K, so that a family of real polynomials, affine in the parameters of the controllers, is Schur. An attractive feature of the paper is the exploitation of the interlacing property of Schur polynomials (based on the characterization in terms of Tchebyshev polynomials) to systematically generate an arbitrarily large large number of sets of linear inequalities in K. The union of the feasible sets of linear inequalities provides an approximation of the set of all controllers, K, which render $P(z, K)$ Schur. Illustrative examples are provided to show the applicability of the proposed methodology.

Keywords: Control design, Controllers, Stabilization, polynomials, discrete-time systems

1. INTRODUCTION

There is renewed interest in the synthesis of fixed-order stabilization of a linear time invariant dynamical system. Surveys by (Syrmos *et al.*, 1997; Bernstein, 1992) show that this problem has attracted significant attention over the last four decades. Application of fixed-order stabilization problem can be found in the work of (Buckley, 1995; Zhu *et al.*, 1995; Bengtsson and Lindahl, 1974). This problem may be simply stated as follows: Given a finite-dimensional LTI dynamical system, is there a stabilizing proper, rational controller of a given order (a causal controller of a given state-space dimension)? The set of all the stabilizing controllers of fixed order is the *basic* set in which all design must be carried out.

Given the widespread use of fixed-order controllers in various applications (see (Goodwin *et al.*, 2001), Ch. 6), it is important to understand whether fixed-order controllers that achieve a specified performance exist and if so, how one can find them and compute the set of all such stabilizing controllers that achieve a specified performance. Unfortunately, the standard optimal design techniques result in controllers of higher order, and provide no control over the order or the structure of the controller. Moreover, the set of all fixed order/structure stabilizing controllers is *non-convex* and in general, *disconnected* in the space of controller parameters, see (Ackermann, 1993). This is a major source of difficulty in its computation.

In this paper, we focus on the problem of determining the set of all real controller parameters, $K = (k_1,\ k_2, \dots k_l)$, which render a real polynomial Schur, where each member of the set is of the form:

$$P(z, K) = P_o(z) + \sum_{l=1}^{N} k_l P_l(z).$$

A good survey of the attempts to solve the fixed order control problem and the related Static Output Feedback (SOF) problem is given in (Syrmos *et al.*, 1997; Blondel *et al.*, 1995; Bernstein, 1992). (Henrion *et al.*, 2003) combine ideas from Strict Positive Realness(SPRness), positive polynomials written as sum of squares (SOS) and LMIs to solve the problem of robust stabilization with fixed order controllers. The LMI approach for synthesizing a Static Output Feedback (SOF) controller is also explored in (Ghaoui *et al.*, 1997; Iwasaki and Skelton, 1995).

(Datta *et al.*, 2000) used the Hermite-Biehler theorem for obtaining the set of all stabilizing PID controllers for SISO plants. Discrete-time PID controller using Tchebyshev representation and by using interlacing property of Schur polynomial has been designed by (Keel *et al.*, 2003). They use root counting formulas and carry out search for the separating frequencies by exploiting the structure of the PID control problem. The interlacing property of real and complex Hurwitz polynomial has been used by (Darbha *et al.*, 2004; Malik *et al.*, Dec. 2004) to generate the set of stabilizing fixed order controllers that achieve certain specified criterion.

This paper is organized as follows: In Section 2, we deal with the Tchebyshev representation of polynomials and we provide the characterization for a polynomial, $P(z)$, to be Schur in terms of its Tchebyshev representation. Section 3, deals with the generation of outer approximation $\mathcal{S}_o$ and inner approximation $\mathcal{S}_i$ of the set of controllers $\mathcal{S}$, of a given structure, that stabilize a given linear time invariant discrete-time system. It is seen that $\mathcal{S}_i \subset S \subset S_o$. Illustrative examples of how the set of fixed structure stabilizing controllers are provided. In section 4, we provide concluding remarks.

2. TCHEBYSHEV REPRESENTATION AND CONDITION FOR A SCHUR POLYNOMIAL

Let $P(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0$ denote a real polynomial, that is the coefficients, a_i are real numbers. We are interested in determining the root distribution of $P(z)$ with respect to the unit circle. The root distribution of $P(z)$ is necessary in characterizing the set of stabilizing controllers for a discrete-time control system. In such systems, $P(z)$ could denote the characteristic polynomial of the given discrete-time control system. The stability of the system is equivalent to the condition that all root of $P(z)$ lie in the interior of the unit circle, i.e. $P(z)$ has to be a *Schur* polynomial.

2.1 Tchebyshev representation of polynomials

We need to determine the image of the boundary of the unit circle under the action of the real polynomial $P(z)$.

$$\left\{P(z) : z = e^{j\theta}, 0 \le \theta \le 2\pi\right\}$$

As the coefficients, a_i, of the polynomial $P(z)$ are real, $P(e^{j\theta})$ and $P(e^{-j\theta})$ are conjugate complex numbers. Hence, it is sufficient to determine the image of the upper half of the unit circle:

$$\left\{P(z) : z = e^{j\theta}, 0 \le \theta \le \pi\right\}$$

By using, $z^k\big|_{z=e^{j\theta}} = \cos k\theta + j \sin k\theta$, we have

$$P(e^{j\theta}) = (a_n \cos n\theta + \cdots + a_1 \cos\theta + a_0) + \\ + j(a_n \sin n\theta + \cdots + a_1 \sin\theta)$$

$\cos k\theta$ and $\sin k\theta / \sin\theta$ can be written as polynomials in $\cos\theta$ using Tchebyshev polynomials. Using $u = -\cos\theta$, if $\theta \in [0, \pi]$ then, $u \in [-1, 1]$. Now,

$$e^{j\theta} = \cos\theta + j\sin\theta = -u + j\sqrt{1-u^2}$$

Let $\cos k\theta = c_k(u)$ and $\sin k\theta/\sin\theta = s_k(u)$, where $c_k(u)$ and $s_k(u)$ are real polynomials in u and are known as the *Tchebyshev polynomials* of the first and second kind, respectively. It is easy to show that,

$$s_k(u) = -\frac{1}{k}\frac{dc_k(u)}{du}, \qquad k = 1, 2, \cdots \tag{1}$$

and that the Tchebyshev polynomials satisfy the recursive relation,

$$c_{k+1}(u) = -uc_k(u) - (1-u^2)s_k(u), \quad k = 1, 2, \cdots \tag{2}$$

Using (1) and (2), we can determine $c_k(u)$ and $s_k(u)$ for all k.

From the above development, we see that

$$P(e^{j\theta})\big|_{u=-\cos\theta} = R(u) + j\sqrt{1-u^2}T(u) =: P_c(u)$$

We refer to $P_c(u)$ as the Tchebyshev representation of $P(z)$. $R(u)$ and $T(u)$ are real polynomials of degree n and $n-1$ respectively, with leading coefficients of opposite sign and equal magnitude. More explicitly,

$$R(u) = a_n c_n(u) + \cdots + a_1 c_1(u) + a_0$$
$$T(u) = a_n s_n(u) + a_{n-1} s_{n-1}(u) + \cdots + a_1 s_1(u)$$

The complex plane image of $P(z)$ as z traverses the upper half of the unit circle can be obtained by evaluating $P_c(u)$ as u runs from -1 to +1.

2.2 Root distribution

Let $\phi_P(\theta) := \arg[P(e^{j\theta})]$ denote the phase of $P(z)$ evaluated at $z = e^{j\theta}$ and let $\Delta_{\theta_1}^{\theta_2}[\phi_P(\theta)]$ denote the net change in phase of $P(e^{j\theta})$ as θ increases from θ_1 to θ_2. Similarly, let $\phi_{P_c}(\theta) := \arg[P_c(u)]$ denote the phase of $P_c(u)$ and $\Delta_{u_1}^{u_2}[\phi_{P_c}(u)]$ denote the net change in phase of $P_c(u)$ as u increases for u_1 to u_2.

Lemma 1. *Let the real polynomial $P(z)$ have i roots in the interior of the unit circle, and no roots on the unit circle. Then*

$$\Delta_0^{\pi}[\phi_P(\theta)] = \pi i = \Delta_{-1}^{+1}[\phi_{P_c}(u)]$$

Proof. From geometric considerations it is easily seen that each interior root contributes 2π to $\Delta_0^{2\pi}[\phi_P(\theta)]$ and therefore because of symmetry of roots about the real axis the interior roots contribute $i\pi$ to $\Delta_0^{\pi}[\phi_P(\theta)]$. The second equality follows from the Tchebyshev representation described above. $\nabla\nabla\nabla$

2.3 Characterization of Schur polynomial in terms of its Tchebyshev representation

Let $P(z)$ be a real polynomial of degree n. This polynomial will be said to be *Schur* if all n roots lie within the unit circle. In this subsection, we characterize the Schur polynomial in terms of its Tchebyshev representation, $P(e^{j\theta}) = \tilde{R}(\theta) + j\tilde{T}(\theta) = R(u) + j\sqrt{1-u^2}T(u)$, where $u = -\cos\theta$. $R(u)$ and $T(u)$ are real polynomials of degree n and $n-1$, respectively.

Theorem 1. *$P(z)$ is Schur if and only if*

(1) $R(u)$ has n real distinct zeros r_i, $i = 1, 2, \cdots, n$ in $(-1, +1)$
(2) $T(u)$ has $n-1$ real distinct zeros t_j, $j = 1, 2, \cdots, n-1$ in $(-1, +1)$
(3) the zeros r_i and t_j interlace, i.e

$$-1 < r_1 < t_1 < r_2 < t_2 < \cdots < t_{n-1} < r_n < +1$$

Proof. Let

$$t_j = -\cos\alpha_j, \quad \alpha_j \in (0, \pi), \quad j = 1, 2, \cdots, n-1$$

or

$$\alpha_j = \cos^{-1}(-t_j), \quad j = 1, 2, \cdots, n-1,$$
$$\alpha_0 = 0, \qquad \alpha_n = \pi$$

and let

$$\beta_i = \cos^{-1}(-r_i), \quad i = 1, 2, \cdots, n, \quad \beta_i \in (0, \pi)$$

Then $(\alpha_0, \alpha_1, \cdots, \alpha_n)$ are the $n+1$ zeros of $\tilde{T}(\theta)$ and $(\beta_1, \beta_2, \cdots, \beta_n)$ are the n zeros of $\tilde{R}(\theta)$. the third condition means that α_i and β_j satisfy

$$0 = \alpha_0 < \beta_1 < \alpha_1 < \cdots < \beta_{n-1} < \alpha_n = \pi$$

This condition means that the plot of $P(e^{j\theta})$ for $\theta \in [0, \pi]$ turns counter-clockwise through exactly $2n$ quadrants. Therefore,

$$\Delta_0^{\pi}[\phi_P(\theta)] = 2n.\frac{\pi}{2} = n\pi$$

and this condition is equivalent to $P(z)$ having n zeros inside the unit circle. $\nabla\nabla\nabla$

3. SYNTHESIS OF SET OF STABILIZING CONTROLLERS

In this section, we seek to exploit the Interlacing Property (IP) of Schur polynomials to systematically generate inner and outer approximation of the set of stabilizing controllers, $\mathcal{S}$. This approach leads to sets of Linear Programs(LPs).

Let $P(z, K)$ be a *real* closed loop characteristic polynomial whose coefficients are affinely dependent on the design parameters K; one can define the Tchebyshev representation through $P(e^{j\theta}, K) = R(u, K) + j\sqrt{1-u^2}T(u, K)$, where $u = -\cos\theta$. $R(u, K)$ and $T(u, K)$ are real polynomials of degree n and $n-1$, respectively and are affine in the controller parameter K. The leading coefficients of $R(u, K)$ and $T(u, K)$ are of opposite sign and are of equal magnitude.

3.1 Inner Approximation

The stabilizing set of controllers, $\mathcal{S}$ is the set of all controllers, K, that simultaneously satisfy the conditions of Theorem 1. The problem of rendering $P(z, K)$ Schur can be posed as a search for $2n-2$ values of u. By way of notation, we represent the polynomials $R(u, K)$ and $T(u, K)$ compactly in the following form:

$$R(u, K) = \begin{bmatrix} 1 & u & \cdots & u^n \end{bmatrix} \Delta_R \begin{bmatrix} 1 \\ K \end{bmatrix} \tag{3}$$

$$T(u, K) = \begin{bmatrix} 1 & u & \cdots & u^{n-1} \end{bmatrix} \Delta_T \begin{bmatrix} 1 \\ K \end{bmatrix} \tag{4}$$

In (3) and (4), Δ_R and Δ_T are real constant matrices that depend on the plant data and the structure of the controller sought; they are respectively of dimensions $(n+1)\times(l+1)$ and $(n)\times(l+1)$, where, n is the degree of the characteristic polynomial and l is the size of the controller parameter vector. For $i = 1, 2, 3, 4$, let C_i and S_i be diagonal matrices of size $2n$; for an integer m, the $(m+1)^{st}$ diagonal entry of C_i is $\cos(\frac{(2i-1)\pi}{4} + \frac{m\pi}{2})$ and the corresponding entry for S_i is $\sin(\frac{(2i-1)\pi}{4} + \frac{m\pi}{2})$.

For any given set of $2n-2$ distinct values of u, $-1 = u_0 < u_1 < \cdots < u_{2n-2} < u_{2n-1} = 1$, and for any integer m define a Vandermonde-like matrix, $V(u_0, u_1, \ldots, u_{2n-1}, m)$, as:

$$V(u_0, u_1, \ldots, u_{2n-1}, m) := \begin{bmatrix} 1 & u_0 & \ldots & u_0^m \\ 1 & u_1 & \ldots & u_1^m \\ 1 & u_2 & \ldots & u_2^m \\ \vdots & \vdots & \vdots & \vdots \\ 1 & u_{2n-1} & \ldots & u_{2n-1}^m \end{bmatrix}$$

We are now ready to characterize the set of stabilizing controllers K in terms of the $(2n-2)$ values:

Theorem 2. *There exists a real control parameter vector $K = (k_1,\ k_2,\ \cdots,\ k_l)$ so that the real polynomial $P(z, K)$*

$$\begin{aligned} P(z, K) &:= P_0(z) + k_1 P_1(z) + \ldots + k_l P_l(z) \\ &= p_n(K) z^n + p_{n-1}(K) z^{n-1} + \cdots + p_0(K) \end{aligned}$$

is Schur if and only if there exists a set of $2n-2$ values, $-1 = u_0 < u_1 < u_2 < \cdots < u_{2n-2} < u_{2n-1} = 1$, so that one of the following two Linear Programs (LPs) is feasible:
LPs :

$$C_k V(u_0, u_1, \ldots, u_{2n-1}, n) \Delta_R \begin{bmatrix} 1 \\ K \end{bmatrix} > 0$$
$$S_k V(u_0, u_1, \ldots, u_{2n-1}, n-1) \Delta_T \begin{bmatrix} 1 \\ K \end{bmatrix} > 0$$

for k=1,3.

Proof. The three conditions of **Theorem 1** is equivalent to the existence of 2n-2 values of u, $-1 < u_1 < u_2 < \cdots < u_{2n-2} < 1$ such that the roots of the Tchebyshev polynomial $R(u, K)$ lie in

$$(-1, u_1), (u_2, u_3), (u_4, u_5)$$

while the roots of the other Tchebyshev polynomial $T(u, K)$ lie in

$$(u_1, u_2), (u_3, u_4), (u_5, u_6)$$

If $R(-1, K) > 0, T(-1, K) > 0$, then the placement of roots will require

$$R(u_1, K) < 0, R(u_2, K) < 0, R(u_3, K) > 0, \cdots$$

and

$$T(u_1, K) > 0, T(u_2, K) < 0, T(u_3, K) < 0, \cdots$$

In other words, the signs of $R(u_i, K)$ and $T(u_i, K)$ are the same as that of $\cos(\frac{\pi}{4} + i\frac{\pi}{2})$ and $\sin(\frac{\pi}{4} + i\frac{\pi}{2})$ respectively. This corresponds to the **LP** for $k = 1$. Similarly for $R(-1, K) < 0$ and $T(-1, K) < 0$ we have the **LP** corresponding to $k = 3$. $\nabla\nabla\nabla$

The essential idea is that the plot of the polynomial $P(e^{j\theta})$ must go through $2n$ quadrants in the counterclockwise direction as θ increases from 0 to π. The conditions given above correspond to the plot starting in the k^{th} quadrant at $\theta = 0$.

The procedure to find the inner approximation is to partition the interval $(-1, 1)$ using more than $(2n-2)$ points (either uniformly or by using appropriate Tchebyshev polynomial) and systematically searching for the feasibility of the obtained set of linear inequalities. Every feasible LP, yields a controller K which makes the polynomial $P(z, K)$ Schur. The union of all the feasible sets of the LPs described above, for all possible sets of $(2n-2)$ points in $(-1, 1)$ is the set of all stabilizing controllers of the given structure. With partitioning $(-1, 1)$, however, one will be able to capture only finitely many of the possible sets of $(2n-2)$ points, $u_1, \ldots, u_{2n-2}$. The feasible sets of the LPs corresponding to these finitely many possible sets will provide an *inner approximation* of the set of all stabilizing controllers. This approximation can be made more accurate by refining the partition - i.e., if K is a stabilizing controller not in the approximate set, then there is refinement [which will separate the roots of $R(u, K)$ and $T(u, K)$] of the partition from which one can pick $2n-2$ points so that one of the four LPs corresponding to these points is feasible. This is the basic procedure for finding the inner approximation.

3.2 Outer Approximation

In the previous subsection, we outlined a procedure to construct LPs whose feasible set is contained in $\mathcal{S}$. Their union $\mathcal{S}_i$ is an inner approximation to $\mathcal{S}$. For computation, it is useful to develop an outer approximation, $\mathcal{S}_o$ that contains $\mathcal{S}$. In this subsection, we will present how to construct an arbitrarily tight outer approximation $\mathcal{S}_o$ as a union of the feasible sets of LPs. We propose to use the Poincare's generalization of Descartes' rule of signs.

Poincare's Generalization: *The number of sign changes in the coefficients of $Q_k(x) := (x+1)^k P(x)$ is a non-increasing function of k; for a sufficiently large k, the number of sign changes in the coefficients equals the number of real, positive roots of $Q(x)$.*
The proof of the generalization due to Poincare is given in (Polya and Szego, 1998).

For the sake of a discussion on outer approximation, we will treat the polynomials, $\hat{R}(\lambda, K)$ and $\hat{T}(\lambda, K)$, as polynomials in λ obtained through the bijective mapping $\lambda = \frac{1+u}{1-u}$. This maps the interval $(-1, +1)$ into the interval $(0, \infty)$. This mapping is applied in the following way:

$$(1+\lambda)^n Q\left(\frac{\lambda - 1}{1 + \lambda}\right) = \hat{Q}(\lambda)$$

The i^{th} roots of $\hat{R}(\lambda, K)$ and $\hat{T}(\lambda, K)$ be represented as $\lambda_{r,i}$ and $\lambda_{t,i}$ respectively. Since the polynomials $\hat{R}$ and $\hat{T}$ must have respectively n and $n-1$ real, positive roots, an application of Poincare's result to the polynomials $\hat{R}$ and $\hat{T}$ yields the following:

Lemma 1. *If K is a stabilizing control vector, then $(\lambda+1)^{k-1}\hat{R}(\lambda, K)$ and $(\lambda+1)^{k-1}\hat{T}(\lambda, K)$ have exactly n and $n-1$ sign changes in their coefficients respectively for every $k \geq 1$.*

The procedure in (Bhattacharyya *et al.*, 1988) corresponds to $k = 1$ of the above lemma.

The following lemma takes care of the interlacing of the roots of two polynomials:

Lemma 2. *Let K render a polynomial $P(z, K)$ Schur. Then the polynomial $\tilde{Q}(\lambda, K, \eta) = \lambda\hat{T}(\lambda, K) - \eta(1+\lambda)\hat{R}(\lambda, K)$ has exactly n real positive roots for all $\eta \in \mathcal{R}$.*

Proof. In the interest of saving space, we only provide a sketch of the proof. The roots $\hat{T}(\lambda, K)$ and $\hat{R}(\lambda, K)$ are real and positive and they interlace *if and only if* $\tilde{Q}(\lambda, K, \eta)$ has exactly n real positive roots for all $\eta \in \mathcal{R}$. To prove sufficiency, we consider the graph of the rational function $y := \frac{\lambda\hat{T}(\lambda)}{(1+\lambda)\hat{R}(\lambda)}$ and consider the intersections with $y = \eta$. To prove necessity, we argue, via a root locus, that if the interlacing of real roots condition is violated, then for some value of $\eta \in \Re$, polynomial $\tilde{Q}(\lambda, K, \eta)$ will have at least a pair of complex conjugate roots. $\nabla\nabla\nabla$

Lemmas 1 and 2 can be put together to show that an arbitrarily tight outer approximation can be constructed.

Example 1 Consider a plant

$$G(z) = \frac{z^2 - 2z + 1}{1.9z^2 + 2.1}.$$

It is desired to calculate the complete set of first order controllers of the form

$$C(z) = \frac{k_1(z-1)}{z + k_2}$$

The characteristic equation is given by

$$(1.9 + k_1)z^3 + (1.9k_2 - 3k_1)z^2 + (2.1 + 3k_1)z + (2.1k_2 - k_1)$$

The Tchebyshev polynomials is found to be:

$$R = -(7.6 + 4k_1)u^3 + (3.8k_2 - 6k_1)u^2 + 3.6u + 2k_1 + .2k_2$$
$$T = (7.6 + 4k_1)u^2 + (6k_1 - 3.8k_2)u + (2k_1 + 0.2)$$

An inner and outer (black color) approximation of the set of gains is shown in Fig. 1. The inner approximation finds an excellent approximation of the complete set of stabilizing controllers.

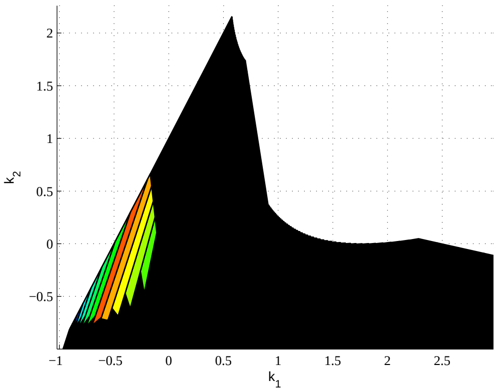

Fig. 1. Set of stabilizing controllers - An inner and outer approximation

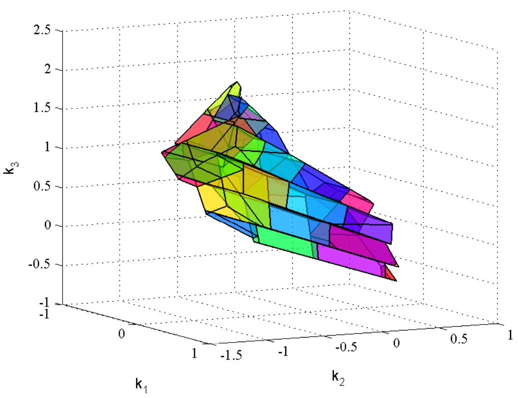

Fig. 2. Solution for Example 2: - An inner approximation

Example 2 Consider a plant:

$$G(z) = \frac{1}{z^2 - 0.25}$$

The controller is considered to be of the following structure:

$$C(z) = \frac{k_3z^2 + k_2z + k_1}{z^2 - z}$$

An inner approximation of the set of controllers is shown in Fig. 2. An inner and outer (yellow color) approximation of the set of controllers is shown in Fig. 3.

4. CONCLUSIONS

In this paper, we consider the problem of the synthesis of fixed order and structure controllers, where the coefficients of the closed loop polynomial are linear in the parameters of the controller. A novel feature of this paper is the systematic exploitation of the interlacing property of Schur polynomials and the use of Poincare's generalization of Descartes' rule of signs to generate LPs in the parameters of a fixed order controller. For real stabilization, the feasible set of any LP generated

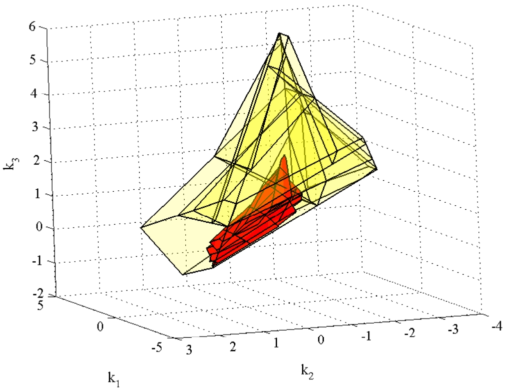

Fig. 3. Set of stabilizing controllers - An inner and outer approximation

for an inner approximation of the set of all stabilizing controllers, can be indexed by a set of $2n-2$ increasing values, $-1 = u_0 < u_1 < u_2 < \cdots < u_{2n-2} < u_{2n-1} = 1$; in particular, any controller in the feasible set of LPs places the roots of the Tchebyshev polynomials of $P(z, K)$ alternately in the intervals (u_i, u_{i+1}), $i = 0, \ldots, 2n-1$. The problem of inner approximation of the set of stabilizing controllers is then posed as the search for all sets of ordered $2n-2$-tuples of points for which the associated LP is feasible; the union of all feasible LPs is an inner approximation for the set of all stabilizing controllers. The proposed methodology naturally extends to the computation of the set of simultaneously stabilizing controllers. We provide examples to illustrate some of the results derived in this paper.

Recent solutions to the PID controller design problem (Datta *et al.*, 2000) requires the even and odd parts of a polynomial to have certain patterns of root separation. In (Datta *et al.*, 2000), the authors carry out a search for the separating frequencies by exploiting the structure of the PID control problem. This method is dependent on the *special* structure of the controller. The method proposed in this paper can find an inner and an outer approximation to the set of stabilizing controllers of any fixed order or structure. To illustrate the methodology developed in this paper, we have used simple examples in which the set of stabilizing controllers can be shown.

REFERENCES

Ackermann, J. (1993). *Robust Control Systems with Uncertain Physical Parameters.* Springer Verlag. Berlin.

Bengtsson, G. and S. Lindahl (1974). A design scheme for incomplete state or output feedback with applications to boiler and power system control. *Automatica* **No.10**, 15–30.

Bernstein, D. (1992). Some open problems in matrix theory arising in linear systems and control. *Linear Algebra and Applications* **No.162-164**, 409–432.

Bhattacharyya, S. P., L. H. Keel and J. Howze (1988). Stabilizability conditions using linear programming. *IEEE Transactions on Automatic Control* **33**, 460–463.

Blondel, V., M. Gevers and A. Lindquist (1995). Survey on the state of systems and control. *European Journal of Control* **1**, 5–23.

Buckley, A. (1995). Hubble telescope pointing control system design improvement study. *Journal of Guidance, Control and Dynamics* **18**, 194–199.

Darbha, S., S. Pargaongkar and S. P. Bhattacharya (2004). A linear programming approach to the synthesis of fixed structure controllers. *American Control Conference.*

Datta, A., M. T. Ho and S. P. Bhattacharyya (2000). *Structure and Synthesis of PID Controllers.* Springer-Verlag. London.

Ghaoui, L. El, F. Oustry and M. AitRami (1997). A cone complementarity linearization algorithm for static output feedback and related problems. *IEEE Transactions on Automatic Control* **42-8**, 1171–1176.

Goodwin, G. C., S. F. Graebe and M. E. Salgado (2001). *Control System Design.* Prentice-Hall. Upper Saddle River, NJ.

Henrion, D., M. Sebek and V. Kucera (2003). Positive polynomials and robust stabilization with fixed-order controllers. *IEEE Transactions on Automatic Control* **48-7**, 1178–1186.

Iwasaki, T. and R. E. Skelton (1995). The xy-centering algorithm for the dual lmi problem: A new approach to fixed order control design. *International Journal of Control* **62-6**, 1257–1272.

Keel, L. H., J. I. Rego and S. P. Bhattacharyya (2003). A new approach to digital pid controller design. *IEEE Transactions on Automatic Control* **48-4**, 687–692.

Malik, W. A., S. Darbha and S. P. Bhattacharya (Dec. 2004). On the synthesis of fixed structure controllers satisfying given performance criteria. *2nd IFAC symposium on system, structure and control.*

Polya, G. and G. Szego (1998). *Problems and Theorems in Analysis II- Theory of Functions, Zeros, Polynomials, Determinants, Number Theory, Geometry.* Springer Verlag. Berlin-Heidelberg.

Syrmos, V. L., C. T. Abdullah, P. Dorato and K. Grigoriadis (1997). Static output feedback - a survey. *Automatica* **33-2**, 125–137.

Zhu, G., K. Grigoriadis and R. Skelton (1995). Covariance control design for the hubble space telescope. *Journal of Guidance, Control and Dynamics* **18, No. 2**, 230–236.