ELECTROACTIVE SMART MATERIALS: NEW CHALLENGES FOR CONTROL

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Abstract: In this paper, the main objective is to provide an overview of a new class of research problems for the area which is broadly termed *smart structures control*. To this end, we describe *electroactive smart materials* and focus on control issues which naturally arise. Such materials, typically nano and micro-polymers, have mechanical properties controlled by an electric field. Unlike existing approaches to control such as those based on piezoelectric actuation, the electroactive materials described here do not require electromechanical apparatus for actuation; i.e., they are self-actuating. Within this setting, in addition to basic modelling issues, a paradigm control problem is also formulated: the so-called *sustained oscillation problem* for an electroactive mass-spring-damper system. For a given electric field E, this formulation leads to a simple nonlinearity involving the product of the modulus of elasticity $\kappa(E)$ and the displacement y. Subsequently, we obtain a bilinear state equation $\dot{x} = A(u)x$ whose control, we conjecture, is highly non-trivial. Within this context, we consider the use of feedback to vary the electric field so as to assure that the desired oscillation is guaranteed with the least possible actuation effort.

1. Introduction

This paper is part of an emerging area of research on dynamics and control of smart structures. Traditionally, in a smart material or structure, the active elements are typically embedded or attached to conventional materials. In contrast, this paper introduces *electroactive smart materials*, describes their self-sensing and self-actuating properties and formulates a first paradigm problem for consideration. Generally speaking, the area of smart structures and materials involves the application of multi-functional capabilities to materials which can sense or respond to external stimuli. That is, these materials exhibit self-sensing or self-actuation. In addition, such materials or structures respond to stimuli in a manner which is directed towards satisfaction of some prescribed performance specification.

It is only quite recently that the control field has started to pay attention to smart structures and smart materials. In this regard, the reader is referred to the recent special issue of *IEEE Control Systems Technology*, see [1] , where an overview of the area is provided; see also [2] where a survey is provided for active noise and vibration control problems. More generally, the area of smart materials research has already received considerable attention in a number of disciplines. This includes polymer science, materials research and engineering efforts involving various applications such as active noise suppression; for example, see [3]-[5] and [7]. To add to the existing literature in the control area, this paper concentrates on smart materials with self-actuation properties which can function in diverse environments such as biological solutions; e.g., see [6]. These materials, typically new classes of micro or nano composite polymers, are characterized by the fact that their mechanical properties are controlled by an applied electric field. For example, as depicted in Figure 1, materials can be locally tailored to enhance electroactive response needed for active damping or sustenance of vibrations.

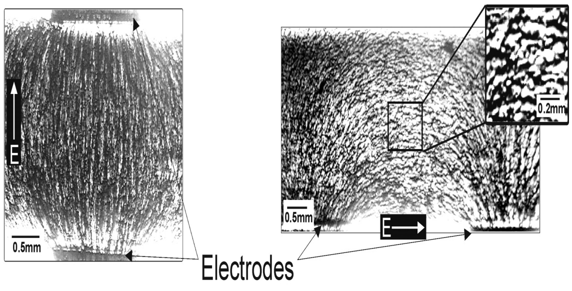

Figure 1: Field-Aided Micro-Tailoring (FAiMTA) technology enables multi-functional materials having variable micro-structure. This enhances self-sensing and self-actuation.

In this paper, we demonstrate that this research area motivates novel and challenging nonlinear control problems. To this end, we introduce a simple paradigm control problem which is quite easy to state but we believe complicated to solve: Given a classical second order mass-spring-damper system with adjustable elastic modulus κ serving as the control variable, the problem is to find the "minimum actuator" required so that sustained oscillations can be guaranteed at the undamped natural frequency of the system? Interestingly, even for this simple second order system, we believe the solution of this problem is quite challenging. In the sequel, this so-called *sustained oscillation problem* is formally defined noting that our intent in this paper is not to claim that this problem should necessarily be a focal point of this new research area; our objective is to use this simple example to motivate the many challenging problems which this area has to offer.

While not covered in this paper, it is also possible to consider material optimization problems concurrently with the control problem. That is, how should the material design parameters be optimized so that the ensuing control algorithms are maximally efficacious? Given the laboratory facilities which we have available, our plan is to focus future work in this closely related topic area. Consideration of such concurrent design problems might lead to an entirely new area of research which might appropriately be called *material-oriented control.*

1.1 Limitations of Traditional Sensing and Actuation: To illustrate limitations which one may encounter in traditional sensing and actuation, we consider the problem of sensing small variations of mass in a fluid. In such a case, a traditional approach for sensing, based on resonance frequency shifts of an oscillating micro or nano-cantilever, has one deficiency: While the quality factor Q, of an oscillating micro-cantilever vibrating in air is approximately in the 30 to 100 range, this value drops dramatically in a liquid environment. In addition, in this setting, the classical approach to sensing suffers from the problem that electrodes can be screened by ions in the liquid. In view of the above, traditional methods for actuation of cantilever beams have very limited application at the micro and nano levels.

Traditional sensing and actuation methods include the use of piezoresistive transducers. For small scale systems used in biomedical applications, heat dissipation, compounded with the electromechanical nature of actuation and associated weight and cost considerations, renders this approach particularly problematic. Another traditional sensing approach requires thick piezoelectric films, which are not compatible with nanoscale technology. Finally, the so-called capacitance method, while successfully used for nanocantilevers working in vacuum or inert gases is not appropriate for an air or liquid environment. In contrast, the use of smart self-sensing and self-actuating electroactive materials presents few obstacles. Multi-functional materials used for this purpose have better weight and energy conversion qualities than traditional systems with discrete material, actuator and sensor elements.

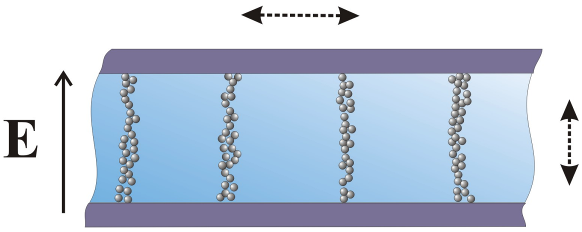

Figure 2: If electric field is directed along a material structure, then the shear elastic modulus is proportional to the applied field; this is called electrorheology. If the electric field causes compressive stress and material deformation in the vertical direction, this effect is called electrostriction.

1.2 Example (Mass Detection in Biological Solutions): Within a control setting, a typical question is: Using feedback, how does one vary the applied electric field E so that the material's dynamic response meets specifications? For example, in the case of a cantilever sensor immersed in a biological solution as depicted in Figure 3, a typical specification is that the control algorithm assures a sustained frequency of vibration.

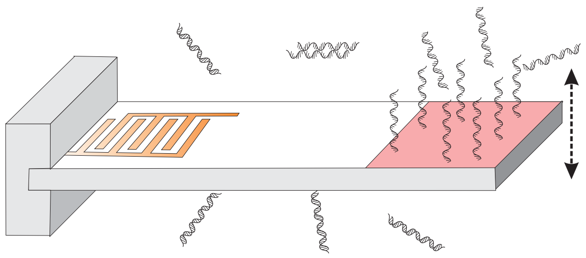

Figure 3: In this illustrative example of bio-sensing, a multi-functional nano-cantilever uses inter-digited electrodes for actuation and/or sensing. These electrodes are isolated from the surrounding ionic solution and utilize electro-active response of the beam material. Species of interest attach to the modified cantilever tip changing its mass and natural frequency.

When the DNA strands adhere to the sensor, a change in mass results. In turn, this leads change in the natural frequency of vibration. Subsequently, in this context, a fundamental control problem involves varying the electric field E so as to "cancel" the damping; i.e., we seek to obtain a system which is oscillating at the undamped natural frequency ω_0 of the "mass augmented" system. Alternatively, a less restrictive version on this problem allows for a time-varying frequency of oscillation whose average is ω_0. By comparing the frequency of this mass augmented system with that of the "zero mass" system, it is possible to obtain an estimate of the DNA mass. A similar concept can be applied to chemical and pollution sensing and a wide range of other environmental applications. In Section 3, we formally describe the so-called sustained oscillations problem — a fundamental control problem which captures a number of the issues described above.

1.3 Idealizing Assumptions: In the sequel, whenever convenient, we introduce some simplifying assumptions. In addition to making the exposition more transparent, there are two additional reasons for imposition of these assumptions: First in a simplified framework, it is easy to demonstrate how smart materials give rise to fundamental control problems without all details of the full fledged application. That is, we describe basic issues in control science which are of interest in their own right. Second, it is felt that once the basic problems are formulated, their more general counterparts are rather obvious. In summary, our objective here is to motivate new problems — not solve them.

Indeed, we henceforth assume that the mass-spring-damper system under consideration describes a cantilever with all its effective mass localized at the tip of the beam. Hence, a classical second order ordinary differential equation is used in lieu of a more general distributed parameter model. Second, we assume perfect sensors providing the beam displacement and velocity. In a more general setting, one might entertain noisy sensors which provide the beam kinematics with different degrees of the accuracy. Finally, we also neglect disturbances such as variation in friction of the surrounding environment and variation of the mass which is attached to the beam.

1.4 Mechanical Parameters: In this paper, the fundamental control problem involves time-variation of the electric field E so as to change the elastic modulus $\kappa(E)$ and the damping $c(E)$ of the cantilever beam. That is, using state feedback, the problem is to vary the field E so as to satisfy a given performance specification. Since these mechanical parameters can only be varied by a limited amount, this leads to questions about the size of the actuator needed to assure satisfaction of system performance specifications; see Section 3 for details.

1.5 Plan for Paper: The plan for the remainder of this paper is as follows: In Section 2, we formulate a simple nonlinear state equation model based on the considerations discussed above. To this end, we model the relationship between applied electric field E and resultant properties of the material. Subsequently, taking these considerations into account, we arrive at an "idealized" scenario involving a second order system. In Section 3, we formulate the paradigm problem of *sustained oscillation* in this smart materials context. In Section 4, exploiting the bilinear structure of the nonlinear system, we outline two approaches to control. Surprisingly, even for the simple second order system model which we consider, a solution to the sustained oscillation problem is not straightforward to obtain. Finally in Section 5, conclusions are drawn and future research directions are discussed.

2. Nonlinear State Space Modelling

To consider a cantilever beam with effective mass assumed to be concentrated at the tip, we begin by modelling small deflections y with a simple second order mass-spring-damper equation. However, in contrast to classical control theoretic formulations for such a system, considerations of self-actuation dictate that we treat the beam stiffness κ as time varying; i.e., it is determined by the applied electric field E which serves as the control variable. In addition, our model includes a time-varying load $f(E)$. Assuming for simplicity an effective beam mass $M = 1$, we arrive at the second order system equation

$$\frac{d^2y}{dt^2} + c(E)\frac{dy}{dt} + \kappa(E)y = f(E).$$

where $c(E)$ and $\kappa(E)$ are respectively nonlinear functions representing the damping and elastic modulus of the beam.

2.1 Saturation Effects: For the mechanical parameters above, an increasing electric field E leads to increasing values of $\kappa(E)$ and $c(E)$ until a limit is reached; i.e., at high electric fields, these parameters reach saturation values and hardly change. Accordingly, the variation of these parameters, consistent with their observed behavior, is modelled as

$$\kappa(E) = \kappa_0(1 + \gamma(E));$$

and

$$c(E) = c_0(1 + \delta(E))$$

where $\gamma(E)$ and $\delta(E)$ are non-negative monotonically increasing functions defined for $E \geq 0$. In addition, these functions satisfy the *zero electric field* condition

$$\gamma(0) = \delta(0) = 0$$

and the *saturation condition*

$$\lim_{E\to\infty} \gamma(E) = \gamma_{max}; \quad \lim_{E\to\infty} \delta = \delta_{max}$$

where γ_{max} and δ_{max} are finite. Finally, it is noted that $\gamma(E)$ and $\delta(E)$ above correspond to percentage increases of the elastic modulus and damping respectively. A typical example of such a γ-function satisfying the conditions above would be

$$\gamma(E) = \gamma_{max}(1 - e^{-E^2}).$$

Notice that the saturation of $\gamma(E)$ above translates into a saturation constraint on the controller.

2.2 Damping and Crosstalk: For most materials of practical interest, it is often the case that the electric field can have large effect on the elastic modulus and little or no effect on the damping. Therefore, in the sequel, for simplicity, we take $\delta_{max} = 0$ which results in fixed damping $c(E) \equiv c_0$ for all levels of the applied electric field E. While not considered here, a more general setting involves inclusion of parameter uncertainty in c_0.

To complete the modelling for the case at hand, we refer to Figure 4 where both the mass-spring-damper with corresponding self-actuating smart material is shown. For the so-called *zero crosstalk* case depicted in the figure, the decoupling between $\kappa(E)$ and $f(E)$ leads to our imposition of the assumption that $f(E) = 0$. In future work, it would be desirable to consider the more general case when $f(E) \neq 0$ and the issue of robustness with respect to parametric uncertainty in c_0. In this regard, smart materials there many problems one can formulate involving the optimization of structure and geometry.

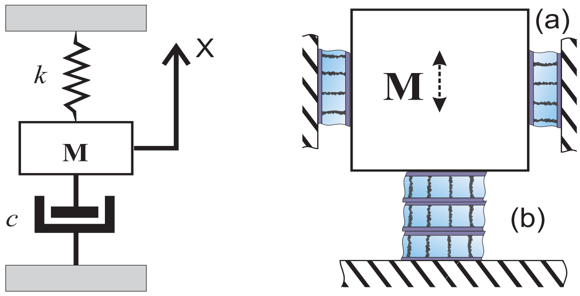

Figure 4: Depiction of classical mass-spring-damper with corresponding smart material actuation. In the figure, (a) corresponds to field controllable stiffness $\kappa(E)$, often called electro-rheology, and (b) corresponds to a field controllable forcing function, often called electrostriction. In this example, the two activation mechanisms are decoupled. More generally, coupled mechanisms and their control implications could be studied.

2.3 Controller Simplification: Since the applied electric field enters the system via the invertible function $\gamma(E)$, without loss of generality, we take $u = \gamma$ as the control with the constraint $0 \leq u(t) \leq \gamma_{max}$. As far as physical realization is concerned, for control value $\gamma = \gamma_0$, the corresponding applied voltage is

$$E = \gamma^{-1}(\gamma_0).$$

2.4 State Space Reformulation: As a first step, we reformulate the model using the state variables $x_1 = y; \quad x_2 = dy/dt$. We now obtain the nonlinear state equations $\dot{x} = A(u)x$ where

$$A(u) \doteq \begin{bmatrix} 0 & 1 \\ -\kappa_0(1+u)) & -c_0 \end{bmatrix}.$$

In turn, we decompose the system above into a linear and nonlinear part by writing $\dot{x} = (A_0 + A_1 u)x$ with

$$A_0 \doteq \begin{bmatrix} 0 & 1 \\ -\kappa_0 & -c_0 \end{bmatrix}; \quad A_1(u) \doteq \begin{bmatrix} 0 & 0 \\ -\kappa_0 & 0 \end{bmatrix} u.$$

In the sequel, the objective is to develop a feedback control law $u = \gamma(x)$ noting that it is desirable to have the control either independent of

κ_0 or insensitive to variations in κ_0 about some nominal value.

3. Sustained Oscillation Problem

As indicated earlier, we now describe a paradigm problem to demonstrate typical nonlinear control issues which may arise in the smart materials context of this paper. Our objective is to find the smallest possible actuator bound, denoted it by

$$\gamma_{max} = \gamma^*_{max},$$

having the following property: For this bound, there exists a corresponding feedback control law

$$u^* = \gamma^*(x)$$

such that for every initial condition $x(0)$, the corresponding state trajectory $x(t)$ is periodic with period

$$T = \frac{2\pi}{\omega_0}$$

where

$$\omega_0 = \sqrt{\kappa_0}$$

is the undamped natural frequency corresponding to zero electric field. That is, with control given by $u = \gamma(0) = 0$, the resulting zero-field transfer function

$$H(s) \doteq \frac{1}{s^2 + c_0 s + \kappa_0}$$

has natural frequency $\omega_0 = \sqrt{\kappa_0}$. A simpler version of the sustained oscillation problem is obtained when the control bound γ_{max} is fixed. In this case, if γ_{max} is large enough to assure oscillation, we say that the system is *oscillable*.

3.1 Remarks: Before proceeding towards solution of the problems above, it is worth noting that with $E = 0$, the output $y(t)$, being the classical mass-spring-damper solution, is damped. Roughly speaking, we note that a system is oscillable if the damping can be "cancelled" using the time-varying electric field $E(t)$. From a smart materials point of view, the attainment of a periodic solution for the state is of particular importance for systems with a high Q factor. Without keeping the state at a significant level, the attenuation of $||x(t)||$ can be so rapid as to preclude accurate estimation of the frequency of oscillation.

4. Two Approaches to Solution

In this section, we outline two approaches to the solution of the sustained oscillation problem above. For typical smart materials one can obtain nominal values c_0 and κ_0 for the damping and stiffness by consideration of the classical second order transfer function

$$H(s) = \frac{1}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

corresponding to $E = 0$. Noting that $\kappa_0 = \omega_0^2$ and that $c_0 = 2\zeta\omega_0$, for typical smart materials under consideration, we work with damping in the range $0.050 \leq \zeta \leq 0.125$. This corresponds to regular cantilever beam sensing.

4.1 Damping Cancellation Approach: Considering a fixed actuator bound γ_{max}, this approach proceeds by common sense in that we try to select a feedback $\gamma(x)$ so that the term $c_0 x_2$ in the state equation is cancelled by the $\gamma(x)\kappa_0 x_1$ term. Intuitively, the argument proceeds as follows: At states for which damping cancellation is achievable, the system reduces to a pure harmonic oscillator at frequency ω_0. Noting that this cancellation is only possible if and only if $c_0 x_2 + \gamma\kappa_0 x_1 = 0$ for some $\gamma \in [0, \gamma_{max}]$, we obtain the equivalent condition

$$-\gamma_{max} \leq \frac{c_0 x_2}{\kappa_0 x_1} \leq 0.$$

For the case when

$$\frac{c_0 x_2}{\kappa_0 x_1} < -\gamma_{max},$$

applying control

$$\gamma(x) = \gamma_{max}$$

produces a partial cancellation of the damping. The final possibility is that

$$\frac{c_0 x_2}{\kappa_0 x_1} > 0.$$

Now, the use of $\gamma(x) \geq 0$ makes matters worse. To make up for undesired damping in this case and the partial cancellation case, it would be natural to "overcompensate" with the controller in the exact cancellation regime. Based on these arguments, if the system is oscillable, we conjecture that there exists a pure gain $\rho \geq 1$ such that use of the modified control

$$\gamma(x) \doteq -\rho\gamma_{max}\frac{c_0 x_2}{\kappa_0 x_1}$$

in the cancellation regime and either $\gamma_(x) = \gamma_{max}$ or $\gamma(x) = 0$ in the other regimes, we conjecture that achieve the desired result is obtained.

It is interesting to note that this intuitive controller construction leads to a result which is consistent with a more formal analysis of this system via the Poincare-Bendixson Theorem. By way of review, for second order nonlinear system $\dot{x} = f(x)$ with $f(x)$ satisfying mild regularity conditions, a necessary condition for existence of a limit cycle in some region $\mathcal{X}$ of the state space is that

$$\text{div } f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2}$$

either vanishes for some (x_1, x_2) pairs in $\mathcal{X}$ or changes sign. Therefore, to satisfy the necessary condition for oscillation, we calculate

$$\text{div } f = -c_0 - \kappa_0 \frac{\partial \gamma}{\partial x_2} x_1$$

and force violation of the Poincare-Bendixson requirement by requiring that

$$\frac{\partial \gamma}{\partial x_2} = -\frac{c_0}{\kappa_0 x_1}$$

for some region of the state space. Then by integrating above and continuing the analysis, one can reach conclusion which are identical to those obtained via the common sense approach above. By way of future research, a natural conjecture presents itself: Noting that the region in state space over which damping cancellation is possible is a cone in the second and fourth quadrants, we conjecture the the "size" of this cone, parameterized in (c_0, κ_0), indicates whether the system is oscillable or not.

4.2 Lyapunov Function Approach: Viewing a quadratic Lyapunov function $V(x))$ as a measure of energy in the system, we use the controller to try and "cancel" decay terms in the time derivative of $V(x)$. More specifically, we first select a 2×2 positive-definite symmetric matrix Q and solve the Lyapunov equation

$$A_0^T P + P A_0 = -Q$$

to obtain a positive-definite symmetric matrix solution P. Now, with Lyapunov function $V(x) = x^T P x$, we look at the time rate of change of $V(x)$ along trajectories of the system. Letting

$$\mathcal{L}(x,t) = \frac{dV}{dt}$$

denote this so-called *Lyapunov derivative*, with feedback control law $u = \gamma(x)$, a straightforward calculation yields

$$\begin{aligned} \mathcal{L}(x,u,t) &= \nabla[V^(x)]^T A(u) x \\ &= -x^T Q x + 2x^T P A_1 x \gamma(x). \end{aligned}$$

Hence, to cancel the effects of the decay term $x^T Q x$ above, a natural choice is to take $\gamma(x) > 0$ when $x^T P A_1 x > 0$ and $\gamma(x) = 0$ otherwise. Accordingly, it is natural to conjecture the following: If the control bound γ_{max} is large enough to offset the damping, the system is oscillable at frequency ω_0 using a control of the form $\gamma(x) = \rho\gamma_{max}$ with $\rho \leq 1$ being a fixed gain when $x^T P A_1 x > 0$ Combining this with the $\gamma(x) = 0$ case, we obtain

$$\gamma(x) = \rho\gamma_{max} \max\{\text{sgn}\{x^T P A_1 x\}, 0\}$$

as a hypothesized form for the controller.

5. Conclusion and Future Research

In addition to the approaches to sustained oscillation outlined above, other methods of attack are possible; e.g., methods based on Pontryagin's Minimum Principle or the Small Gain Theorem. As indicated throughout the paper, several simplifying assumptions have been made. Possible continuation of this research could involve more general problem formulations which considers factors such as distributed cantilever mass, noisy sensor measurement, optimization of sensor and actuator positions or even related application areas such as the control of two dimensional film-mirrors in space communications.

1. REFERENCES

[1] S. O. R. Moheimani and G. C. Goodwin (Guest Editors), Dynamics and Control of Smart Structures, *IEEE Transactions on Control Systems Technology*, vol 9, 2001.

[2] S. O. R. Moheimani, I. R. Petersen and H. R. Pota (Guest Editors), Emerging Technologies for Active Noise and Vibration Control Systems, *Control Engineering Practice*, vol. 12, 2004.

[3] Y. M. Shkel and D. J. Klingenberg, " Continuum Approach to Electrorheology," *Journal of Rheology*, vol. 43, pp. 1307-1322, 1999.

[4] D. J. Segalman, W. R. Witkowski, D. B. Adolf, and M. Shahinpoor, M., "Theory and Application of Electrically Controlled Polymeric Gels," *Smart Materials and Structures* vol. 1, pp 95-100, 1992.

[5] T. R. Filanc-Bowen, G. H. Kim, and Y. M. Shkel, "Novel Sensor Technology for Shear and Normal Strain Detection with Generalized Electrostriction," *Proceedings of IEEE*, pp. 1648 - 1653, Orlando, 2002.

[6] Y. M. Shkel and B. R. Barmish, "Self-Sensing Structures for Control of Micro and Nano Cantilevers," to appear in *SPIE Symposium on Smart Materials and Non-Destructive Evaluation*, San Diego, 2005.

[7] H. P. Gavin, "Control of Seismically Excited Vibration Using Electrorheological Materials and Lyapunov Methods," *IEEE Transactions on Control Systems Technology*, vol 9, pp. 22 - 36, 2001.