# AVOIDING CONTROLLER SINGULARITIES IN ADAPTIVE RECURRENT NEURAL CONTROL

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Abstract: In this paper, to overcome the controller singularity problems, a novel neural parameters adaptive law for on-line identification is proposed, such strategy avoid specific adaptive weights zero-crossing. Using a *priori* knowledge about the real plant, a recurrent neural network is proposed as identifier. Based on the neural identifier model, a discontinuous control law is derived, which combines Block Control and Sliding Modes. The proposed scheme is tested in a induction motor via simulations. *Copyright* <sup>(R)</sup>*IFAC 2005* 

Keywords: Induction Motors, Sliding Modes, Neural Control, Block Control

### INTRODUCTION

Although the large number of success applications of neural networks for control and identification systems, one important drawback of such neural approaches (Rovithakis and Christodolou, 1994), (Kosmatopoulus et. al., 1995) is the requirement of full-connected recurrent neural networks. This usually implies a large number of synaptic connections, becoming such schemes unacceptable for real time applications. To alleviate this situation, certain level of insight about the system is utilized to improve the empirical modelling. For example, in Loukianov et al. (2002), the Nonlinear Block Controllable form (NBC-form) (Loukianov, 1998) and the relative degree are taken into account to design a dynamic neural network to identify the plant; based on such neural identifier, a control law is derived combining the Block Control and Sliding Modes techniques (Utkin, 1999), yielding the so called Neural Block Control (NBC).

Comparing with others neural control techniques (see: Sanchez, Perez and Ricalde (2003) and Rovithakis and Christodolou (1994)) that require full-state full-connected neural identifiers, the NBC strategy, has the advantage that only a partial-state partially-connected neural identifier is required, reducing significantly the mathematical analysis and the computational burden.

Nevertheless, as well as several feedback linearization like controllers (Ge and Wang , 2002), the NBC may present singularities, yielding frequently, closed-loop system instability. In this paper, to overcome such controller singularity problem, *a priori* information about the parameters of the neural model is used to design the update law; such strategy avoids not only controller singularities, but also the drift parameter phenomenon.

### 1. HIGH ORDER RECURRENT NEURAL NETWORKS

In this paper, for the identification task, expansions of the first order Hopfield model called High Order Recurrent Neural Networks (RHONN) are used (Kosmatopoulus et. al. , 1995). Additionally, the RHONN model is very flexible and allows to incorporate to the neural identifier  $a \ priori$  information about the plant structure.

A recurrent high-order recurrent neural network of n neurons and m inputs is defined as

$$\dot{x}_i = -a_i x_i + \sum_{k=1}^{L_i} w_{ik} \prod_{j \in I_k} \eta_k^{d_j(k)}, \ i = 1, ..., n \quad (1)$$

where  $x_i$  is the *i*-th neuron state,  $L_i$  is the number of high order connections,  $\{I_1, I_2, ..., I_L\}$  is a collection of non-ordered subsets of  $\{1, 2, ..., m + n\}$ ,  $a_i > 0$ ,  $w_{ik}$  are the adjustable weights of the neural network,  $d_j(k)$  are non-negative integers, and  $\eta$  is a vector defined as

$$\boldsymbol{\eta} = [S(x_1), \dots, S(x_n), x_1, \dots, x_n, u_1, \dots, u_m]^\top$$
$$= [\eta_1, \dots, \eta_n]^\top$$

with  $\mathbf{u} = [u_1, ..., u_m]^{\top}$  being the input to the neural networks, and  $S(\cdot)$  a smooth hyperbolic tangent function formulated by

$$S(x) = \frac{2}{1 + \exp(-\beta x)} - 1$$

Defining the high order terms vector as

$$\boldsymbol{\rho}_i = \left[\prod_{j \in I_1} \eta_j^{d_j(1)}, \ \prod_{j \in I_2} \eta_j^{d_j(2)}, \dots, \prod_{j \in I_L} \eta_j^{d_j(L_i)}\right],$$

the system (1) can be rewritten as

$$\dot{x}_i = -a_i x_i + \mathbf{w}_i^\top \boldsymbol{\rho}_i(\mathbf{x}, \mathbf{u}), \, i = 1, ..., n \quad (2)$$

where  $\mathbf{w}_{i} = [w_{i,1}...w_{i,L_{i}}]^{\top}$ .

#### 2. ON-LINE IDENTIFICATION

In this section, we consider the problem of identifying a nonlinear system given by

$$\dot{\boldsymbol{\chi}} = \mathbf{f}(\boldsymbol{\chi}, \mathbf{u}) \tag{3}$$

where  $\boldsymbol{\chi} \in \Re^n$ ,  $\mathbf{u} \in \Re^m$ ,  $\mathbf{f}$  is a smooth vector field and  $f_i(\boldsymbol{\chi}, \mathbf{u})$  its entries. In order to identify (3), as discussed in Kosmatopoulus et. al. (1995), we assume that this system is fully described by a RHONN, with each neuron state given by

$$\dot{\chi}_i = -a_i \chi_i + \mathbf{w}_i^{*\top} \boldsymbol{\rho}_i(\boldsymbol{\chi}, \mathbf{u}) + \nu_i(t), \ i = 1, ..., n \ (4)$$

with  $\mathbf{w}_i^*$ ,  $\boldsymbol{\rho}_i \in \Re^{L_i}$ , the optimal unknown parameters vector  $\mathbf{w}_i^*$  is defined as

$$\mathbf{w}_{i}^{*} = \arg \min_{\mathbf{w}_{i}} \left\{ \sup_{\boldsymbol{\chi}, \mathbf{u}} \left| \begin{array}{c} f_{i}(\boldsymbol{\chi}, \mathbf{u}) + a_{i} \chi_{i} \\ -\mathbf{w}_{i}^{\top} \boldsymbol{\rho}_{i}(\boldsymbol{\chi}, \mathbf{u}) - \nu_{i}(t) \end{array} \right| \right\}$$
(5)

where the modelling error term  $\nu_i$  is defined as

$$\nu_i(t) = f_i(\boldsymbol{\chi}, \mathbf{u}) + a_i \chi_i - \mathbf{w}_i^{*\top} \boldsymbol{\rho}_i(\boldsymbol{\chi}, \mathbf{u}).$$
 (6)

To develop the weight update law, the seriesparallel model is used:

$$\dot{x}_i = -a_i x_i + \mathbf{w}_i^\top \boldsymbol{\rho}_i(\boldsymbol{\chi}, \mathbf{u}), \qquad i = 1, ..., n$$
 (7)

where  $x_i$  is the *i*-th component of the RHONN, and  $\boldsymbol{\chi}$  is the plant state.

#### 2.1 On-Line Update Law for Constrained Weights

In this Section a on-line update law is developed to constrain adaptive parameters trajectories, into a compact set. First, let define the *i*-th identification error  $e_i = x_i - \chi_i$  and the *i*-th parameter error  $\mathbf{\tilde{w}}_i = \mathbf{w}_i - \mathbf{w}_i^*$ .

Assuming that the modelling error term  $\nu_i$  is zero, from (4) and (7) the identification error dynamics is obtained as

$$\dot{e}_i = -a_i e_i + \tilde{\mathbf{w}}_i^\top \boldsymbol{\rho}_i. \tag{8}$$

Consider a Lyapunov function candidate of the form

$$V_i = \frac{1}{2} (e_i^2 + \tilde{\mathbf{w}}_i^\top \boldsymbol{\Gamma}_i \tilde{\mathbf{w}}_i)$$
(9)

where  $\Gamma_i = diag\{\gamma_{i1}, \gamma_{i2}, ..., \gamma_{i,L_i}\}$  is a diagonal positive definite matrix. Differentiating (9) along the trajectories of (8), yields

$$\dot{V}_i = -a_i e_i^2 + e_i \tilde{\mathbf{w}}_i^\top \boldsymbol{\rho}_i + \tilde{\mathbf{w}}_i^\top \boldsymbol{\Gamma}_i \dot{\tilde{\mathbf{w}}}_i.$$
(10)

Additionally to plant structure, we consider the following a priori knowledge about the optimal weights:  $\mathbf{w}_i^* \in W_i^* = W_{i1}^* \times W_{i2}^* \times \ldots \times W_{i,L_i}^* \subset \Re^{L_i}$ , where  $W_{ik}^* = \{w_{ik}^{\min} \leq w_{ik}^* \leq w_{ik}^{\max}\}, k = 1, 2, \ldots, L_i$  is called the constrain set of  $w_{ik}$ . We assume that, for every optimal weight, its upper and lower bounds are known. This fact is used to design an update law that avoids the adaptive weights drift and/or zero crossing. The aforementioned assumption implies that when the *i*-th parameter error is outside of  $W_{ik}^*$ , its corresponding parametric error sign can be calculated as follows:

$$sign(\tilde{w}_{ik}) = \begin{cases} 1, & \text{if } w_{ik} > w_{ik}^{\max} \\ -1, & \text{if } w_{ik} < w_{ik}^{\min} \end{cases}, \ k = 1, 2, ..., L_{k}$$

The weight adaptive law is defined as

$$\dot{\mathbf{w}}_i = \mathbf{\Gamma}_i^{-1}(-e_i \boldsymbol{\rho}_i - \mathbf{D}_i \mathbf{sign}(\tilde{\mathbf{w}}_i)) \qquad (11)$$

where  $\mathbf{D}_i = diag\{d_{i1}, d_{i2}, ..., d_{i,L_i}\}$  with  $d_{i,k} = \sigma_{ik}(|e_i\rho_{ik}| + c), k = 1, 2, ..., L_i, c > 0$  and

$$\sigma_{ik} = \begin{cases} 0, & \text{if } w_{ik}^{\min} \le w_{ik} \le w_{ik}^{\max} \\ 1, & \text{if } w_{ik}^{\min} > w_{ik} \text{ or } w_{ik} < w_{ik}^{\max} \end{cases}.$$

Then, equation (10) becomes

$$\begin{aligned} \dot{V}_i &= -a_i e_i^2 - \tilde{\mathbf{w}}_i^\top \mathbf{D}_i \mathbf{sign}(\tilde{\mathbf{w}}_i) \\ &\leq -a_i e_i^2 - c \sum_{k=1}^{L_i} \sigma_{ik} \tilde{w}_{ik} sign(\tilde{w}_{ik}) \\ &= -a_i e_i^2 - c \sum_{k=1}^{L_i} \sigma_{ik} |\tilde{w}_{ik}| \leq -a_i e_i^2 \end{aligned}$$

The next lemma (Khalil, 1996) is needed to prove the weight adaptive law convergence.

Lemma 1. Consider the system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \tag{12}$$

where  $\mathbf{x} \in \Re^n$  and  $\mathbf{f}(\mathbf{x})$  is locally Lipschitz. Assume that there exists a function  $V : \Re^n \to \Re^+$  radially unbounded and continuously differentiable such that

$$\dot{V} = \frac{\partial V}{\partial \mathbf{x}} \mathbf{f}(\mathbf{x}) \le -W(\mathbf{x}) \le 0$$

 $\forall \mathbf{x} \in \Re^n$ , where  $W(\mathbf{x})$  is a positive semidefinite radially unbounded function. Then all trajectories of (12) are ultimately and uniformly bounded for  $t \ge 0$  and  $\mathbf{x}(0) \in \Re^n$ , moreover

$$\lim_{t \to \infty} W(\mathbf{x}(t)) = 0.$$

Using Lemma 1, with  $\mathbf{x} = [e_i, \mathbf{\tilde{w}}_i^{\top}]^{\top}$ ,  $V(\mathbf{x}) = V_i(e_i, \mathbf{\tilde{w}}_i)$  and  $W(\mathbf{x}) = -a_i e_i^2$ , the adaptive law (11) ensures the convergence of  $e_i$  to zero and the boundness of  $\mathbf{\tilde{w}}_i$ .

Lemma 2. The adaptive vector  $\mathbf{w}_i$  converges to  $W_i^*$ .

**Proof.** Let assume that  $w_{ik}$  is outside  $W_{ik}^*$  at t = 0, then, from (11), the dynamics of  $w_{ik}$  is given by

$$\dot{w}_{ik} = \gamma_{ik} (e_i \rho_{ik} - (|e_i \rho_{ik}| + c) sign(w_{ik})) \quad (13)$$

Now, we analyze the following cases.

**Case 1**.  $w_{ik} > w_{ik}^{\text{max}}$ . Equation (13) becomes

$$\dot{w}_{ik} = \gamma_{ik} (e_i \rho_{ik} - (|e_i \rho_{ik}| + c)) \le -c \gamma_{ik}$$

Using the comparison principle (Khalil , 1996) we have

$$w_{ik}(t) \le w_{ik}(0) - c\gamma_{ik}t$$

which means  $w_{ik}(t) \leq w_{ik}^{\max}$  for  $t > t_s$ , where  $t_s$  is a finite time.

**Case 2.**  $w_{ik} < w_{ik}^{\min}$ . Equation (13) is rewritten as

$$\dot{w}_{ik} = \gamma_{ik} (e_i \rho_{ik} + (|e_i \rho_{ik}| + c)) \ge c \gamma_{ik}$$

Again, using the comparison principle we have

$$w_{ik}(t) \ge w_{ik}(0) + c\gamma_{ik}t$$

which means  $w_{ik}(t) \ge w_{ik}^{\min}$  for  $t > t_s$ , where  $t_s$  is a finite time.

In both cases  $w_{ik}$  converges to  $W_{ik}^*$ . Due to the definition of  $W_i^*$ , we conclude that  $\mathbf{w}_i$  converges into  $W_i^*$ .

Let define  $\Delta w_{ik}^* = |w_{ik}^{\max} - w_{ik}^{\min}|$ , and  $\Delta \mathbf{w}_i^* = [\Delta w_{i1}^*, \Delta w_{i2}^*, ..., \Delta w_{i,L_i}^*]^\top$ . By Lemma 2 it is easy to see that the parameter error  $\mathbf{\tilde{w}}_i$  converges into the set  $\tilde{W}_i = {\{\mathbf{\tilde{w}}_i : |\mathbf{\tilde{w}}_{ik}| \leq \Delta w_{ik}^*, k = 1, 2, ..., L_i\}}.$ 

The update law forces the trajectories of  $\mathbf{w}_i$  to converge into the constrain set  $W_i^*$ ; we might select this set such that not only all weights remain bounded, but also some of them dot not change their signs. In Section 5, this feature is used to avoid controller singularities.

It is worth to mention, that the selection of the constrain set  $W_i^*$  remains as an open problem. In this paper, such selection was made based on the observation of experimental results, which were published in Loukianov et al. (2002).

# 2.2 On-Line Weight Update with no zero modelling error

When the modelling error term is not zero, any standard adaptive laws can not guarantee either the boundness of the parameters or the convergence of the identification error to zero. Furthermore, the parameters drift phenomenon could occur. The " $\sigma$ -modification" (Kosmatopoulus et. al., 1995) is often used to overcome this situation and to assure, at least, that the identification error and the weights are bounded. In this work the update law (11) guarantees the converge of  $e_i$  and  $w_i$  into a bounded set, which is stated in the following Lemma.

Lemma 3. Consider the system (4) and the RHONN identifier (7), with weight adaptation law (11), and assume the modelling error (6) is not zero. Then,  $e_i$  and  $w_i$  converge to a bounded set

**Proof.** The derivative of  $V_i$  along the trajectories of (4) and (11) is given by

$$\dot{V}_i = -a_i e_i^2 - \tilde{\mathbf{w}}_i^\top \mathbf{D}_i \mathbf{sign}(\tilde{\mathbf{w}}_i) + e_i \nu_i(t).$$

Applying the triangular inequality and defining  $d_0 = \max_{t \leq 0}(\nu_i(t))$ , yields

$$\dot{V}_i \le -a_i e_i^2 - c \sum_{k=1}^{L_i} \sigma_{ik} |\tilde{w}_{ik}| + \frac{e_i^2}{2} + \frac{d_0^2}{2}$$

Selecting  $a_i$ , such that  $\alpha_i = a_i - \frac{1}{2} > 0$ , yields

$$\dot{V}_i \le -\alpha_i e_i^2 + \frac{d_0^2}{2}.$$

Substituting  $e_i$  from (9) in the above inequality, it is rewritten as

$$\dot{V}_i \leq -2\alpha_i V_i + \alpha_i \tilde{\mathbf{w}}_i^{\top} \mathbf{\Gamma}_i \tilde{\mathbf{w}}_i + \frac{d_0^2}{2}$$

Since  $\tilde{\mathbf{w}}_i^{\top}$  converges to  $\tilde{W}_i$ , we conclude that  $e_i$  and  $\tilde{\mathbf{w}}_i$  converge into the residual set

$$D_i = \left\{ \{e_i, \ \tilde{\mathbf{w}}_i\} : V_i \le \frac{\alpha_i \Delta \mathbf{w}_i^{*\top} \Gamma_i \Delta \mathbf{w}_i^* + \frac{d_0^2}{2}}{2\alpha_i} \right\}$$

and the proof is completed.  $\blacksquare$ 



Fig. 1. Block Control Scheme 3. NEURAL BLOCK CONTROL

In this scheme, the control law based is on the neural network (7). The RHONN parameters are updated according to (11). Fig. 1 explains the proposed control scheme, which is based on the following proposition.

Proposition 4. Given a desired output trajectory  $\mathbf{y}_r$ , a dynamic system with output  $\mathbf{y}_P$ , and a neural network with output  $\mathbf{y}_N$ , then it is possible to establish the following inequality:

$$\|\mathbf{y}_r - \mathbf{y}_P\| \le \|\mathbf{y}_N - \mathbf{y}_P\| + \|\mathbf{y}_r - \mathbf{y}_N\|$$

where  $\mathbf{y}_r - \mathbf{y}_P$  is the system output tracking error,  $\mathbf{y}_N - \mathbf{y}_P$  is the output identification error, and  $\mathbf{y}_N - \mathbf{y}_r$  is the RHONN output tracking error.

Based on this proposition, it is possible to divide the tracking problem in two parts:

- (1) Minimization of  $\|\mathbf{y}_N \mathbf{y}_P\|$ , which can be achieved by the proposed on-line identification algorithm (11).
- (2) Minimization of  $\|\mathbf{y}_N \mathbf{y}_r\|$ , for that a tracking algorithm is developed on the basis of the neural identifier (7).

The second goal can be reached by designing a control law based on the RHONN model. To design such controller we propose to use the NBC methodology (Loukianov et al., 2002).

### 4. INDUCTION MOTOR APPLICATION

In order to illustrate the application of the proposed approach let consider the induction motors control. The  $\alpha - \beta$  coordinate system model is

$$\dot{\chi}_1 = c_1(\chi_2\chi_5 - \chi_3\chi_4) - c_0T_L \tag{14}$$

$$\dot{\chi}_2 = -c_2\chi_2 - n_p\chi_1\chi_3 + c_3\chi_4 \tag{15}$$

$$\dot{\chi}_3 = -c_2\chi_3 + n_p\chi_1\chi_2 + c_3\chi_5 \tag{16}$$

$$\dot{\chi}_5 = c_4 \chi_2 + c_5 n_p \chi_1 \chi_3 - c_6 \chi_4 + c_7 u_\alpha \quad (17)$$

$$\dot{\chi}_6 = c_4 \chi_3 - c_5 n_p \chi_1 \chi_2 - c_6 \chi_5 + c_7 u_\beta \quad (18)$$

Where  $\chi_1$  represents the angular velocity of the motor shaft,  $\chi_2$  and  $\chi_3$  are, the rotor magnetic

flux leakage components,  $\chi_4$  and  $\chi_5$  are the stator current components,  $u_{\alpha}$  and  $u_{\beta}$  stand, respectively, for the voltage applied on the stator windings, and  $T_L$  represents the load torque perturbation. The constants  $c_i$ , i = 0, ..., 7 are defined as follows:  $c_0 = \frac{b}{J}$ ,  $c_1 = \frac{Mn_p}{JL_r}$ ,  $c_2 = \frac{R_r}{L_r}$ ,  $c_3 = \frac{R_r M}{L_r}$ ,  $c_4 = \frac{R_s L_r^2 + R_r M^2}{L_s (L_s L_r - M^2)}$ ,  $c_5 = \frac{R_r}{L_r} \frac{M}{L_s L_r - M^2}$ ,  $c_6 = \frac{M}{L_s L_r - M^2}$ ,  $c_7 = \frac{L_r}{L_s L_r - M^2}$ . Where  $L_s$ ,  $L_r$  and M, are the stator, rotor and mutual inductances, respectively,  $R_s$  and  $R_r$ , are the stator and rotor resistances, J is the rotor moment of inertia and  $n_p$  is the number of stator winding pole pairs.

Commonly, induction motor applications require not only shaft speed regulation, but also flux magnitude  $\phi = \chi_2^2 + \chi_3^2$  regulation. Since the currents and velocity are the only measurable variables, the rotor fluxes estimation is required for neural networks identification. In this work, we use the flux observer proposed in Loukianov et al. (2002); it is a partial state observer with adjustable convergence rate. This features enables to reduce the number of calculations comparing with a full state observer. For the rest of the calculations on this paper, the estimated fluxes are considered as the real ones.

#### 4.1 Neural Model for Induction Motors

Let assume that the partial model (14-16) has the RHONN representation, without modelling error terms, given by

$$\dot{\chi}_1 = -a_1 \chi_1 + \mathbf{w}_1^{*\top} \boldsymbol{\rho}_1$$
  

$$\dot{\chi}_2 = -a_2 \chi_2 + \mathbf{w}_2^{*\top} \boldsymbol{\rho}_2$$

$$\dot{\chi}_3 = -a_3 \chi_3 + \mathbf{w}_3^{*\top} \boldsymbol{\rho}_3$$
(19)

where  $\mathbf{w}_1^* = [w_{11}^*, w_{12}^*, w_{13}^*]^\top$ ,  $\mathbf{w}_2^* = [w_{21}^*, w_{22}^*, w_{23}^*]^\top$ and  $\mathbf{w}_3^* = [w_{31}^*, w_{32}^*, w_{33}^*]^\top$ , are the optimal weight vectors, which are constant and unknown, and  $\boldsymbol{\rho}_1 = [S(\chi_1), S(\chi_3)\chi_4, S(\chi_2)\chi_5]^\top$ ,  $\boldsymbol{\rho}_2 = [S(\chi_2), S(\chi_1)S(\chi_3), \chi_4]^\top$  and  $\boldsymbol{\rho}_3 = [S(\chi_3), S(\chi_1)S(\chi_2), \chi_5]^\top$  are the high order term vectors.

Based on the mathematical model (19), the following reduced order neural identifier is proposed

$$\dot{x}_1 = -a_1 x_1 + \mathbf{w}_1^\top \boldsymbol{\rho}_1$$
  

$$\dot{x}_2 = -a_2 x_2 + \mathbf{w}_2^\top \boldsymbol{\rho}_2$$
  

$$\dot{x}_3 = -a_3 x_3 + \mathbf{w}_3^\top \boldsymbol{\rho}_3$$
(20)

For this model  $\mathbf{w}_1 = [w_{11}, w_{12}, w_{13}]^{\top}$ ,  $\mathbf{w}_2 = [w_{21}, w_{22}, w_{23}]^{\top}$  and  $\mathbf{w}_3 = [w_{31}, w_{32}, w_{33}]^{\top}$  are the adaptive RHONN parameters, which are adapted using (11).  $x_1$  is the neural speed, and  $x_2$  and  $x_3$  are the neural fluxes, these neural states are used to identify  $\chi_1$ ,  $\chi_2$  and  $\chi_3$  respectively.

The output variables to be controlled are the speed  $\chi_1$  and the flux magnitude  $\phi$ , respectively. Now, let define the neural flux magnitude as  $\varphi = x_2^2 + x_3^2$ . Then, the plant output is  $\mathbf{y}_P = [\chi_1 \ \phi]^\top$ , the neural output is  $\mathbf{y}_N = [x_1 \ \varphi]^\top$  and the reference signal is  $\mathbf{y}_r = [\omega_r \ \varphi_r]^\top$ .

### 4.2 Neural Block Controller Design

In this section, based on the neural identifier (20), a control law is developed using the Neural Block Control strategy (Loukianov et al., 2002). The neural model (20) and the stator currents model (15) are combined to obtain a quasi NBC-form, consisting of two blocks:

$$\dot{\mathbf{x}}_1 = \dot{\mathbf{f}}_1 + \dot{\mathbf{B}}_1 \boldsymbol{\chi}_2$$
$$\dot{\boldsymbol{\chi}}_2 = \mathbf{f}_2 + \mathbf{B}_2 \mathbf{u}$$
(21)

with  $\mathbf{x}_1 = [x_1, x_2, x_3]^{\top}, \ \boldsymbol{\chi}_2 = [\chi_4, \chi_5]^{\top}, \ \mathbf{u} = [u_{\alpha}, u_{\beta}]^{\top},$ 

$$\begin{split} \tilde{\mathbf{f}}_{1} &= \begin{bmatrix} -a_{1}x_{1} + w_{11}S(\chi_{1}) + w_{14} \\ -a_{2}x_{2} + w_{21}S(\chi_{2}) + w_{22}S(\chi_{1})S(\chi_{3}) \\ -a_{3}x_{3} + w_{31}S(\chi_{3}) + w_{32}S(\chi_{1})S(\chi_{2}) \end{bmatrix}, \\ \tilde{\mathbf{B}}_{1} &= \begin{bmatrix} -w_{12}S(\chi_{3}) w_{13}S(\chi_{2}) \\ w_{23} & 0 \\ 0 & w_{33} \end{bmatrix}, \\ \mathbf{f}_{2} &= \begin{bmatrix} c_{4}\chi_{2} + c_{5}n_{p}\chi_{1}\chi_{3} - c_{6}\chi_{4} \\ c_{4}\chi_{3} - c_{5}n_{p}\chi_{1}\chi_{2} - c_{6}\chi_{5} \end{bmatrix}, \\ \mathbf{B}_{2} &= \begin{bmatrix} c_{7} & 0 \\ 0 & c_{7} \end{bmatrix}. \end{split}$$

For shorter notation all the weights are ordered in one vector  $\mathbf{w} = [\mathbf{w}_1^\top \ \mathbf{w}_2^\top \ \mathbf{w}_3^\top]^\top$ . This model can be reduced to the NBC-form (Loukianov , 1998), and therefore the Block Control methodology is applied. At first, the tracking error for the neural output is rewritten as

$$\mathbf{z}_1 = \mathbf{y}_N - \mathbf{y}_P = \begin{bmatrix} x_1 - \omega_r \\ \varphi - \varphi_r \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}. \quad (22)$$

Then, the tracking error dynamics can be expressed as the first block of the NBC-form:

$$\dot{\mathbf{z}}_1 = \bar{\mathbf{f}}_1 + \bar{\mathbf{B}}_1 \boldsymbol{\chi}_2 \tag{23}$$

where  $\mathbf{\bar{f}}_1 = \begin{bmatrix} \bar{f}_{11} \\ \bar{f}_{12} \end{bmatrix}$ ,  $\mathbf{\bar{B}}_1 = \begin{bmatrix} w_{12}S(\chi_3) & w_{13}S(\chi_2) \\ 2w_{23}\chi_2 & 2w_{33}\chi_3 \end{bmatrix}$ , with  $\bar{f}_{11} = -a_1x_1 + w_{11}S(\chi_1) + w_{14} - \dot{\omega}_r$  and

$$\begin{split} \bar{f}_{12} = & 2x_2 \left( -a_2 x_2 + w_{21} S(\chi_2) + w_{22} S(\chi_1) S(\chi_3) \right) + \\ & 2x_3 (-a_3 x_3 + w_{31} S(\chi_3) + w_{32} S(\chi_1) S(\chi_2)) - \dot{\varphi}_{\eta} \end{split}$$

Following the block control strategy, the quasicontrol vector  $\chi_2$  is selected as

$$\boldsymbol{\chi}_{2} = \begin{bmatrix} \chi_{4} \\ \chi_{5} \end{bmatrix} = \bar{\mathbf{B}}_{1}^{-1} \left( -\bar{\mathbf{f}}_{1} + \mathbf{K} \mathbf{z}_{1} \right) + \mathbf{z}_{2} \qquad (24)$$

where  $\mathbf{K} = diag\{-k_1, -k_2\}, \mathbf{z}_2 = [z_4, z_5]^{\top}$  and  $\bar{\mathbf{B}}_{\mathbf{1}}^{-\mathbf{1}} = \frac{1}{\delta} \begin{bmatrix} 2w_{33}\chi_3 & -2w_{23}\chi_2\\ -w_{13}S(\chi_2) & w_{12}S(\chi_3) \end{bmatrix}$ , with  $\delta =$   $2w_{12}w_{33}\chi_3 S(\chi_3) - 2w_{13}w_{23}\chi_2 S(\chi_2)$ , and  $k_1, k_2 > 0$ . Then, (23) can be rewritten as

$$\dot{\mathbf{z}}_1 = \mathbf{K}\mathbf{z}_1 + \mathbf{B}_1\mathbf{z}_2.$$

Now, the new variables  $\mathbf{z}_2$  are expressed from (24) as

$$\mathbf{z}_2 = \bar{\mathbf{B}}_1^{-1} \left( \bar{\mathbf{f}}_1 - \mathbf{K} \mathbf{z}_1 \right) + \boldsymbol{\chi}_2 = \boldsymbol{\alpha}_2.$$
 (25)

Considering the time derivative of (25), the second block of the NBC-form for the variables  $z_4$  and  $z_5$ is presented as

$$\dot{\mathbf{z}}_2 = \overline{\mathbf{f}}_2 + \mathbf{B}_2 \mathbf{u} \tag{26}$$

where  $\mathbf{\bar{f}}_2 = \mathbf{f}_2 - \left(\frac{\partial \boldsymbol{\alpha}_2}{\partial \mathbf{x}_1} \mathbf{\tilde{f}}_1 + \frac{\partial \boldsymbol{\alpha}_2}{\partial \boldsymbol{\chi}_1} \mathbf{f}_1 + \frac{\partial \boldsymbol{\alpha}_2}{\partial \boldsymbol{\chi}_1} \mathbf{f}_2 + \frac{\partial \boldsymbol{\alpha}_2}{\partial \mathbf{w}} \mathbf{\dot{w}} + \frac{\partial \boldsymbol{\alpha}_2}{\partial \mathbf{y}_r} \mathbf{\dot{y}}_r + \frac{\partial \boldsymbol{\alpha}_2}{\partial \mathbf{\dot{y}}_r} \mathbf{\ddot{y}}_r \right)$ . Now, the Sliding Modes control strategy formulated as

$$\mathbf{u} = -u_0 \mathbf{sign} \left( \mathbf{z}_2 \right), \quad u_0 > 0$$

under the condition  $c_7u_0 > \max(|\mathbf{\bar{f}}_2|)$ , a sliding mode on the surface  $z_4 = 0$ ,  $z_5 = 0$  is guarantees in a finite time. Then the sliding dynamics, for the tracking errors variables  $z_1$  and  $z_2$ , are governed by the second order linear system

$$\dot{z}_1 = -k_1 z_1$$
$$\dot{z}_2 = -k_2 z_2$$

with desired eigenvalues  $k_1$  and  $k_2$ . Hence, we conclude that the neural output tracks the reference.

Since  $\mathbf{z}_1$  tends to zero in the manifold  $\mathbf{z}_2 = 0$  and considering zeros the modelling error terms, it can be appreciated that  $\mathbf{y}_P \to \mathbf{y}_N$  and  $\mathbf{y}_N \to \mathbf{y}_r$ . By proposition 4 we conclude that  $\mathbf{y}_P \to \mathbf{y}_r$ .

Due to time varying nature of RHONN weights, we need to guarantee that rank( $\mathbf{\bar{B}}_1$ )= 2 for all time. Notice that if the so-called controllability weights  $w_{12}$ ,  $w_{13}$ ,  $w_{23}$  or  $w_{33}$  are zeros, the matrix  $\mathbf{\bar{B}}_1$  may lose rank, making the identifier uncontrollable and the controller singular. Hence, we select constrain sets  $W_{12}^*$ ,  $W_{13}^*$ ,  $W_{23}^*$  and  $W_{33}^*$  with  $w_{12}^{\min} < 0$ ,  $w_{13}^{\min} > 0$ ,  $w_{23}^{\min} > 0$ ,  $w_{33}^{\min} > 0$ , and define the initial value  $w_{12}(0) \in W_{12}^*$ ,  $w_{13}(0) \in W_{13}^*$ ,  $w_{23}(0) \in W_{23}^*$ ,  $w_{33}(0) \in W_{33}^*$ , which guarantees that such weights do not cross by zero, keeping  $\mathbf{\bar{B}}_1$  as a full rank matrix.

#### 5. SIMULATIONS

r. The nominal values of the induction motor parameters as:  $R_s = 12.53\Omega$ ,  $L_s = 0.2464H$ , M = 0.2219H,  $R_r = 11.16\Omega$ ,  $L_r = 0.2464H$ ,  $n_p = 2$ , J = 0.01Kgm. The design parameters for the fluxes observer are  $l_1, l_2 = 3500$  and  $\delta = 0.1$ ; for the neural network, we selected  $a_1 = 18$ ,  $a_2 = a_3 = 500$ ,  $\beta = 0.1$ ,  $\Gamma_1^{-1} = diag\{140, 350, 350\}$ ,  $\Gamma_2^{-1} = \Gamma_3^{-1} = diag\{200, 200, 1600\}$ , and  $k_1 = 350$  $k_2 = 100$ . In order to test the proposed scheme performance, a variation of 2 Ohm per second is



Fig. 2. Simulations without constrained weights



Fig. 3. Simulations with constrained weights

added to the stator resistance and a constant load torque  $T_L = 3Nm$ . The bounds for the constrain sets are selected as:  $w_{12}^{\min} = -20000$ ,  $w_{12}^{\max} = -50$ ,  $w_{13}^{\min} = 50$ ,  $w_{13}^{\min} = 20000$ ,  $w_{23}^{\min} = w_{33}^{\min} = 5$ ,  $w_{23}^{\max} = w_{33}^{\max} = 20000$ .

For sake of completeness, we have included a simulation using (11) without sign term (Loukianov et al., 2002) (no constrained weights), whose results can be appreciated in Fig. 2. Notice that the close-loop system becomes unstable; soon after t = 2.3s, both, the real and the neural speed diverge from the reference. For the simulations with constrained parameters, Fig. 3 a) shows that the system remains stable and the speed reference  $\omega_r$  is tracked by the real plant speed  $\chi_1$  and the neural speed  $x_1$ . Controllability weights are plotted for both simulations; notice that for the simulation without constrained weights (see Fig. 2 b), the system becomes unstable just when the parameters cross zero. In contrast, the constrained weights do not change their signs (see Fig. 3) b), avoiding controller singularity and closed-loop system instability.

#### 6. CONCLUSIONS

In this paper, we have presented an improved version of the Neural Block Control with a new weight update law. By using a priori knowledge about the real plant, the proposed scheme avoids completely the controller singularity problem and the parameter drift phenomenon. All signals of the closed-loop system remain bounded. The new online identification and control scheme is tested on an induction motor; simulation illustrated its advantages against the previous results in Loukianov et al. (2002).

Acknowledgments — To CONACYT, Mexico, on grants 39866Y and 36960A.

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