DYNAMICAL MODELLING USING C^k SPLINE FUNCTIONS WITH APPLICATION TO WEIGHT SENSORS

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Abstract: We present in this article an identification and signal processing of weight sensors. The dynamical modeling of these sensors is nonlinear and can be described in the state space representation by a bilinear algebraic structure. This system can be transformed by local diffeomorphisms by using an elimination method. The identification process is based upon a projection of experimental data in the Sobolev $H_k(\Omega)$ space (C^k spline approximation). The experimental results show the good accordance of our projection with the real response of the sensor.

Keywords: C^k spline functions, identification, modelling, sensors, response curves, bilinear systems, mechatronics.

1. INTRODUCTION

Weight sensors generally use mechanical transducers which transpose weight in a displacement variable measured by an electronic device. We can quote for example technologies based on strain gauge, mechanical resonators, optical interferometer or tunneling effect. These mechanical transducers must be generally designed in order to be deterministic (no random or stochastic effect), and to avoid dynamical and/or nonlinear effects. In spite of this expensive design over a given threshold of precision; creep, relaxation and thermal dilatation of these mechanical transducers modify strongly the measure limiting in the same way the precision of these technologies. One way to overcome this problem is to model these phenomena in order to create an appropriate signal processing of the measure. This new approach has the advantage of studying these behaviors with as its final goal, the design of inexpensive and precise mechanical transducers. In this way we have studied two different technologies; a strain gauge resistor one, and a mechanical resonator one (Rouff, *et al.*, 1992).

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These two technologies like the others are limited in precision by the creep and relaxation of the metallic transducers, and usually it is not possible to obtain without compensations a precision better than 10^{-2}. To overcome this fundamental limitation of this kind of technology, numerical corrections for the electronic responses of the sensors are needed. Two steps of numerical corrections can be considered; a static one which involves algebraic equations on the measure itself, to compensate the nonlinear static responses of the measured weight versus deformation or displacement and the thermal fluctuations at low frequencies (Rouff and Konieczka, 1993). These techniques can be generally used up to a precision of 10^{-3}. Further advances in precision pass through a dynamical correction, i.e., the analysis of the temporal response of the sensor. In fact for precisions over10^{-4} it is difficult to define the measure itself, because the temporal response of the sensor is not asymptotically constant but logarithmic, which poses the mathematical problem of the measure definition. The solution of the problem, which first and foremost goes through a nonlinear deconvolution process, supposed a complete dynamical modelling of the sensor. This technique leads to a precision better than 10^{-5}. Clearly these systems are nonlinear and this dynamical modelling approach began by a black box approach by using bilinear formalism (Zhang, 1993).

The plan of the paper is as follows. Section 2, is devoted to presenting a review of some control theory. Section 3 presents the elimination process for general bilinear systems. The identification process using C^k spline functions is formalized in Section 4. Section 5 presents experimental identification of weight sensors. Finally, the paper ends with concluding remarks in section 6.

2. REVIEWING SOME CONTROL THEORY

2.1 Notion of Equivalence

Let us consider a general system description of the form

$$P(\omega,\dot{\omega},\omega^{(2)},...,\omega^{(k)},\eta,\dot{\eta},\eta^{(2)},...,\eta^{(l)})=0 \quad (1)$$

where $\omega \in \mathbb{R}^n$ are called the external variables (also called input-output variables), $\eta \in \mathbb{R}^S$ are called latent variables, and P is a nonlinear $\mathbb{C}^\infty$ function defined from $\mathbb{R}^{n+S} \longrightarrow \mathbb{R}^r$.

The system (1) is generally called an external differential system, and the latent variables can be state, partial state or other characteristic variables of some behavior of the system. Two external differential systems as (1) are called equivalent if the set of trajectories that they allow for the external variables are the same.

2.2 The Elimination and Realisation Formalism

Let us consider a differential system in its general state space representation

$$\begin{aligned} q &= F(q,u,t) \\ y &= h(q,u,t) \end{aligned} \quad (2)$$

where q is the state vector of dimension N, u is the input control vector of dimension m, t is the time variable, and F and h are two $\mathbb{C}^\infty$ functions defined from $\mathbb{R}^{N+m+1} \longrightarrow \mathbb{R}^N$ and from $\mathbb{R}^{N+m+1} \longrightarrow \mathbb{R}^r$, respectively.

Let us consider the following differential system, depending only on the input-output variables and the time variable

$$S(y,\dot{y},...,y^{(N)},u,\dot{u},...,u^{(k)},t)=0 \quad (3)$$

where y, u, t, N are defined as in (2) and S is a $\mathbb{C}^\infty$ function defined from $\mathbb{R}^{N+m+1} \longrightarrow \mathbb{R}^r$.

For a given differential system like (2), the elimination problem consists of finding its equivalent (in the sense defined in II. *A*) in the form by (3). Inversely, the realisation problem can be also defined as for a system like (3) to find its suitable equivalent in (2).

3. THE ELIMINATION PROCESS FOR GENERAL BILINEAR SYSTEMS

Let

$$\begin{aligned} \dot{q} &= Aq + \sum_{i=1}^{m} u_i B_i q + Cu \\ y &= Dq + Eu \end{aligned} \quad (4)$$

be a generalized bilinear system, where q is the state vector of dimension N, u is the control vector of dimension m (u_i is the *i-th* coordinate of u), y is the output vector of dimension r and $A,B_1,B_2,...,B_m,C,D,E$ are m+4 matrices of dimension $N\times N$, $N\times N$, …, $N\times N$, $N\times m$, $r\times N$, and $r\times m$, respectively. These m+4 matrices can be time dependent and/or parametrised.

By eliminating 2 or 3 suitable matrices, it is easy to find, from (4), a classical bilinear or linear system. Our elimination procedure consists of finding, from (4), its equivalent, in the following form

$$\sum_{i=0}^{N} Q_{is}(u,\dot{u},...,u^{(k)})\frac{d^i y_s}{dt^i} = Q_{00s}(u,\dot{u},...,u^{(k)}) \quad (5)$$

for $s\in[1,r]$, where $Q_{00s},Q_{0s},Q_{1s},...,Q_{Ns}$ are $r*(N+2)$ rational functions of $u,\dot{u},...,u^{(k)}$. For any of the specified parameters and/or the time variable t, the Q_i are defined from $\mathbb{R}^{k+m+1} \longrightarrow \mathbb{R}^r$, for $i\in[0,N]$ and by

$$Q_{is} = \frac{P_{is}(u,\dot{u},...,u^{(k)},t)}{P_{is}^{'}(u,\dot{u},...,u^{(k)},t)} \quad (6)$$

for $i\in[0,N]$. P_{is} and $P_{is}^{'}$ are $2*r*(N+2)$ polynomials of order N, at most, of the variables $u,\dot{u},...,u^{(k)}$.

4. IDENTIFICATION PROCESS USING C^K SPLINE FUNCTIONS

The process identification is obtained through the following steps

- Projection of experimental data in K-Sobolev space (C^k spline approximation),
- Identification of a bilinear system
- Estimate the residual (validity test results)

4.1 Brief Description of C^k Spline Functions

C^k spline functions expansions have the remarkable property that for a considered function, the coefficients of its C^k spline functional expansion are only the set of all the derivatives (partial or total) up to k at each point of discretization on the open set Ω. This means that we can include in the $H_k(\Omega)$, the Sobolev space on Ω, generated by C^k spline functions, the set of all the differential constraints, value conditions and boundary conditions as simple exact algebraic relations.

This implies that for a given process the algebraic inclusion of the various differential invariants leads to rebuild the functional space $H_k(\Omega)$ in a well defined appropriate functional space (or manifold), specific to the given process.

In this way, Sobolev spaces generated C^k spline functions, can be compared for nonlinear differential equations, to Fourier space in which all linear differential equations are represented as algebraic relations of frequencies. Moreover, algebraic properties of C^k spline functions allow us to introduce sophisticated and efficient algorithmic formulations of the classical nonlinear differential problems.

Let us consider Ω a one-dimensional open set composed by I+1 nodes, respectively, $x_{00}, x_{01}, ..., x_{0(I-1)}, x_{0I}$, we have

$$\Omega =]x_{00}, x_{0I}[$$
$$\omega_i =]x_{0(i-1)}, x_{0(i+1)}[$$
$$\omega_i \cap \omega_{(i+1)} =]x_{0i}, x_{0(i+1)}[$$

for $i \in [1, I-1]$ and with $\omega_0 =]x_{00}, x_{01}[$ and $\omega_I =]x_{0(I-1)}, x_{0I}[$. According to reference [1], the set of C^k spline functions defined on Ω is a set of *2I(k+1)* polynomial functions define as follows

$$S_i^{k,\nu}(x) = \sum_{j=0}^{2k+1} a_{Rj}^{\nu} \left(\frac{x - i\Delta x}{\Delta x} \right)^j, \quad \forall x \in]x_{0i}, x_{0(i+1)}[$$
$$S_i^{k,\nu}(x) = \sum_{j=0}^{2k+1} a_{Lj}^{\nu} \left(\frac{x - i\Delta x}{\Delta x} \right)^j, \quad \forall x \in]x_{0(i-1)}, x_{0i}[$$

with Δx the distance between two nodes and $a_{Rj}^{\nu}, a_{Lj}^{\nu}$, *2(2k+1)(k+1)* constants. Each spline $S_i^{k,\nu}$ is associated to an open subset ω_i of Ω and satisfies the following relation

$$\left. \frac{d^l S_i^{k,\nu}}{dx^l} \right|_{x=x_{0j}} = \delta[\nu - l]\delta[i - j]$$

with $S_i^{k,\nu}$ the ν^{th} component of the spline of order k defined on the open subset ω_i , δ is the Kronecker symbol. For more details see (Rouff, 1996).

4.2 Identification Process

The dynamical modeling of weight sensors presented in this article is described in the state space representation by a bilinear algebraic structure, for our application, we suppose that the input of the system is an expansion of Heaviside functions, i.e., that measuring load are constants by pieces in time, this is the typical behavior of weighing machines, in this case we have $\dot{u} = u^{(2)} .... = u^{(k)} = 0$. Then Q_{is} and Q_{00s} depends only on *u*. Equation (5) can be written as

$$\sum_{j=0}^{N} a_j \frac{d^j y}{dt^j} = c \qquad (7)$$

where *a* and *c* are the parametrised real constants.

Our identification program is based upon this relationship. According to (5), the relations between a_j and c and the input are given by polynomial systems

We consider the projection of (7) in a k Sobolev space of k time continuous and derivable real signals on [0,*T*], the vectorial presentation of *y* in Sobolev space is defined as

$$\tilde{y} = \begin{vmatrix} y_0^0 \\ y_0^1 \\ y_0^2 \\ . \\ y_0^{\nu} \\ y_1^0 \\ . \\ y_1^{\nu} \\ . \\ y_m^0 \\ . \\ . \\ y_m^{\nu} \end{vmatrix} \qquad (8)$$

y_j^{ν} for $j \in [0, N_s]$ with N_s the points number on [0,*T*], $\nu \in [0, k]$ is the ν^{th} value of the temporal derivative of *y* at the j^{th} discretization point on [0,*T*]

We have in the real space the following relations :

$$\tilde{y}(t) = \sum_{j=0}^{N} \sum_{\nu=0}^{k} y_j^{\nu} S_j^{k,\nu}(t)$$
$$\tilde{u}(t) = \sum_{j=0}^{N} \sum_{\nu=0}^{k} u_j^{\nu} S_j^{k,\nu}(t) \qquad (9)$$

with $S_j^{k,\nu}(t)$ is the ν^{th} C^k spline function and *j* is the discretization index.

The k associated canonical topology is inducing by the following distance

$$d(\tilde{x}, \tilde{y}) = \sum_{\nu=0}^{k} \int_0^T \left[\frac{d^{\nu} x}{dt^{\nu}} - \frac{d^{\nu} y}{dt^{\nu}} \right]^2 dt \qquad (10)$$

For our computer programming, we have chosen the simplified distance defined as

$$d(\tilde{x}, \tilde{y}) = \int_0^T \left[x(t) - y(t) \right]^2 dt$$

Let us consider

$$\sum_{j=0}^{N} a_j \mathcal{O}^j \tilde{y} = c \qquad (11)$$

the representation of (9) in the Sobolev space, $\mathcal{O}^j$ is j^{th} iterative temporal derivative operator.

The identification principle leads to minimize the distance between $\sum_{j=0}^{N} a_j \mathcal{O}^j \tilde{y}$ and *c*.

Taking into account the known experimental data *y* and *u*, we choose the a_j coefficients in order to minimize the distance between $\sum_{j=0}^{N} a_j \mathcal{O}^j \tilde{y}$ and *c*.

4.3 Validity Test Results

In this case, our algorithm leads to minimize the following residual $\Re_e$

$$\Re_e = \int_0^T \left[\sum_{n=0}^{N} \sum_{j=0}^{M} \sum_{\nu=0}^{k} a_n y_j^{\nu} S_j^{(n)_k,\nu}(t) - c \sum_{j=0}^{M} \sum_{\nu=0}^{k} u_j^{\nu} S_j^{k,\nu}(t) \right]^2 dt \quad (12)$$

it can be written as

$$\nabla_{a_i} \Re_e = 2\int_0^T \left[a_i \sum_{j=0}^{M} \sum_{\nu=0}^{k} y_j^{\nu} S_j^{(n)_k,\nu}(t) - c \sum_{j=0}^{M} \sum_{\nu=0}^{k} u_j^{\nu} S_j^{k,\nu}(t) \right]^2 * \sum_{j=0}^{M} \sum_{\nu=0}^{k} y_j^{\nu} S_j^{(n)_k,\nu}(t)\, dt$$

$$= 2a_i \sum_{\substack{j1=0\\ \nu1=0}}^{M,k} \sum_{\substack{j2=0\\ \nu2=0}}^{k} y_{j1}^{\nu1} u_{j2}^{\nu2} \int_0^T S_{j1}^{(i)_k,\nu1}(t) S_{j2}^{(i)_k,\nu2}(t) dt$$

$$- 2c \sum_{\substack{j1=0\\ \nu1=0}}^{M,k} \sum_{\substack{j2=0\\ \nu2=0}}^{k} y_{j1}^{\nu1} u_{j2}^{\nu2} \int_0^T S_{j1}^{k,\nu1}(t) S_{j2}^{k,\nu2}(t) dt = 0$$

We solve this problem by a linear system.

$$\nabla_{a_i} \Re_e = 0 \quad (13)$$

This equation leads to the following linear system

$$\begin{vmatrix} . & . & . & . & . \\ . & . & . & . & . \\ . & . & \left\langle \sum_{\substack{j1=0\\ \nu1=0}}^{M,k} \sum_{\substack{j2=0\\ \nu2=0}}^{M,k} M_{\substack{j1,j2\\ \nu1,\nu2}}^{l,c} y_{j1}^{\nu1} u_{j2}^{\nu2} \right\rangle & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{vmatrix} \begin{pmatrix} a_0 \\ a_1 \\ . \\ a_i \\ a_N \end{pmatrix} =$$

$$\begin{vmatrix} . & . & . & . & . \\ . & . & . & . & . \\ . & . & \left\langle c \sum_{\substack{j1=0\\ \nu1=0}}^{M,k} \sum_{\substack{j2=0\\ \nu2=0}}^{M,k} N_{\substack{j1,j2\\ \nu1,\nu2}}^{l} y_{j1}^{\nu1} u_{j2}^{\nu2} \right\rangle & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{vmatrix}$$

with

$$M_{\substack{j1,j2\\ \nu1,\nu2}}^{l,c} = \left\langle S_{j1}^{(l)_k,\nu1}(t) S_{j2}^{(c)_k,\nu2}(t) \right\rangle$$

c is the row and column index of the matrix defined above and

$$N_{\substack{j1,j2\\ \nu1,\nu2}}^{l} = \left\langle S_{j1}^{(l)_k,\nu1}(t) S_{j2}^{k,\nu2}(t) \right\rangle$$

is the l^{th} temporal derivative of $\nu1$ component of C^k spline of *j1* point. By writing the equation mentioned above in its matrix form, we have

$$M_a \cdot A = M_b \quad (14)$$

If the matrix M_a is invertible, then we have the solution of the vector A with $A = (a_0, a_1, a_2, ..., a_N)^T$

5. EXPERIMENTAL IDENTIFICATION AND SIGNAL PROCESSING OF WEIGHT SENSORS

5.1 Presentations of the sensor and Experimental System

In this paper we present the identification results of the only gauge strain sensor (Fig. 1).

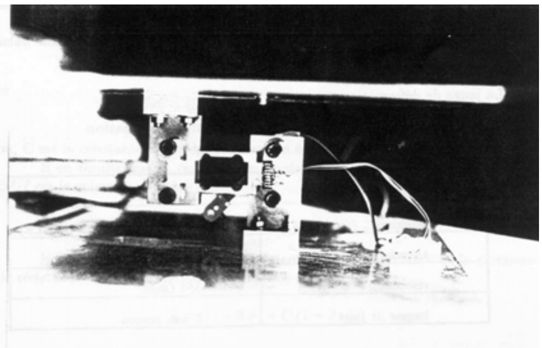

Fig. 1. Presentation of the experimental strain gauge sensor

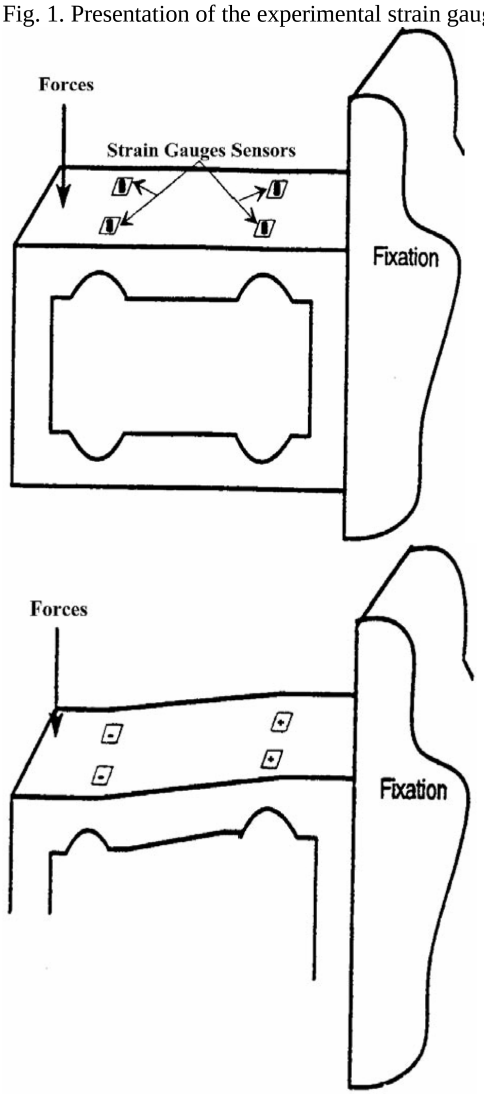

Fig. 2. Schematic representation of the deformation of the mechanical transducer used in the technology of the weighing machine

In these figures we can find the action of the weight as localized forces applied to the transducer and the strain gauge sensors. This strain gauge sensor uses the deformation of a parallelogram mechanical transducer of aluminium alloy, with four strain gauge sensors placed as shown in Fig. 2. These four strain gauge sensors are used in wheastone bridges in order to improve the sensitivity of the system.

We have used an experimental system Fig. 3. composed by a high sensitivity thermal box which is able to maintain the temperature of the sensor at a fixed value with 0.1 °C of uncertainty, high sensitivity electronic measure (10^{-9}) and a mechanical device of automatic loading of the tested weighing sensors. All these experimental devices are controlled by computer in order to plan long term automatic identification process.

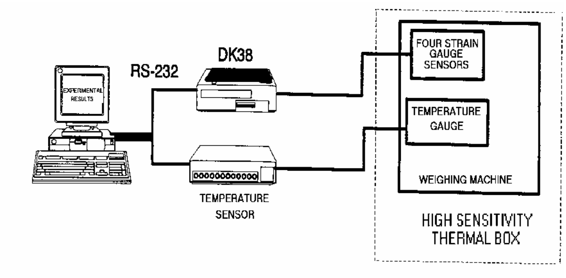

Fig. 3. Experimental System.

5.2 Identification Results

In our case, we have used the C^k spline functions. In order to obtain an experimental data projection in a K-Sobolev space we define the following approximation with k=3 (equation 9):

$$f(t) = \sum_{i=0}^{Ns} \left\{ f(t)_i S_i^{3,0} + f(t)_i^{(1)} S_i^{3,1} + f(t)_i^{(2)} S_i^{3,2} + f(t)_i^{(3)} S_i^{3,3} \right\}$$

The Fig. 4. shows the comparison of our projection (in continuous and regular line) with the time response of our weighing sensor (the noisy signal), for a weight of 3 kg at 25 °C. We can easily see how well our projection corresponds to the real response of the sensor.

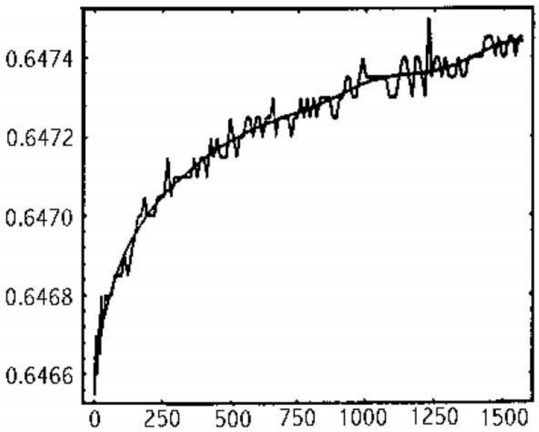

Fig. 4. Real and estimated time responses of our weighing sensor

The dynamical modeling of weight sensors is described in the state space representation by a bilinear algebraic structure, for our application, we suppose that the input of the system is an expansion of Heaviside functions, in this case we have the Equation (7) can be written as

$$\sum_{j=0}^{N} a_j \frac{d^j y}{dt^j} = 1$$

For our identification, *N=4*, the differential equation which presents the creep behavior defined as

$$a_0 y + a_1 y' + a_2 y'' + a_3 y^3 + a_4 y^4 = 1$$

Figures. 5-9 give the parameter variations $\{a_0, a_1, a_2, a_3, a_4\}$ of our identification process for creep behavior. The abscissa is in Kilogram

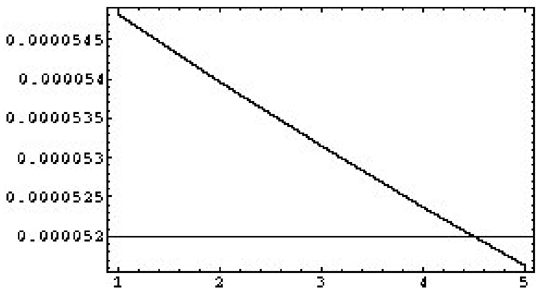

Fig. 5. Variation of a_0 parameter

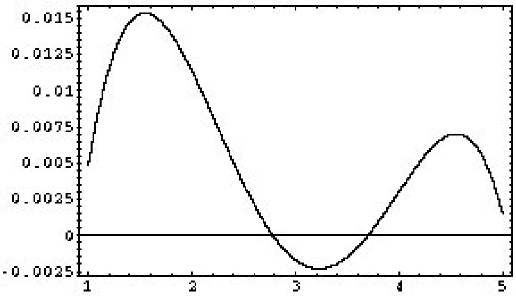

Fig. 6. Variation of a_1 parameter

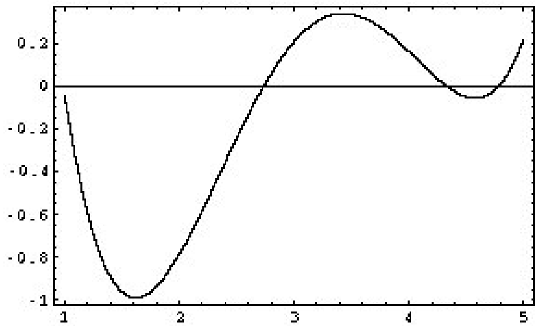

Fig. 7. Variation of a_2 parameter,

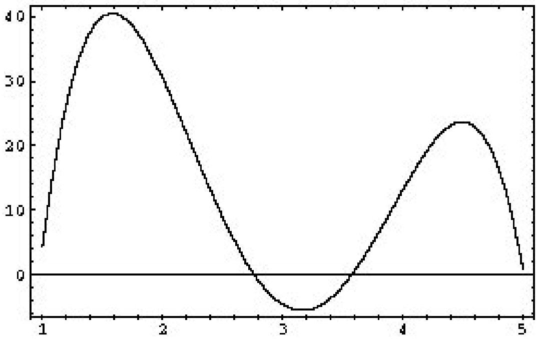

Fig. 8. Variation of a_3 parameter

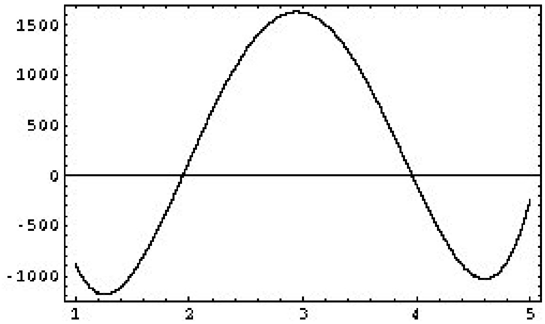

Fig. 9. Variation of a_4 parameter

The validity test results of our identification are defined by the residual equation 12.

The residual results are given in table. 1. These residuals show that the models are in good accordance with the experimental data and can be used for these kind of sensors to correct creep and relaxation processes.

Table 1 The residuals Results

Kilogram	Residual
1	0.000045358247442
2	0.000039089867331
3	0.000039453085677
4	0.000011964371113
5	0.000027456429087

6. CONCLUSION

In this article we have presented an identification and signal processing in order to correct creep and relaxation for the technology of strain gauge force and weight sensors. These results depend strongly on the design and on the technology used by the considered force and weight sensors. For the technology presented in this paper, we have shown that creep and relaxation can be modelled with a sufficient precision to improve this technology. This opens the way to the framework of intelligent sensors and to the design of inexpensive mechanical transducers for these kind of weight sensors.

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