ROBUST FDI WITH MIXED $\mathcal{H}_2/\mathcal{H}_\infty$ CRITERIA FOR DISCRETE-TIME LINEAR SYSTEMS

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Abstract: In this work, the problem of robust fault detection in discrete time uncertain linear systems is studied. The robustness is analyzed using the characterization of two types of perturbation signals: energy signals and bounded power signals. For these two types of signals, $\mathcal{H}_2/\mathcal{H}_\infty$ mixed performance approaches are used for fault detection filter design. The robust filter synthesis is obtained generating a convex optimization problem, whose numeric solution is reached by means of linear matrix inequalities (LMI). The approach is a simple procedure for the design of robust fault detection filter. *Copyright ©2005 IFAC*

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1. INTRODUCTION

A fundamental issue in a fault supervision, detection and diagnostic system is the fault Detection and isolation (FDI) filter, which should be designed for operating under adverse conditions due to the presence of external unknown signals, uncertainties and to diverse operation conditions. Such a filter should be able to produce residual signals that allow: 1) To determinate the fault presence under adverse conditions (Robust detection). 2) To determinate the fault origin (Robust separation).

Due to the adverse conditions (perturbations, uncertainties), the robust filter performance index is defined from a robustness measure from the sensitivity to faults sensitivity to perturbations. In this direction, the sensitivity measure of a filter FDI can be characterized as follows:

$$\mathcal{S}_2 = \frac{||H_{e_z\nu}||_2}{||H_{e_z\omega}||_2}; \quad \text{o} \quad \mathcal{S}_\infty = \frac{||H_{e_z\nu}||_\infty}{||H_{e_z\omega}||_\infty};$$

where H_{-} is the transfer function, ν_i are the faults, ω the disturbances and e_z the residual signal. In this way, the robust filters design should offer an excellent faults sensitivity and, if it is possible, the disturbances rejection (Edelmayer *et al.*, 1994).

For this study, let consider the following structured uncertainties diagnosis model:

$$\begin{aligned} x(k+1) &= (A+\Delta A(k))x(k) + B_1\omega(k) + \\ &\quad (B_2+\Delta B(k))u(k) + F_1\nu(k), \\ z(k) &= C_1x(k) + D_{11}\omega(k) + F_3\nu(k), \\ y(k) &= (C_2+\Delta C(k))x(k) + D_{21}\omega(k) + F_2\nu(k), \end{aligned} \quad (1)$$

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where $x \in \Re^n$ are the states, $z \in \Re^m$ is a signal to measure the performance and is a combination of the available measurements, $y \in \Re^p$ the measured output signals; $\omega \in \mathcal{L}_2$ is the disturbance signal and ν is the unknown fault vector. Matrices A, B_1, C_2, D_{11} y D_{21} have appropriate dimensions. Uncertainties are characterized using

$$\begin{aligned}\Delta A(k) &= G_1 E(k) H_1, \quad \Delta B(k) = G_1 E(k) H_2,\\ \Delta C(k) &= G_2 E(k) H_1,\end{aligned}$$

where G_1, G_2, H_1, H_2, are known constant matrices with appropriate dimensions. $E(k)$ is a bounded unknown scalar function.

F_1, F_2, F_3 are the fault distribution matrices, which are assumed as known.

$F_1\nu(k)$ represents the actuators or components faults and $F_2\nu(k), F_3\nu(k)$ are the sensors faults. It is assumed that the pair (C_2, A) is detectable.

The robust fault detection problem consist in generating a residual signal $e_z(k)$ that satisfies:

$$\begin{aligned}\digamma(e_z(k)) &\leq T_h \quad \text{si} \quad \nu(k) = 0,\\ \digamma(e_z(k)) &> T_h \quad \text{si} \quad \nu(k) \neq 0,\end{aligned} \tag{2}$$

where $\digamma(e_z(k))$ is a residual *size* measurement, for example a norm, and T_h is a threshold value.

Thus, using the diagnosis model it is possible to construct a filter for the dynamic system as follows:

$$\Sigma_2 \begin{cases} \hat{x}(k+1) = A\hat{x}(k) + B_2 u(k) + \mathbf{K} e_y(k),\\ \hat{z}(k) = C_1 \hat{x}(k),\\ \hat{y}(k) = C_2 \hat{x}(k),\end{cases} \tag{3}$$

where $\mathbf{K}$ is the gain to be selected and $e_y(k) = y(k) - \hat{y}(k)$. If it is defined $e_x(k) = x(k) - \hat{x}(k)$ and the residual signal $e_z(k) = z(k) - \hat{z}(k)$, then

$$\begin{aligned} e_x(k+1) &= (A - \mathbf{K}C_2)e_x(k) + \mathbb{B}_1\tilde{\omega}(k) +\\ &\quad \mathbb{B}_2\omega(k) + (F_1 - \mathbf{K}F_2)\nu(k),\\ e_z(k) &= C_1 e_x(k) + D_{11}\omega(k) + F_3\nu(k),\end{aligned} \tag{4}$$

where $\tilde{\omega}(k) = \begin{pmatrix} x(k)\\ u(k)\end{pmatrix}$, the matrix $\mathbb{B}_1 = (\Delta A(k) - \mathbf{K}\Delta C_2(k) \quad \Delta B(k))$ and the matrix $\mathbb{B}_2 = B_1 - \mathbf{K}D_{21}$.

For fault detection, it is necessary to design $\mathbf{K}$ in order to find that the residual signal sensibility e_z due to disturbance is *small*, and *large* for fault presence. This is, given the diagnostic system (1) then the filter Σ_2 allows the fault detection if:

(1) The dynamic system (4) is asymptotically stable.
(2) The gain relation *fault-noise* $\mathcal{S} = \gamma_\nu/\gamma_{\bar{\omega}}$ is *large*, where $\bar{\omega} = \omega, \tilde{\omega}$ and $\gamma_\nu > 0$, $\gamma_{\bar{\omega}} > 0$.

In summary, the main problem of the filtering sensitivity is to find a trade-off between the minimization of the magnitude of the transfer function from unknown inputs to the residual from the one hand and maximization of the magnitude of the transfer function from the failure modes to the residual from the other in an attempt to achieve a desired minimum amplification rate of a particular failure mode in the filter residual.

Some results in the design of robust detection filters has been presented for continuous systems: in (Edelmayer *et al.*, 1994), (Chen and Patton, 1999) and (Patton and Hou, 1997) there are considered actuators faults and the solutions are presented in the context of $\mathcal{H}_\infty$. In (Ríos-Bolívar and García, 2001) it is also approached the problem from the perspective of the norm $\mathcal{H}_\infty$ but in this case the problem of fault detection is transformed as an optimal control problem and capabilities are given for the robust fault separation. In (Chen *et al.*, 1996) is presented the design of filters based on unknown input observers, which presents strong design conditions. (Zhong *et al.*, 2003) presents a method based on joining $\mathcal{H}_\infty$ models, whose solution is based on LMI optimization. In a similar way, (Wang *et al.*, 2003) proposes a robust filter by means of an iterative method of LMI optimization, under a strong condition in the structure of the diagnosis model. (Khosrowjerdi *et al.*, 2003; Khosrowjerdi *et al.*, 2004) considers the simultaneous design of filter and control under a formulation of $\mathcal{H}_2/\mathcal{H}_\infty$ mixed optimization. General characteristics of these methods are the search of an appropriate detection, by means of the fault sensibility improvement, having small consideration in the fault separation problem, which can be seen as a problem of multiple filtrate.

For designing robust filters in discrete time systems, (Ríos-Bolívar *et al.*, 1999) considers the disturbance rejection by means of a generalized observer. This method is restricted to some conditions concerning the disturbance distribution regarding the faults in order to guarantee the separability. (Nobrega *et al.*, 2000) considers the design of LMI-based filters. There are given synthesis conditions based on joining range restrictions for certain matrices; also, the estimation error is formulated based directly on the faults, and this make harder the implementation. (Wang and Lam, 2002) presents a method for the case of structured disturbances. This method is presented as a gradient-based non restricted optimization problem. A certain sensibility level is reached. (Zhong *et al.*, 2001) proposes an approach that consists of two steps: first, a stable weighting function matrix is selected in order to improve the fault sensitivity. Second, the formulation of the fault detection filter design as a model-matching problem, whole solution is obtained by LMI optimization. In (Ríos-Bolívar and Garcia, 2004), it is presented a design method by means of the transformation of the robust detection problem being an optimal robust control problem in $\mathcal{H}_2$-

$\mathcal{H}_\infty$. Under certain conditions the robust faults detection and separation is reached.

In this paper, the robust filters synthesis by means of considering mixed performance criteria $\mathcal{H}_2/\mathcal{H}_\infty$ is proposed. The problem arises when it is considered the uncertainties like bounded power disturbance signals, while the noise is considered as well-known fixed spectral density signals. These two types of signals induce to use mixed performance indexes for obtaining the dynamic filter gain $\mathbf{K}$. The solution of the synthesis problem for the filter can be seen as a convex LMI optimization.

2. ROBUST FDI PROBLEM

As can be seen in (4), the residual signal $e_z(k)$ depends on two types of disturbance signals: $\omega(k)$ and $\tilde{\omega}(k)$. The signal $\omega(k)$ is considered as a white noise. Due the system (1) is assumed stable then, $\|\tilde{\omega}(k)\| \leq \|x(k)\| + \|u(k)\|$, if the input $u(k)$ is bounded. Since the uncertainties are bounded, ($E(k)$ is delimited), then the error dynamics will be influenced by bounded disturbances. This makes us consider, for fault detection effects, that the diagnosis model (1) can be influenced by two types of disturbances, as follows:

$$\begin{aligned} x(k+1) &= Ax(k) + \tilde{B}_1\tilde{\omega}(k) + B_1\omega(k) \\ &\quad +B_2u(k) + F_1\nu(k), \\ z(k) &= C_1x(k) + D_{11}\omega(k) + F_3\nu(k), \\ y(k) &= C_2x(k) + \tilde{D}_{21}\tilde{\omega}(k) \\ &\quad +D_{21}\omega(k) + F_2\nu(k), \end{aligned} \tag{5}$$

where $\tilde{B}_1 = (\Delta A(k) \quad \Delta B(k))$ and $\tilde{D}_{21} = (\Delta C(k) \quad 0)$, which characterizes the bounded power disturbance signal.

Considering the detection filter Σ_2, the error dynamics corresponds to the system (4). The proposed formulation is the design of a robust fault detection filter in a $\mathcal{H}_2/\mathcal{H}_\infty$ mixed framework. The robust fault detection is reached if (4) is asymptotically stable and if a fault high sensibility is maintained, while the sensitivity to the disturbances is minimized. In this context, the following definitions can be established.

Definition 2.1. Let us consider the diagnosis model (5). The fault F_i is said detectable if there exist a filter such that the generated residual signal $e_z(k)$ satisfies $\digamma(e_z(k)) > T_h$.

In the sensor faults particular case, the fault is detectable if the fault distribution matrix belongs to the system observable sub-space.

Definition 2.2. Let us consider the diagnosis model (5). The fault F_i is said robustly detectable if there exist a filter, minimizing the disturbance effect, such that the generated residual signal $e_z(k)$ satisfies $\digamma(e_z(k)) > T_h$.

Besides belonging to the observable sub-space, the fault distribution matrices and the disturbance signal direction matrices should belong to different sub-spaces. Thus, the minimizing the disturbance effect on the residual signal, the condition that the *size* of the residual be bigger than the threshold can be guaranteed.

2.1 Problem Formulation

In order to guarantee the robust fault detection and diagnosis there should be designed $\mathbf{K}$ considering that: a) The dynamic system (4) is asymptotically stable. b) The gain relationship *fault-noise* $\mathcal{S} = \gamma_\nu/\gamma_{\tilde{\omega}}$ is large.

This problem can be postulate as a mixed $\mathcal{H}_2/\mathcal{H}_\infty$ filtering problem which amounts to find a filter gain which minimizes the $\mathcal{L}_2$ norm of the transfer function from unknown inputs to the residual of the filter subject to the $\mathcal{H}_\infty$ norm of the transfer function from failure modes to the filter error.

Based on the uncertainties models, seen as disturbance signals, and of the external disturbances nature, the detection filter synthesis problem can be outlined as follows:

Problem: *Given the diagnosis model (5), we want to design* $\mathbf{K}$*, for the filter* Σ_2*, considering:*

(1) The close loop system (4) should be asymptotically stable.
(2) The disturbance effect $\tilde{\omega}$ *and the* ω *effect over the residual signal* e_z *should be minimized in any sense.*

For guaranteeing an high fault sensibility, minimizing the disturbance effect, then the separability condition presented in (Massoumnia, 1986) should be applied to the distribution matrices of those signals. Then, $(\mathrm{Im}B_1 \bigcup \mathrm{Im}\ \tilde{B}_1) \bigcap \mathrm{Im}F_1 = \emptyset$ should be satisfied, which is a necessary condition for guaranteeing fault detectability. Thus, minimizing the effects of the disturbance over the residual signal, the gain relationship fault-noise $\mathcal{S}$ will be large.

3. FILTER SYNTHESIS USING A MIXED FRAMEWORK WITH $\mathcal{H}_2/\mathcal{H}_\infty$

It will be presented the $\mathbf{K}$ design using the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms mixed, supported by the LMI technique. Let's consider the dynamic system error (4) using only the disturbance and residual signals. So, the transfer matrices are defined as follows:

$$H_{e_z\tilde{\omega}}(z) = \left[\begin{array}{c|c} \mathbb{A} & \mathbb{B}_1 \\ \hline \mathbb{C} & 0 \end{array}\right], \quad H_{e_z\omega}(z) = \left[\begin{array}{c|c} \mathbb{A} & \mathbb{B}_2 \\ \hline \mathbb{C} & \mathbb{D} \end{array}\right],$$

where

$$\mathbb{A} = A - \mathbf{K}C_2, \quad \mathbb{B}_1 = \tilde{B}_1 - \mathbf{K}\tilde{D}_{21},$$
$$\mathbb{B}_2 = B_1 - \mathbf{K}D_{21}, \quad \mathbb{C} = C_1, \quad \mathbb{D} = D_{11};$$

So, $\mathbf{K}$ should be designed in order to let $||H_{e_z\tilde{\omega}}||_2^2 < \mu$, $\mu > 0$ and $||H_{e_z\omega}||_\infty^2 < \gamma$, $\gamma > 0$, as optimization problem, this is

$$\min_{\mathbf{K}} \{||H_{e_z\tilde{\omega}}||_2^2 \ : \ ||H_{e_z\omega}||_\infty^2 < \gamma\}$$

Next lemmas are well known results, which completely characterize the norms $\mathcal{H}_2$ and $\mathcal{H}_\infty$ as LMI restrictions (Scherer *et al.*, 1997; Oliveira *et al.*, 1999).

Lemma 3.1. Inequality $||H_{e_z\tilde{\omega}}||_2^2 < \mu$ is satisfied if and only if, there exist symmetrical matrices $\mathbb{X}$, $\mathbb{W}$ that make $\mathrm{tr}(\mathbb{W}) < \mu$ and

$$\begin{bmatrix} \mathbb{X} & \mathbb{X}\mathbb{A} & \mathbb{X}\mathbb{B}_1 \\ (\circ)^T & \mathbb{X} & 0 \\ (\circ)^T & (\circ)^T & \mathbb{I} \end{bmatrix} > 0, \quad \begin{bmatrix} \mathbb{W} & \mathbb{C} \\ (\circ)^T & \mathbb{X} \end{bmatrix} > 0, \qquad (6)$$

factible.

Lemma 3.2. Inequality $||H_{e_z\omega}||_\infty^2 < \gamma$ is satisfied if and only if there exists a symmetrical matrix $\mathbb{X}$, that make

$$\begin{bmatrix} \mathbb{X} & \mathbb{X}\mathbb{A} & \mathbb{X}\mathbb{B}_2 & 0 \\ (\circ)^T & \mathbb{X} & 0 & \mathbb{C}^T \\ (\circ)^T & (\circ)^T & \mathbb{I} & \mathbb{D}^T \\ (\circ)^T & (\circ)^T & (\circ)^T & \gamma\mathbb{I} \end{bmatrix} > 0, \qquad (7)$$

factible.

Using lemma 3.1 and Lemma 3.2 it can be established the following result:

Proposition 3.1. Let consider the diagnosis model (1), with the equivalent (5). Such a model admits a filter Σ_2 that $||H_{e_z\tilde{\omega}}||_2^2 < \mu$ and $||H_{e_z\omega}||_\infty^2 < \gamma$ if and only if, there exist n-order symmetrical matrices $\mathbf{X} > 0$ and $\mathbf{W} > 0$; and matrix $\mathbf{Y}$ that satisfy the following LMIs:

$$\begin{bmatrix} \mathbf{X} & \mathbf{X}A - \mathbf{Y}C_2 & \mathbf{X}\tilde{B}_1 - \mathbf{Y}\tilde{D}_{21} \\ (\circ)^T & \mathbf{X} & 0 \\ (\circ)^T & (\circ)^T & \mathbb{I} \end{bmatrix} > 0, \qquad (8)$$

$$\begin{bmatrix} \mathbf{W} & C_1 \\ (\circ)^T & \mathbf{X} \end{bmatrix} > 0, \qquad (9)$$

$$\mathrm{tr}(\mathbf{W}) < \mu, \qquad (10)$$

$$\begin{bmatrix} \mathbf{X} & \mathbf{X}A - \mathbf{Y}C_2 & \mathbf{X}B_1 - \mathbf{Y}D_{21} & 0 \\ (\circ)^T & \mathbf{X} & 0 & C_1^T \\ (\circ)^T & (\circ)^T & \mathbb{I} & D_{11}^T \\ (\circ)^T & (\circ)^T & (\circ)^T & \gamma\mathbb{I} \end{bmatrix} > 0, \qquad (11)$$

The filter gain is given by

$$\mathbf{K} = \mathbf{X}^{-1}\mathbf{Y}. \qquad (12)$$

Proof

This proof is based on the matricial inequalities linearization procedures using variable substitution (Scherer *et al.*, 1997; Oliveira *et al.*, 2002). Let $\mathbb{X} = \mathbf{X}$, $\mathbb{W} = \mathbf{W}$. Substituting the original matrices in inequalities (6) and (7) it is obtained non linear inequalities. Considering $\mathbf{Y} = \mathbf{XK}$ the linearization is obtained. ■

Fault detection is guaranteed when the disturbance effects over the residual signal are decremented. For that, there are obtained attenuation levels μ and γ, which, added to the fault and disturbance separability condition (Massoumnia, 1986), will be always possible to find that $F(e_z(k)) > T_h$ if $\nu(k) \neq 0$.

The presented procedure, let use mixed performance criteria $\mathcal{H}_2/\mathcal{H}_\infty$ according to disturbance signals conditions in similar way to continuous case (Khosrowjerdi *et al.*, 2003), but using a less conservative methodology. This represents a generalization for designing discrete time linear systems fault detection robust filters.

4. NUMERICAL EXAMPLE

Let consider the discrete model for a vertical movement airplane, presented in (Wang and Lam, 2002):

$$A = \begin{pmatrix} 0.9813 & 0.0083 & -0.0454 & -0.2459 \\ 0.0117 & 0.5813 & -0.3898 & -1.6662 \\ 0.0457 & 0.1274 & 0.8230 & 0.4803 \\ 0.0117 & 0.0358 & 0.4433 & 1.1361 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.2 \\ 0.2 & 0 \\ 0 & 0.2 \end{pmatrix},$$

$$B_2 = \begin{pmatrix} 0.2664 & 0.0365 \\ 1.7629 & -3.2664 \\ -2.3152 & 1.7209 \\ -0.6083 & 0.4660 \end{pmatrix}, \quad C_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix},$$

$$D_{11} = \begin{pmatrix} 0 & 0.2 \\ 0 & 0 \\ 0 & 0.2 \\ 0 & 0 \end{pmatrix}, \quad C_2 = C_1, \quad D_{21} = D_{11}.$$

The uncertainties models are defined by

$$G_1 = \begin{pmatrix} 0.25 & 0 \\ 0 & 0.25 \\ 0.25 & 0 \\ 0 & 0.25 \end{pmatrix}, \quad G_2 = \begin{pmatrix} 0 & 0.25 \\ 0.25 & 0 \\ 0 & 0.25 \\ 0.25 & 0 \end{pmatrix}, \quad E(k) \ \text{random}.$$

$$H_1 = \begin{pmatrix} 0.2 & 0 & 0 & 0.2 \\ 0 & 0.2 & 0.2 & 0 \end{pmatrix}, \quad H_2 = \begin{pmatrix} 0.05 & 0 \\ 0 & 0.05 \end{pmatrix}.$$

Fault distribution matrices correspond to

$$F_1 = 2B_2, \quad F_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0.2 & 0 \\ 0.2 & 0.2 \end{pmatrix}$$

Using the LMIs numerical solutions, it can be obtained the following filter gain:

$$\mathbf{K} = \begin{pmatrix} 3.1584 & -0.9593 & -3.1511 & 0.9236 \\ 0.3798 & 1.3827 & 0.6178 & -1.3740 \\ 0.7259 & -0.8698 & -0.7239 & 0.8654 \\ -3.0967 & 0.5844 & 4.0860 & -0.5269 \end{pmatrix}.$$

The attenuation level found corresponds to $\mu = 0.1213$ and $\gamma = 0.1825$. Robust fault detection is guaranteed considering that the fault attenuation level corresponds to $\gamma_\nu = 22.2353$. Fig. 1 depicts a comparative diagram showing the maximum singular values for the diverse transfer functions for residual signal e_z, disturbances ω, uncertainties $\tilde{\omega}$, and faults ν. It can be seen the effectiveness of the presented method, because the fault detection is always possible.

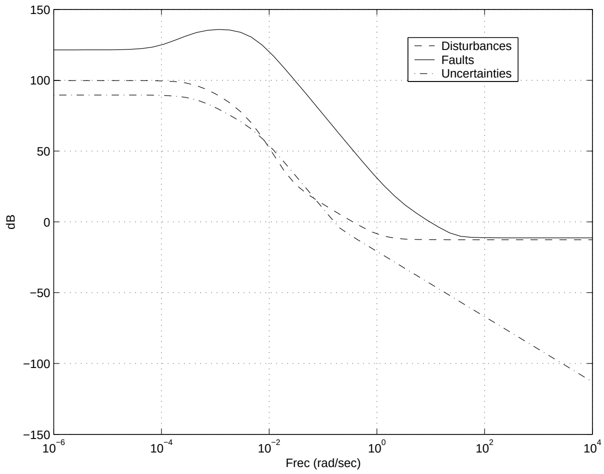

Fig. 1. Maximum singular values Diagram.

Concerning the temporal response, Fig. 2 depicts the diagnosis model measured output $y(k)$. After the setting time of the temporal dynamic, it was produced a sinusoidal type fault $k = 200s$. As

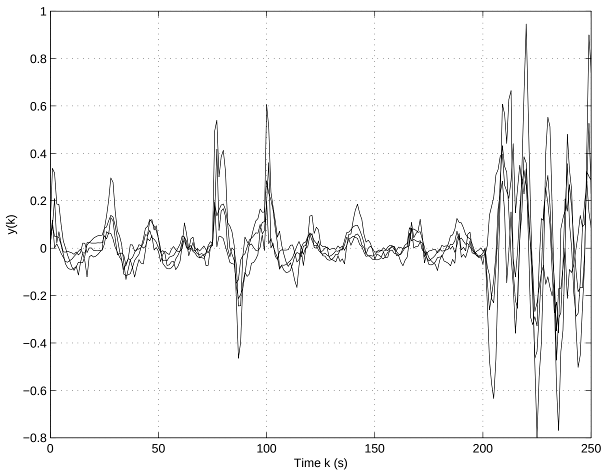

Fig. 2. System output $y(k)$.

it can be seen in output $y(k)$, it is not easy to distinguish the fault presence accurately. The disturbance and uncertainties presence make it difficult.

On the other hand, considering the residual signal $e_z(k)$, that is shown in Fig. 3, it can be established a clear difference under the fault presence when the fault patterns are shown.

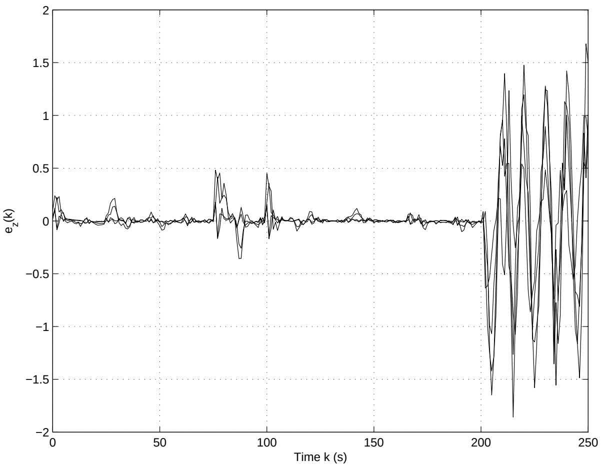

Fig. 3. Residuals $e_z(k)$.

This simulation results can confirm the capabilities of the proposed method. Robust fault detection for uncertain discrete-time systems can be obtained using the proposed filter.

5. CONCLUSIONS

Considering the system uncertainties as bounded disturbance signals, a method for designing robust detection and isolation filters for discrete time linear systems with both model uncertainty and disturbances has been proposed. Two kind of signals, energy and bounded power, are used. The filter synthesis is given using the norm criterions $\mathcal{H}_2/\mathcal{H}_\infty$ mixed, expressed as LMI restrictions. Thus, the filter gain is obtained solving an LMI-based optimization problem. This guarantee an attenuation level for disturbances and uncertainties effects over the residual signal. The faults residual sensitivities is guaranteed. This technique is simpler than some known methods.

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