FIXED-ORDER CONTROL OF ACTIVE SUSPENSION: A HYBRID APPROACH

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Abstract: This paper considers the design of a controller for an active suspension benchmark problem. For a plant model of 27th order, a 5th order controller that meets performance specifications in terms of sensitivity and control sensitivity is designed, using a novel Hybrid Evolutionary-Algebraic approach to low-order mixed-sensitivity design. This method combines evolutionary techniques with a Riccati approach by splitting the problem into a convex and a non-convex sub-problem. Simulation results demonstrate that a performance superior to that of previously published results can be achieved.

Keywords: H_∞ control, Low order control, Riccati Equation, Genetic algorithms.

1. INTRODUCTION

The problem considered in this paper is control of an active suspension system; this problem was proposed in November 2001 as a benchmark problem for the *European Journal of Control*. The challenge was to design a low-order discrete-time controller that meets given design specifications. More than thirteen solutions were published, refer to (Landau *et al.* 2003) for more details and a full reference list.

The design specifications for this problem are given in terms of closed-loop sensitivity functions, which suggests to use a H_∞ approach. Because H_∞ design techniques - unlike LQG control - offer a unified framework in which both robustness and performance of the closed loop system can be addressed, they are widely used for advanced applications. Nevertheless, PID-type controllers still dominate practical applications, in particular when low-order or fixed structure (e.g. decentralized) controllers are desired (Mizumoto *et al.* 1999). In this respect model-based design techniques fail to provide reliable techniques that can achieve performance levels comparable with classical techniques. The main reason for the difficulties facing all model-based techniques is the non-convexity of problems when the controller does not have the most general structure and the same order as the (generalized) plant. Consequently, developing efficient design techniques that allow the controller structure or order to be fixed and at the same time use the full potential of H_∞ techniques is a significant step towards bridging the gap between theory and practice in control.

In (Farag and Werner 2004*b*), (Farag and Werner 2004*a*) a novel approach based on a combination of Riccati solvers and Genetic Algorithms (GA) was proposed. The main idea was to split the original non-convex problem into a convex sub-problem that can involve a large number of decision variables, and a non-convex sub-problem with a small number of decision variables. The former problem can be solved with efficient Riccati solvers, while the latter one is solved using a global search algorithm, in this case GA. It is important to observe that the use of Riccati solvers gives a huge advantage in speed over other techniques such as those based on Linear Matrix Inequalities (LMI) (Boyd *et al.* 1994). Using LMI for fixed structure controller design is an active field of research, see e.g. (Iwasaki 1999). But even for plants of moderately high orders, iterative techniques that require to solve a sequence of LMI problems become quickly unpractical due to long computation times.

The active suspension system benchmark problem has been used in (Hol *et al.* 2004) to compare different approaches to low order H_∞ design. The goal was to compare three recently developed design techniques, considering both computational efficiency and achieved performance. The techniques are the cone complementary method (El Ghaoui *et al.* 1997), a posteriori reduction method (Wortelboer *et al.* 1999) and the Nonlinear Semi-Definite Programming (Hol *et al.* 2004). In this paper, the Hybrid Evolutionary-Algebraic (HEA) approach proposed in (Farag and Werner 2004*b*) is applied to the benchmark problem and compared with the three fixed-order design techniques mentioned above.

The paper is organized as follows. Section 2 gives a description of the active suspension system and the design objectives. A brief review of the HEA approach is given in section 3. The application to the benchmark problem and results are given section 4, and conclusions are drawn in section 5.

2. SYSTEM DESCRIPTION

The structure of the active hydro-suspension system used to reduce machine vibrations is shown in Figure 1. The main parts of the system are the elastomere cone that encloses the main chamber filled with silicon oil (1), an inertia chamber (2), a piston (3), and an orifice (4) that allows the oil to flow between chambers. The control input drives the position of the piston via an actuator and measured output is the residual force. The key idea of active suspension is to change the elasticity of the closed loop system in such a way that the vibration generated by the machine is absorbed.

The primary control objective is to attenuate disturbances in a large frequency band and in the presence of load variation. The specifications are presented in terms of constraints on the closed-loop sensitivity functions, so using a H_∞ performance measure seems natural for this problem. The motivation for using a low-order controller comes from the fact that a fast sampling rate is desired and the control action of higher order controller requires more time to be executed.

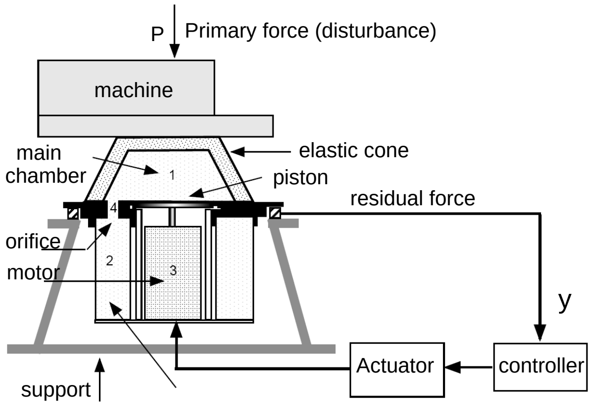

Fig. 1. Schematic diagram of the active suspension system

To simulate the vibrations generated by the machine experimentally, a computer driven shaker is used to generate artificial vibrations. Two transfer functions are defined, the first one describing the dynamics between the excitation of the shaker and the residual force, and the second one the dynamics between the control input and the residual force. These transfer functions are referred to as primary transfer function $D(s)$ and secondary transfer function $G(s)$, respectively, see Figure 2. A discrete-time version of both transfer functions can be downloaded from the benchmark problem web site:

http://iawww.epfl.ch/News/EJC_Benchmark/index.html.

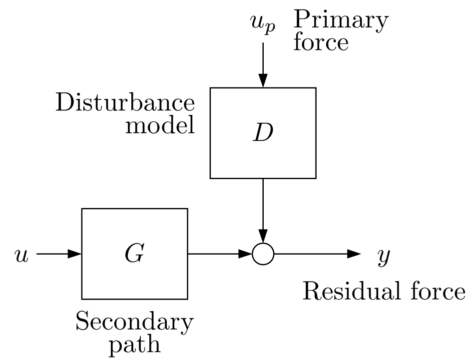

Fig. 2. Open-loop system

The frequency responses of the primary and secondary path are shown in Figure 3; the sampling frequency is $f_s = 800$ Hz, and the model order is 17.

The frequency responses shown in Figure 3 show that the primary path has several resonant modes, at 31.5 Hz, 160 Hz, 240 Hz, 275 Hz, and 370 Hz. The control objective can be stated as follows.

Control Problem: Find a discrete-time LTI controller of lowest possible order that achieves a small residual force around the first and the second vibration modes of the primary path model, and distributes the amplification of disturbances over the higher frequencies. In addition, the con-

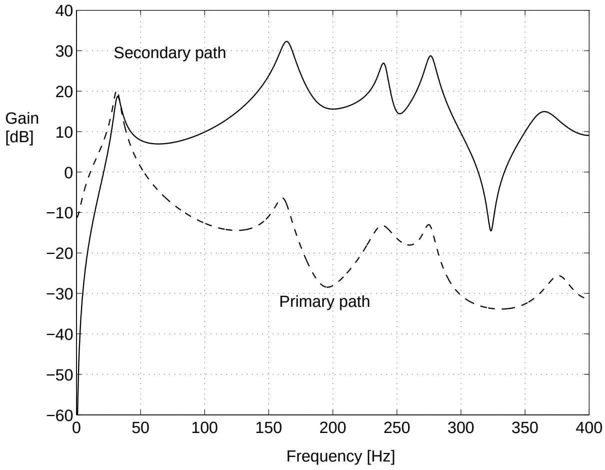

Fig. 3. Frequency responses of primary and secondary path

troller gain should equal zero at the frequency $0.5f_s$. For more details see (Landau *et al.* 2003).

More precisely, the control objectives are provided in terms of constraints on the closed loop sensitivity function S and the controller sensitivity KS, as shown in Figures 4 and 5 respectively.

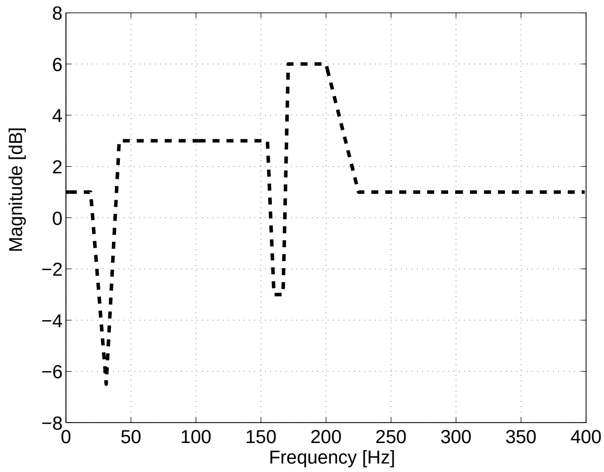

Fig. 4. Constraints on output sensitivity

For comparison purposes the H_∞ design strategy used in this paper is identical to that proposed in (Hol *et al.* 2004). In particular, the same four-block structure and the same shaping filters (weights) are used here, as shown in Figure 6.

The objective is to minimize the H_∞ norm of the closed-loop system from $w^T = [w_1^T \; w_2^T]$ to $z^T = [z_1^T \; z_2^T]$ shown in Figure 6, i.e.

$$\min_{K(s)} \left\| \begin{matrix} W_1SV_1 & W_1SGV_2 \\ W_2KSV_1 & W_2KSGV_2 \end{matrix} \right\|_\infty \qquad (1)$$

where

$$S = \frac{1}{1+KG}$$

is the sensitivity and V_1, V_2, W_1 and W_2 are shaping filters.

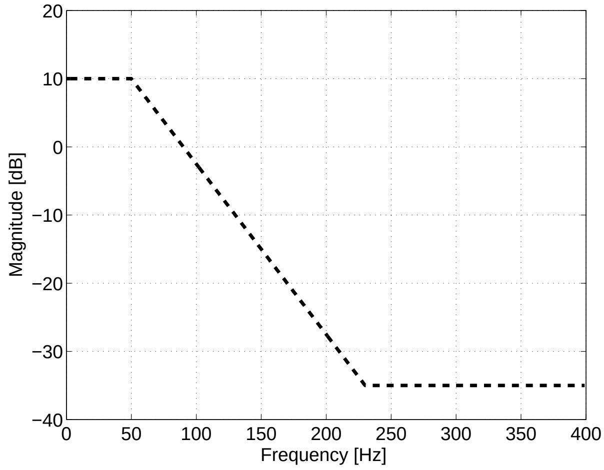

Fig. 5. Constraints on control sensitivity

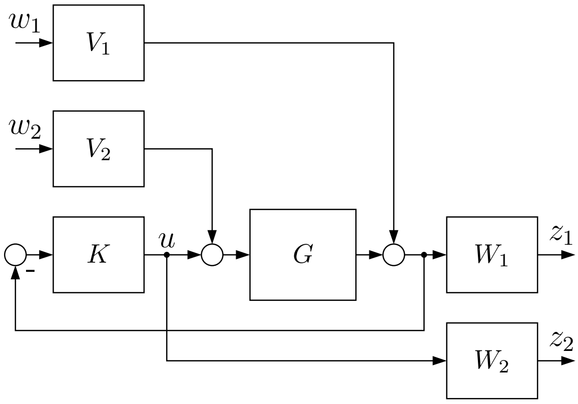

Fig. 6. Four block structure

A continuous state space model of the plant with matrices A, B, and C was obtained by applying using the Tustin approximation

$$z = \frac{1 + T_s s/2}{1 - T_s s/2}$$

3. HYBRID EVOLUTIONARY-ALGEBRAIC APPROACH

In this section we discuss the application of the hybrid evolutionary-algebraic approach proposed in (Farag and Werner 2004*b*) to the design problem in the previous section. Introduce a state space realization of the generalized plant model

$$\begin{aligned} \dot{x} &= Ax + B_w w + Bu \\ z &= C_z x + D_{zw} w + D_z u \\ y &= Cx + D_w w \end{aligned} \qquad (2)$$

where $x \in R^n$ is the state vector, $u \in R^{n_u}$ is the control input, $w \in R^{n_w}$ is a unit intensity white noise process, $y \in R^{n_y}$ is the measured output, $z \in R^{n_z}$ is the performance output. The four-block structure shown in Figure 6 can be easily rearranged in the form of the generalized plant representation shown in Figure 7, where z and w are as defined above.

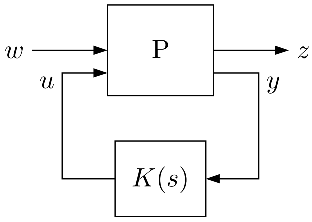

Fig. 7. Generalized plant

Introduce the feedback controller $K(s)$ from y to u with state space realization

$$\begin{aligned}\dot{\zeta}(t) &= A_K\zeta(t) + B_K y(t)\\ u(t) &= C_K\zeta(t) + D_K y(t)\end{aligned} \tag{3}$$

where $\zeta(t) \in R^{n_c}$ is the controller state vector, and $n_c < n$ is the order of the controller.

Let $T(s)$ denote the closed-loop transfer function from w to z in Figure 7, with state space realization

$$\begin{aligned}\dot{x}_{cl} &= \bar{A}x_{cl} + \bar{B}w\\ z &= \bar{C}x_{cl} + \bar{D}w\end{aligned}$$

where

$$\bar{A} = \begin{bmatrix} A + BD_KC & BC_K\\ B_KC & A_K\end{bmatrix}, \bar{B} = \begin{bmatrix} B_w + BD_KD_w\\ B_KD_w\end{bmatrix}$$

$$\bar{C} = \begin{bmatrix} C_z + D_zD_KC & D_zC_K\end{bmatrix},\ \bar{D} = D_{zw} + D_zD_KD_w$$

The problem now is to find a controller $K(s)$ that minimizes γ subject to

$$\|T(s)\|_\infty < \gamma$$

We will use the following result.

Theorem 3.1. The matrix $\bar{A}$ is stable and $\|T\|_\infty < \gamma$ if and only if there exists a solution $P = P^T > 0$ to the LMI

$$\begin{bmatrix} \bar{A}^TP + P\bar{A} + \gamma^{-1}\bar{C}^T\bar{C} & P\bar{B} + \gamma^{-1}\bar{C}^T\bar{D}\\ \bar{B}^TP + \gamma^{-1}\bar{D}^T\bar{C} & -\gamma I + \gamma^{-1}\bar{D}^T\bar{D}\end{bmatrix} \leq 0 \tag{4}$$

Proof: See (Boyd *et al.* 1994).

Define the set $\mathcal{K}$ as

$$\left\{K(s) : K(s) = \frac{b_{n_c}s^{n_c} + b_{n_c-1}s^{n_c-1} + ... + b_0}{a_{n_c}s^{n_c} + a_{n_c-1}s^{n_c-1} + ... + a_0}\right\}$$

It is clear that the above set covers all SISO nc-order controllers. Expressing the controllers as transfer function rather than state space model reduces the number of the decision variables of the controller, which is important when using the HEA approach.

The design problem can now be formulated as

$$\min_{K(s)\in\mathcal{K}} \gamma \quad \text{such that (4) holds}$$

Using the Schur complement, the inequality (4) holds if and only if

$$\begin{aligned}&\bar{A}^TP + P\bar{A} + \gamma^{-1}\bar{C}^T\bar{C} -\\ &(P\bar{B} + \gamma^{-1}\bar{C}^T\bar{D})R^{-1}(\bar{B}^TP + \gamma^{-1}\bar{D}^T\bar{C}) \leq 0\end{aligned} \tag{5}$$

and

$$R = -\gamma I + \gamma^{-1}\bar{D}^T\bar{D} \leq 0 \tag{6}$$

Since the optimal solution P subject to the above inequality constraint is always on the boundary, the inequality can be replaced by an equation. Instead of inequality (5), we can thus consider

$$\begin{aligned}&\bar{A}(\theta)^TP + P\bar{A}(\theta) + \gamma^{-1}\bar{C}^T(\theta)\bar{C}(\theta) -\\ &(P\bar{B}(\theta) + V(\theta))R(\theta)^{-1}(\bar{B}^T(\theta)P + V(\theta)^T) = 0\end{aligned} \tag{7}$$

where

$$V = \gamma^{-1}\bar{C}^T\bar{D}$$

and

$$\theta^T = [b_{n_c},\ b_{n_c-1}, ...,\ b_0,\ a_{n_c},\ a_{n_c-1},\ a_0]$$

is a vector containing the controller variables.

The problem now is to find θ and $P = P^T > 0$ that satisfy (7). This problem is non-convex; however, for any fixed value $\theta = \theta_o$ the problem is convex and solvable via standard Riccati solvers. These considerations motivate the following usage of a genetic algorithm.

Algorithm

- Generate an initial random population of controllers $\{K_1(s),\ K_2(s),\ \ldots,\ K_\mu(s)\}$
- Use the objective function

$$f(K_i) = \begin{cases}\gamma, & \text{if } \bar{A} \text{ is stable}\\ \kappa(\bar{A}) + \beta, & \text{if } \bar{A} \text{ is unstable}\end{cases}$$

  where $\kappa(\bar{A})$ stands for maximum real part of the eigenvalues of $\bar{A}$, and β is a penalty (e.g. 10^3) for destabilizing controllers
- Use ranking to determine fitness

Note that starting with a complete random papulation - which may contain controllers that do not stabilize the plant - creates no problem at all for the HEA algorithm. The above penalty based approach enables the HEA algorithm to search for stabilizing controllers in early generations, and to turn to the task of norm minimization later, for more details refer to (Farag and Werner 2004*b*).

This property gives the HEA approach an advantage over design techniques that cannot be started with random initialization.

Note also that since the plant order is 17, the weighting filters order is 10 and the controller order is 5, the Lyaponuv matrix P contains 496 decision variable. So, if GA alone is used to search for $K(s)$ and P, the number of decision variables is $(496 + 10 = 506)$. On the other hand, using the HEA approach, GA is used to search for θ only (i.e. just 10 variables). In other words, the large non-convex search space (506-variables) is divided into a small non-convex part (10-variables) solved with GA, and large convex part (496-parameter) solved with Riccati solvers.

4. DISCUSSION AND RESULTS

This section compares the HEA approach proposed in the previous section with three other design techniques. All HEA computations are performed on a standard desktop computer (Pentium-IV 2.0G, 512MB Ram). The HEA method is implemented using standard GA operations with the following preferences: population size $\mu = 20$, maximum number iteration $N = 100$, floating point representation, selection performed using stochastic universal sampling, elitism (with two individuals), cross-over (80% of population) and mutation (20% of population) implemented in the standard manner.

The three design techniques used for comparison are the: Curved Line-search interior point method (CLIP) (Hol *et al.* 2004), Cone Complementary method (CC) (El Ghaoui *et al.* 1997), and finally order reduction of a full-order H_∞ controller designed using the standard Ricatti solvers, using the posteriori reduction method (Wortelboer *et al.* 1999). All low order controllers (except one HEA controller) are of 5th order, all controllers have been discretized and an extra term $\frac{z+1}{z}$ has been appended to each controller to satisfy the requirement of zero gain at 0.5 f_s.

Table 1 shows the comparison of the computation times as well as the performance values obtained with each design techniques. The full order H_∞ controller yields a closed loop performance of $\gamma = 2.48$, which is the minimum possible value that can be achieved with a linear rational controller. It is well known that using model order reduction techniques does usually not preserve performance or stability properties achieved with the full-order controller. The frequency responses of all controllers are shown in Figure 8 and Figure 9.

Computing the 2nd order controller required significantly less computation time (1.42 min) than any of the other high-performance low-order controllers, and yet achieves an H_∞ norm only 5% greater than the full-order controller. The transfer functions of this controller in discrete and continuous time are

$$K(s) = \frac{0.0078s^2 + 8.508s - 3.462}{0.644s^2 + 3.538s + 4.715}$$

$$K_d(z) = \frac{0.0203 - 0.02417z^{-1} + 0.003856z^{-2}}{1 - 1.993z^{-1} + 0.9932z^{-2}}$$

Table 1. Performance and computation times of controllers

Controller	γ	Computation time
Full order (27th-ord)	2.476	11.9 sec
Balanced reduced (5th-ord)	3.405	12.8 sec
Curved Line search (5th-ord)	2.506	4h23min45sec
Cone complem. (5th-ord)	2.630	14min33sec
HEA (5th-ord)	2.589	2.7 min
HEA (2th-ord)	2.596	1.42 min

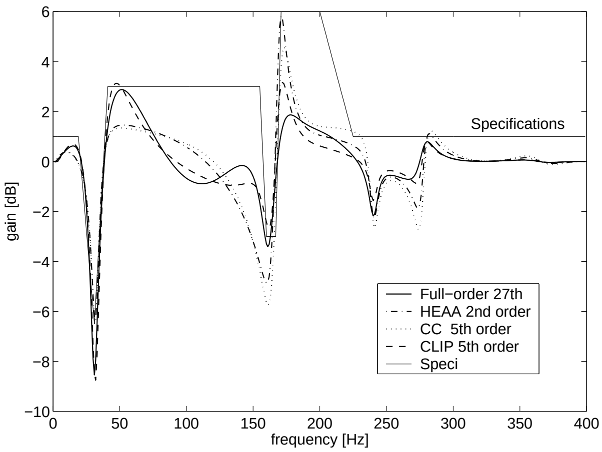

Fig. 8. Frequency response of the sensitivity S

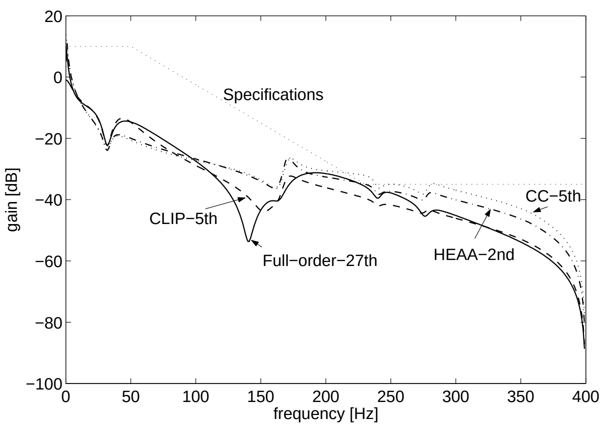

Fig. 9. Frequency response of the controller sensitivity KS

5. CONCLUSION

This paper presents an application of a novel hybrid evolutionary-aglebraic approach, proposed in (Farag and Werner 2004*b*), to a benchmark problem that has been used as a testbed for many fixed-order design techniques. The case study given here here shows that the HEA approach compares favorably with other recently proposed techniques and can produce controllers that achieve high performance levels. For example, the HEA design procedure for the 2nd order controller is almost 200 times faster than the "curved line-search interior point method", and yet achieves almost the same performance. On the other hand HEA algorithm is 10 times faster than the "cone complementary method" while delivering a better performance. Moreover, in case of MIMO systems the HEA approach can be extended to decentralized controller design in a straightforward manner, see (Farag and Werner 2004*a*).

Among the attractive features of the HEA approach is its built-in ability of finding stabilizing controllers; experience has shown that this usually happens quickly over a few early generations. This property is important when no simple initialization approach is available.

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