

FEEDBACK LINEARIZATION OF PIECEWISE LINEAR SYSTEMS

M.K. Çamlıbel^{*,**} İ. Üstoğlu^{***}

** Dept. of Mechanical Eng., Eindhoven University of
Technology, P.O. Box 513, 5600 MB Eindhoven
The Netherlands*

*** Dept. of Electronics and Communication Eng., Dogus
University, Acibadem 81010, Kadikoy-Istanbul, Turkey
kcamlibel@dogus.edu.tr*

**** Dept. of Electrical Eng., Istanbul Technical University,
Maslak, Istanbul, Turkey*

Abstract: One of the classical problems of nonlinear systems and control theory is feedback linearization. Its obvious motivation is that one can utilize linear control theory if the nonlinear system at hand is linearizable by feedback. This problem is well-understood for the smooth nonlinear systems. In the present paper, we investigate feedback linearizability of a class of piecewise linear, and hence nonsmooth, systems. *Copyright©2005 IFAC*

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1. INTRODUCTION

Feedback linearization is a topic that is well-studied in the context of smooth nonlinear systems (see e.g. (Sastry, 1999; Sontag, 1998; Khalil, 2002)). Its obvious motivation stems from the fact that the linear systems theory can be employed for both analysis and synthesis for feedback linearizable systems. As far as the authors knowledge, the non-smooth case has not been studied yet in the depth in the literature.

This paper aims at investigating feedback linearization problem for a class of piecewise linear systems that are called conewise linear systems. Basically, these systems consist of a number of linear dynamics that are active on certain cones in the state-space. The papers (Çamlıbel

et al., 2003; Çamlıbel *et al.*, 2004; Çamlıbel *et al.*, 2005) studied the controllability properties of these systems in an increasing level of generality and provided algebraic necessary and sufficient conditions. In this paper, we show that these results on the controllability make it possible to solve the feedback linearization problem for this class of systems.

The organization of the paper is as follows. The next section introduces the notations and the conventions in force throughout the paper. This is followed by a quick review of the feedback linearization problem in the context of smooth nonlinear systems. The main results of the system are presented in two subsequent sections. The first one is devoted to the relatively easier case of bimodal piecewise linear systems. This section

gives a flavor of the proof of the presented results and will be followed by a section on conewise linear systems. Finally, the conclusions section closes the paper.

2. NOTATION

The symbol \mathbb{R} denotes the set of real numbers, \mathbb{R}^n n -tuples of real numbers, $\mathbb{R}^{n \times m}$ $n \times m$ real matrices.

For a matrix $A \in \mathbb{R}^{n \times m}$, A^T stands for its transpose, $\ker A$ for its kernel, i.e. the set $\{x \in \mathbb{R}^m : Ax = 0\}$, $\text{im } A$ for its image, i.e. the set $\{y \in \mathbb{R}^n : y = Ax \text{ for some } x \in \mathbb{R}^m\}$.

The notation $\langle A \mid \text{im } B \rangle$ denotes the linear space $\text{im } B + A \text{im } B + \dots + A^{n-1} \text{im } B$ where $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$. We say that the pair (A, B) is controllable if the relation $\langle A \mid \text{im } B \rangle = \mathbb{R}^n$ holds.

The set of all locally integrable functions are denoted by $\mathcal{L}_1^{\text{loc}}$. Sometimes we say that a function is sufficiently smooth meaning that the function is sufficiently many times continuously differentiable.

A set \mathcal{Y} is said to be a cone if $\alpha x \in \mathcal{Y}$ for all $x \in \mathcal{Y}$ and $\alpha \geq 0$. For a nonempty set \mathcal{Y} (not necessarily a cone), we define its dual cone as the set $\{x \mid x^T y \geq 0 \text{ for all } y \in \mathcal{Y}\}$. It is denoted by \mathcal{Y}^* .

3. FEEDBACK LINEARIZATION OF NONLINEAR SYSTEMS

The problem of rendering a nonlinear system to a linear system by means of a state feedback is called *feedback linearization problem*. The motivation of this problem comes from the obvious fact that one can employ linear systems theory for the analysis and the synthesis of feedback linearizable nonlinear systems.

To formulate the problem precisely, consider the single-input nonlinear systems of the form

$$\dot{x} = f(x) + g(x)u \quad (1)$$

where $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$, $g : \mathbb{R}^n \rightarrow \mathbb{R}^n$ are sufficiently smooth. We say that the system (1) is *feedback linearizable* if there exist sufficiently smooth functions $\alpha : \mathbb{R}^n \rightarrow \mathbb{R}^n$, $\beta : \mathbb{R}^n \rightarrow \mathbb{R}^n$, and $\phi : \mathbb{R}^n \rightarrow \mathbb{R}^n$ such that the feedback transformation

$$u = \alpha(x) + \beta(x)v \quad (2)$$

and the state transformation

$$z = \phi(x) \quad (3)$$

render the system (1) to the linear system

$$\dot{z} = Az + bv \quad (4)$$

where $A \in \mathbb{R}^{n \times n}$, $b \in \mathbb{R}^n$, and the pair (A, b) is controllable.

The solutions of the feedback linearization problem and its variations are among the classical results of the nonlinear system theory (see e.g. (Khalil, 2002; Sastry, 1999)). It is well-known that the problem is closely related to the notion of relative degree.

Consider the single-input nonlinear system (1) together with the single-output

$$y = h(x) \quad (5)$$

where $h : \mathbb{R}^n \rightarrow \mathbb{R}$ is a sufficiently smooth function. The system (1) and (5) is said to have *relative degree k* at x_0 if

$$L_g L_f^i h(\bar{x}) = 0 \quad (6a)$$

for all $\bar{x} \in \mathbb{R}^n$ and $i = 0, 1, \dots, k-2$, and

$$L_g L_f^{k-1} h(x_0) \neq 0. \quad (6b)$$

Here $L_f h(\bar{x})$ denotes the Lie derivative of h with respect to the vector field f at the point \bar{x} , i.e.

$$L_f h(x) = \sum_{i=1}^n \frac{\partial h}{\partial x_i}(\bar{x}) f_i(\bar{x}). \quad (7)$$

In the linear case, i.e. when $f(x) = Ax$, $g(x) = b$, and $h(x) = c^T x$, one gets

$$L_g L_f^i h(\bar{x}) = c^T A^i b.$$

Hence, the usual definition of relative degree for linear systems is compatible with the above definition.

The following theorem presents necessary and sufficient conditions for the solvability of the feedback linearization problem.

Theorem 3.1. (Thm. 9.8 of (Sastry, 1999)). *The system (1) is feedback linearizable if and only if there exists a sufficiently smooth function h such that the system (1) and (5) has relative degree n at a point x_0 .*

This paper aims at studying a special class of nonlinear systems: piecewise linear systems. As the smoothness assumptions are not satisfied by these systems, one cannot directly apply the available results.

4. BIMODAL PIECEWISE LINEAR SYSTEMS

For the moment, we focus on the bimodal piecewise linear systems given by

$$\dot{x} = \begin{cases} A_1 x + bu & \text{if } c^T x \leq 0 \\ A_2 x + bu & \text{if } c^T x \geq 0 \end{cases} \quad (8)$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the input, $A_1, A_2 \in \mathbb{R}^{n \times n}$, $b \in \mathbb{R}^n$, and $0 \neq c \in \mathbb{R}^n$. We

assume that the overall vector field is continuous across the hypersurface $\ker c^T = \{x \mid c^T x = 0\}$, i.e.,

$$c^T x = 0 \Rightarrow A_1 x = A_2 x. \quad (9)$$

Equivalently,

$$A_2 - A_1 = e c^T \quad (10)$$

for some n -vector e .

The right hand side of (8) is Lipschitz continuous in the variable x . Hence, it follows from the theory of ordinary equations that for each initial state $x_0 \in \mathbb{R}^n$ and input $u \in \mathcal{L}_1^{\text{loc}}$ there exists a unique absolutely continuous function x satisfying (8) almost everywhere.

4.1 Controllability

In what follows we will discuss the controllability of the piecewise linear systems as it play an important role in our treatment.

The fundamental concept of controllability in the state space framework was introduced by (Kalman, 1960). The system (8) is said to be *completely controllable* if given any pair of states (x_0, x_f) there exists an input $u \in \mathcal{L}_1^{\text{loc}}$ such that the solution of (8) with $x(0) = x_0$ satisfies $x(\tau) = x_f$, for some $\tau > 0$.

For (finite-dimensional) linear systems, the notion of controllability is well-understood. In this case, algebraic necessary and conditions have been provided by Kalman, Popov, Belevitch, and Hautus. We refer to (Kailath, 1980; Sontag, 1998) for the historical remarks. When it comes to piecewise linear systems, however, necessary and sufficient conditions are hard to come by. In (Blondel and Tsitsiklis 1999), the authors showed that the controllability problem is undecidable even for very simple piecewise linear systems. Nevertheless, the bimodal systems of the form (8) posses a certain structure which can be exploited in order to derive necessary and sufficient conditions.

Theorem 4.1. (Thm. 5 of (Çamlibel et al., 2004)). *Let e be as in (10). The bimodal piecewise linear system (8) is completely controllable if and only if the following conditions hold.*

- (1) *The pair $(A_1, [b \ e])$ is controllable,*
- (2) *The implication*

$$[w^T \ \mu_i] \begin{bmatrix} \lambda I - A_i & b \\ c^T & 0 \end{bmatrix} = 0,$$

$$\lambda \in \mathbb{R}, w \neq 0, i = 1, 2 \Rightarrow \mu_1 \mu_2 > 0.$$

holds.

Remark 4.2. In case $e \in \text{im } b$ (i.e. $e = fb$ for some real number f), the first condition implies

the second. To see this, let $w \neq 0$ and $\mu_i \ i = 1, 2$ be such that

$$[w^T \ \mu_i] \begin{bmatrix} \lambda I - A_i & b \\ c^T & 0 \end{bmatrix} = 0$$

where λ is a real number. Since $e \in \text{im } b$, one gets $(\mu_1 - \mu_2)c^T = 0$. As c is a nonzero vector, we get $\mu_1 = \mu_2$. The first condition, i.e. the fact that (A_1, b) is controllable, guarantees that $\mu_1 \neq 0$. To see this, suppose that $\mu_1 = 0$. Then, one has $w^T(\lambda I - A_1) = 0$ and $w^T b = 0$. Since (A_1, b) is controllable, we get $w = 0$: contradiction! The second condition is satisfied as $\mu_1 = \mu_2 \neq 0$.

4.2 Feedback linearization

The bimodal piecewise linear system (8) is said to be *feedback linearizable* if there exists a control of the form

$$u = \begin{cases} k_1^T x + v & \text{if } c^T x \leq 0 \\ k_2^T x + v & \text{if } c^T x \geq 0 \end{cases} \quad (11)$$

with the property

$$c^T x = 0 \Rightarrow k_1^T x = k_2^T x \quad (12)$$

such that the closed loop system is linear, i.e. of the form

$$\dot{x} = Ax + bu$$

for some matrix A and vector b with the property that (A, b) is controllable.

Theorem 4.3. *The bimodal piecewise linear system given in (8) is feedback linearizable if, and only if, $e \in \text{im } b$ and (A_1, b) is controllable.*

Proof: Suppose that $e \in \text{im } b$ and (A_1, b) is controllable. Let $e = fb$ where $f \in \mathbb{R}$. Consider the feedback transformation

$$u = \begin{cases} k_1^T x & \text{if } c^T x \leq 0 \\ k_2^T x & \text{if } c^T x \geq 0 \end{cases}$$

where k_1 is arbitrary and $k_2^T = k_1^T + fc^T$. Note that the condition (12) is satisfied. Straightforward calculations show that the system (8) takes the form

$$\dot{x} = (A_1 + bk_1^T)x + bv \quad (13)$$

in the closed loop. Since (A_1, b) is controllable, so is $(A_1 + bk_1^T, b)$. Thus, we can conclude that the system (8) is feedback linearizable in view of Remark 4.2. Suppose, now, that the system (8) is feedback linearizable, i.e. there exists a feedback transformation of the form (11) which render the system into the form

$$\dot{x} = Ax + bv \quad (14)$$

for some matrix $A \in \mathbb{R}^{n \times n}$ where (A, b) is controllable. This would mean that

$$A = A_1 + bk_1^T = A_2 + bk_2^T.$$

Hence, one gets

$$A_1 - A_2 = bk_1^T - bk_2^T \quad (15)$$

$$ec^T = fbc^T \quad (16)$$

for some $f \in \mathbb{R}$. The last equation is satisfied only if $e = fb$, i.e., $e \in \text{im } b$. Then, it is enough to show that the pair (A_1, b) is controllable in order to conclude the proof. Note that $A = A_1 + bk_1^T$ for some $k_1 \in \mathbb{R}^n$. Since (A, b) is controllable, so is (A_1, b) .

One can relate Theorem 3.1 to Theorem 4.3 in the following way.

Theorem 4.4. There exists a vector $h \in \mathbb{R}^n$ such that the relative degree of the systems

$$\begin{aligned} \dot{x} &= A_i x + bu \\ y &= h^T x \end{aligned}$$

for $i = 1, 2$ is n if, and only if, the bimodal system (8) is feedback linearizable.

5. CONEWISE LINEAR SYSTEMS

We can extend the above results to a class of piecewise linear systems which consist of a number of linear dynamics that are active on some cones in the input-state space. More specifically, they are systems of the form

$$\dot{x}(t) = Ax(t) + Bu(t) + f(Cx(t)) \quad (17a)$$

$$Cx(t) \in \mathcal{Y} \quad (17b)$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the input, $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{p \times n}$, $\mathcal{Y} \subseteq \mathbb{R}^p$ is a cone, and f is a *conewise linear function* on \mathcal{Y} , i.e. there exist an integer r , cones \mathcal{Y}_i , and matrices $M^i \in \mathbb{R}^{n \times p}$ for $i = 1, 2, \dots, r$ such that

$$\cup_{i=1}^r \mathcal{Y}_i = \mathcal{Y}, \quad (18a)$$

$$f(y) = M^i y \text{ if } y \in \mathcal{Y}_i. \quad (18b)$$

These systems will be called *conewise linear systems* (CLS). Throughout the paper, we assume that

- (A₁) the cones \mathcal{Y}_i are closed, convex, and solid, and
- (A₂) the cones $M^i \mathcal{Y}_i$ are closed.

For polyhedral cones, Assumption A₁ implies A₂. However, this does not happen in general (see e.g. (Craven, 1978, Example 2.2.8)).

The simplest examples of CLSs, except the trivial case of linear systems, are the bimodal piecewise linear systems, i.e. systems of the form (8). To fit the system (8) into the framework of CLS (17), one can take $A = A_1$, $B = b$, $C = c^T$, $r = 2$, $\mathcal{Y}_1 = -\mathbb{R}_+$, $M^1 = 0$, $\mathcal{Y}_2 = \mathbb{R}_+$, and $M^2 = e$.

An interesting example of CLSs arise in the context of linear complementarity systems. Consider the linear system

$$\dot{x} = Ax + Bu + Ez \quad (19a)$$

$$w = Cx + Du + Fz \quad (19b)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, and $(z, w) \in \mathbb{R}^{p+p}$. When the external variables (z, w) satisfy the so-called complementarity relations

$$0 \leq z \perp w \geq 0 \quad (19c)$$

the overall system (19) is called a *linear complementarity system* (LCS). A wealth of examples, from various areas of engineering as well as operations research, of these piecewise linear (hybrid) systems can be found in (Çamlıbel *et al.*, 2004; Schumacher, 2004; Van der Schaft and Schumacher, 2000). For the work on the analysis of general LCSs, we refer to (Çamlıbel *et al.*, 2003; Heemels *et al.*, 2002; Çamlıbel *et al.*, 2002; Van der Schaft and Schumacher, 1996; Van der Schaft and Schumacher, 1998; Heemels *et al.*, 2000). A special case of interest emerges when all the principal minors of the matrix F are positive. Such matrices are called *P*-matrices in the literature of the mathematical programming. It is well-known (see for instance (Cottle *et al.*, 1992, Thm. 3.1.6 and Thm. 3.3.7)) that every positive definite matrix is in this class. *P*-matrices enjoy several interesting properties. One of the most well-known is in the context of linear complementarity problem, i.e. the problem of finding an *p*-vector z satisfying

$$0 \leq z \perp q + Fz \geq 0. \quad (20)$$

for a given *p*-vector q and a $p \times p$ matrix F . It is denoted by $\text{LCP}(q, F)$. When the matrix F is a *P*-matrix, $\text{LCP}(q, F)$ admits a unique solution for any $q \in \mathbb{R}^p$. This is due to a well-known theorem (see (Cottle *et al.*, 1992, Thm. 3.3.7)) of mathematical programming. Moreover, for each q there exists an index set $\alpha \subseteq \{1, 2, \dots, p\}$ such that

- (1) $-(F_{\alpha\alpha})^{-1}q_\alpha \geq 0$ and $q_{\alpha^c} - F_{\alpha^c\alpha}(F_{\alpha\alpha})^{-1}q_\alpha \geq 0$,
- (2) the unique solution z of the $\text{LCP}(q, F)$ is given by $z_\alpha = -(F_{\alpha\alpha})^{-1}q_\alpha$ and $z_{\alpha^c} = 0$

where α^c denotes the set $\{1, 2, \dots, p\} \setminus \alpha$. This shows that the mapping $q \mapsto z$ is a conewise linear function. In fact, one can come up 2^p (i.e. $r = 2^p$) (polyhedral) cones \mathcal{Y}_i and matrices M^i satisfying A₁-A₂ by using the above relations.

5.1 Controllability

Controllability properties of the CLSs are addressed in (Çamlıbel *et al.*, 2005).

Theorem 5.1. ((Çamlıbel *et al.*, 2005)). *Consider the CLS (17). Suppose that the transfer matrix $C(sI -$*

$A)^{-1}B$ is invertible as a rational matrix. The CLS (17) is completely controllable if, and only if,

(1) the relation

$$\sum_{i=1}^r \langle A + M^i C \mid \text{im } B \rangle = \mathbb{R}^n \quad (21)$$

is satisfied and

(2) the implication

$$\begin{aligned} \lambda \in \mathbb{R}, z \in \mathbb{R}^n, \text{ and } w_i \in \mathbb{R}^m \\ \begin{bmatrix} z^T w_i^T \\ \lambda I - A - M^i C - B \end{bmatrix} = 0 \text{ and} \\ w_i \in \mathcal{Y}_i^* \text{ for all } i = 1, 2, \dots, r \Rightarrow z = 0 \end{aligned}$$

holds.

Remark 5.2. Similar to the bimodal case, the first condition implies the second if $\mathcal{Y} = \mathbb{R}^p$ and $\text{im } M^i C \subseteq \text{im } B$ for all $i = 1, 2, \dots, r$. To see this, $\lambda \in \mathbb{R}$, $z \in \mathbb{R}^n$, and $w_i \in \mathcal{Y}_i^*$ for $i = 1, 2, \dots, r$ be such that

$$\begin{bmatrix} z^T w_i^T \\ \lambda I - A - M^i C - B \end{bmatrix} = 0 \quad (22)$$

holds. Since $z^T B = 0$ and $\text{im } M^i C \subseteq \text{im } B$, one gets

$$z^T (\lambda I - A) + w_i^T C = 0 \quad (23)$$

for all $i = 1, 2, \dots, r$. Since $\mathcal{Y} = \mathbb{R}^p$, for each $x \in \mathbb{R}^n$ there exists $i \in \{1, 2, \dots, r\}$ such that $Cx \in \mathcal{Y}_i$. By right-multiplying (23) by x , we get $z^T (\lambda I - A)x \geq 0$ as $w_i \in \mathcal{Y}_i^*$. This means that $z^T (\lambda I - A) = 0$. Consequently, $z \in \langle A + M^i C \mid \text{im } B \rangle^\perp$ for all $i = 1, 2, \dots, r$. Thus, we get $z = 0$ from the first condition.

5.2 Feedback linearization

The CLS (17) is said to be *feedback linearizable* if there exists a feedback transformation of the form

$$u = K_i x + v \text{ if } Cx \in \mathcal{Y}_i \quad (24)$$

where $K_i \in \mathbb{R}^{m \times n}$ and

$$K_i x = K_j x \text{ if } x \in \mathcal{Y}_i \cap \mathcal{Y}_j.$$

such that the closed loop system is linear, i.e.

$$\dot{x} = A' x + B u \quad (25)$$

for some matrices $A' \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$ with (A', B) is controllable.

The following theorem presents necessary and sufficient conditions for the feedback linearizability of CLSs.

Theorem 5.3. Consider a CLS of the form (17) with $\mathcal{Y} = \mathbb{R}^p$. Assume that $M^1 = 0$. Then, (17) is feedback linearizable if, and only if, $\text{im } M^i C \subseteq \text{im } B$ for all $i = 1, 2, \dots, r$ and $\sum_{i=1}^r \langle A + M^i C \mid \text{im } B \rangle = \mathbb{R}^n$.

Proof: For the ‘if’ part, suppose that $\text{im } M^i C \subseteq \text{im } B$ for all $i = 1, 2, \dots, r$ and $\sum_{i=1}^r \langle A + M^i C \mid \text{im } B \rangle = \mathbb{R}^n$. Note that the CLS takes the form

$$\dot{x} = (A + M^i C + BK_i)x + Bv \text{ if } Cx \in \mathcal{Y}_i \quad (26)$$

with the application of the feedback law (24). Since $\text{im } M^i C \subseteq \text{im } B$, one can choose K_i such that $M^i C + BK_i = 0$. Then, the CLS (26) becomes a linear system of the form

$$\dot{x} = Ax + Bv. \quad (27)$$

To conclude the proof, it is enough to show that the pair (A, B) is controllable. Note that $\langle A + M^i C \mid \text{im } B \rangle = \langle A + M^i C + BK_i \mid \text{im } B \rangle = \langle A \mid \text{im } B \rangle$. Since $\sum_{i=1}^r \langle A + M^i C \mid \text{im } B \rangle = \mathbb{R}^n$, one gets $\langle A \mid \text{im } B \rangle = \mathbb{R}^n$, i.e. (A, B) is controllable.

For the ‘only if’ part, suppose that the CLS (17) is feedback linearizable. This means that there exists a feedback law of the form (24) such that the closed loop system is of the form (25) where (A', B) is controllable. Note that the feedback law (24) yields

$$\dot{x} = (A + M^i C + BK_i)x + Bv \text{ if } Cx \in \mathcal{Y}_i \quad (28)$$

in the closed loop. Since the cones \mathcal{Y}_i are all solid, one gets

$$A + M^i C + BK_i = A' \quad (29)$$

for all $i = 1, 2, \dots, r$. Therefore, one gets

$$M^i C + BK_i = M^j C + BK_j \quad (30)$$

for all $i, j \in \{1, 2, \dots, r\}$. Since $M^1 = 0$ by the hypothesis, we reach $M^i C = B(K_1 - K_i)$. In turn, this results in $\text{im } M^i C \subseteq \text{im } B$. Note that $\langle A + M^i C \mid \text{im } B \rangle = \langle A + M^i C + BK_i \mid \text{im } B \rangle = \langle A' \mid \text{im } B \rangle$ for all i . Since (A', B) is controllable, we get $\sum_{i=1}^r \langle A + M^i C \mid \text{im } B \rangle = \mathbb{R}^n$.

Note that the structure of (17) reveals that the assumption $M^1 = 0$ can be made without loss of generality.

6. CONCLUSIONS

We studied the problem of feedback linearization for a class of piecewise linear systems for which the state space partitioned into cones and on each of these cones the dynamics is linear. Earlier work on the controllability of these systems led us to state necessary and sufficient conditions for the solvability of the feedback linearization problem. We also made connections between the results on the smooth nonlinear case and on the nonsmooth piecewise linear case. To extend the results of this paper to more general classes of piecewise linear systems is one of the possibilities for future work.

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