IMPROVING THE TILT CONTROL PERFORMANCE OF HIGH-SPEED RAILWAY VEHICLES: AN LQG APPROACH

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Abstract: The paper discusses on the use of an optimal *LQG* control design to improve the vehicle curving performance at increased running speed, employing only local (rail vehicle-based) signal measurements. It addresses the fundamental problem related with straightforward feedback control, and introduces the commercially-used *command-driven with precedence* scheme. A combination of simulation results and, a recently proposed, *tilt control system assessment method* are utilised for assessing the overall performance of the tilt controller.

Keywords: LQG control, Estimation, Train control, Railways, Tilt control, Control applications

1. INTRODUCTION

The concept of tilting train technology is rather straightforward: the bodies of the vehicles are leaned inwards on curves, thereby reducing the lateral acceleration experienced by the passengers and enabling higher vehicle speed operation. These were researched in the 1960s and 1970s, developed for production during the 1980s, and increasingly introduced into service operation during the 1990s. Most new high-speed trains in Europe now are fitted with tilt and there is a growing interest for its use in regional express train (Goodall and Brown, 2001).

Early tilt control systems were based primarily on a feedback measurement from a lateral accelerometer, applied separately for each vehicle (Figure 1), although it proved impossible at the time to get an appropriate combination of straight track and curve transition performance. However, interactions between suspension and controller dynamics (due to the sensor being within the control loop) led to stability problems. Since then, tilt controllers have evolved in an incremental sense, the end result of which is a control structure which is not optimised from a system point of view. Surprisingly however there has never been a rigorous study of control strategies.

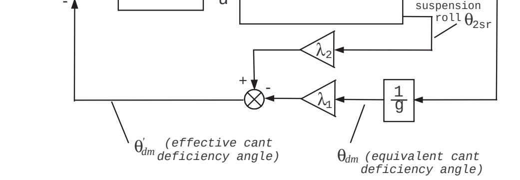

Fig. 1. Early-type partial-nulling control

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The industrial standard nowadays is the use of *precedence* control schemes (Goodall, 1999) devised in the early 1980s as part of the *Advanced Passenger Train* development (Boocock and King, 1982). In this scheme a bogie-mounted accelerometer is used to develop a tilt command signal by measuring the curving acceleration on a non-tilting part of the vehicle. However, because the accelerometer also measures higher frequency movements associated with lateral track irregularities, it is necessary to filter the signal. This filtering action (time delay) creates a detrimental performance on the transition from the straight track to the curve section. The usual solution is to use the accelerometer signal from the vehicle in front to provide "precedence", carefully designed so that the delay introduced by the filter compensates for the preview time corresponding to a vehicle length (Figure 2).

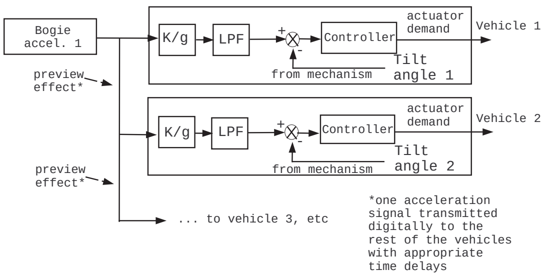

Fig. 2. Command-driven with precedence control

Nevertheless achieving a satisfactory local tilt control strategy remains an important research target because of the system simplifications and more straightforward failure detection. This paper presents results from a research study (Zolotas, 2002) which investigates advanced control approaches with the particular objective of identifying effective strategies which can be applied to each vehicle independently, i.e. to avoid the added complexity of precedence control.

2. VEHICLE MODELLING

The modelling is based upon a linear four degree-of-freedom end-view vehicle model (Figure 3), which includes the lateral and roll dynamics of both the body and the bogie. Modelling of the vertical secondary suspension (a pair of airsprings) only contributes to the roll mode of the vehicle (vertical degrees of freedom are ignored). The model also contains the stiffness of an anti-roll bar connected between the body and the bogie frame. To provide active tilt a rotational displacement actuator, assumed to be an *ideal actuator* in this case, is included in series with the roll stiffness, i.e. the concept of an 'active anti-roll bar (ARB)' (Pearson *et al.*, 1998). The advantages of active ARBs result from their relative simplicity, i.e.

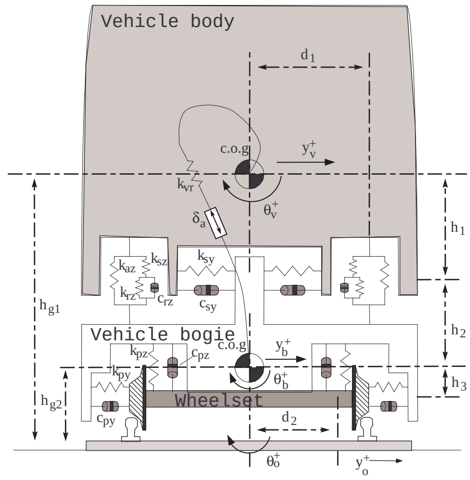

Fig. 3. End-view of a 4 DOF vehicle structure

small weight increase, low cost, easily fitted as an optional extra during manufacture or as a retrofit.

Mathematical models of increasing complexity,via the Newtonian approach, were developed to encapsulate the lateral and roll dynamics of the tilting vehicle system, equations 1-4 (Zolotas and Goodall, 2000).

$$\begin{aligned} m_v\ddot{y}_v = &-2k_{sy}(y_v - h_1\theta_v - y_b - h_2\theta_b)\ldots \\ &- 2c_{sy}(\dot{y}_v - h_1\dot{\theta}_v - \dot{y}_b - h_2\dot{\theta}_b)\ldots \\ &- \frac{m_v v^2}{R} + m_v g\theta_o - h_{g1}m_v\ddot{\theta}_o \end{aligned} \tag{1}$$

$$\begin{aligned} i_{vr}\ddot{\theta}_v = &-k_{vr}(\theta_v - \theta_b - \delta_a)\ldots \\ &+ 2h_1\{k_{sy}(y_v - h_1\theta_v - y_b - h_2\theta_b)\ldots \\ &+ c_{sy}(\dot{y}_v - h_1\dot{\theta}_v - \dot{y}_b - h_2\dot{\theta}_b)\}\ldots \\ &+ m_v g(y_v - y_b) + 2d_1\{-k_{az}(d_1\theta_v\ldots \\ &- d_1\theta_b) - k_{sz}(d_1\theta_v - d_1\theta_r)\} - i_{vr}\ddot{\theta}_o \end{aligned} \tag{2}$$

$$\begin{aligned} m_b\ddot{y}_b = &\ 2k_{sy}(y_v - h_1\theta_v - y_b - h_2\theta_b)\ldots \\ &+ 2c_{sy}(\dot{y}_v - h_1\dot{\theta}_v - \dot{y}_b - h_2\dot{\theta}_b)\ldots \\ &- 2k_{py}(y_b - h_3\theta_b - y_o)\ldots \\ &- 2c_{py}(\dot{y}_b - h_3\dot{\theta}_b - \dot{y}_o)\ldots \\ &- \frac{m_b v^2}{R} + m_b g\theta_o - h_{g2}m_b\ddot{\theta}_o \end{aligned} \tag{3}$$

$$\begin{aligned} i_{br}\ddot{\theta}_b = &\ k_{vr}(\theta_v - \theta_b - \delta_a)\ldots \\ &+ 2h_2\{k_{sy}(y_v - h_1\theta_v - y_b - h_2\theta_b)\ldots \\ &+ c_{sy}(\dot{y}_v - h_1\dot{\theta}_v - \dot{y}_b - h_2\dot{\theta}_b)\}\ldots \\ &- 2d_1\{-k_{az}(d_1\theta_v - d_1\theta_b) - k_{sz}(d_1\theta_v\ldots \\ &- d_1\theta_r)\} + 2d_2(-d_2k_{pz}\theta_b - d_2c_{pz}\dot{\theta}_b)\ldots \\ &+ 2h_3\{k_{py}(y_b - h_3\theta_b - y_o)\ldots \\ &+ c_{py}(\dot{y}_b - h_3\dot{\theta}_b - \dot{y}_o)\} - i_{br}\ddot{\theta}_o \end{aligned} \tag{4}$$

Remarks: Equation 2 includes an end moment effect $m_v g\ (y_v - y_b)$, which models the roll effect of the body weight due to the lateral displacement of its centre of gravity, an effect which would

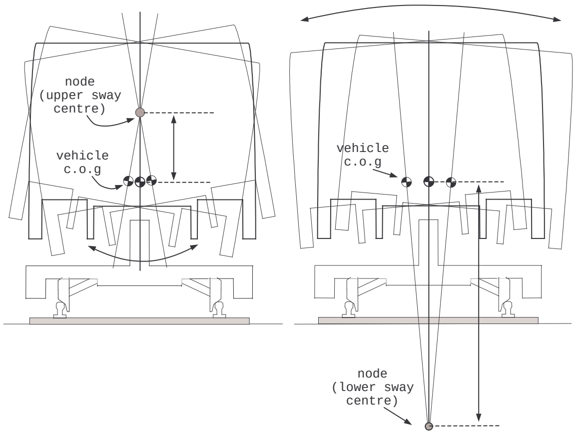

Fig. 4. Rail vehicle sway modes

arise naturally if the vertical suspension was fully included in the model. The equivalent effect is neglected in the bogie equation 4 due to the high primary suspension stiffness. The airspring model is presented in Appendix A. The symbols and the parameters used are listed in Appendix C.

There exists *substantial coupling* between the lateral and roll motions. This results in two modes, known as 'sway modes', which combine both lateral and roll movement, and their centres are located at points other than the vehicle centre of gravity (c.o.g.) (Figure 4): an 'upper sway' mode with a node above the body c.o.g. giving predominantly roll movement; and a 'lower sway' mode with a node located below the body c.o.g., characterised primarily by a lateral motion. The node location is very sensitive to slight parameter variations, and its assessment is useful in the selection of appropriate model parameter values for acceptable passive suspension performance.

3. TILT CONTROL REQUIREMENTS

The performance of the tilt control system on the curve transitions is critical. Primarily the passenger ride comfort provided by the tilting vehicle should not be (significantly) degraded compared to the non-tilting vehicle speeds.

The main objective of a tilt control system can be summarised as follows:

(1) to provide an acceptably fast response to changes in track cant and curvature (deterministic track features)
(2) not to react substantially to track irregularities (stochastic track features)

Any tilt control system directly controls the secondary suspension roll angle and not the vehicle lateral acceleration. Hence, there is a fundamental trade-off between the vehicle curve transition response and straight track performance.

It is also worth noting that for reasons of human perception, designers utilise *partial tilt compensation.* In such a case the passenger will still experience a small amount of acceleration on steady curve, in order to minimise motion sickness phenomena.

From a control point of view the objectives of the tilt control system can be translated as: increasing the response of the system at low frequencies (deterministic track features), while reducing the high frequency system response (stochastic track features) and maintaining stability.

4. ASSESSMENT APPROACH

The assessment of tilt controllers on curve transition is based upon a combination of the 'P_{CT} *factors*' and the '*ideal tilting*' assessment (Goodall *et al.*, 2000).

The former is based upon a comprehensive experimental/empirical study which provides the percentage of (both standing and seated) passengers who feel uncomfortable during the curve transition. The latter method principally assesses the control system performance by determining the deviations from the idea of "ideal tilting", i.e. where the tilt action follows the specified tilt compensation in an ideal manner, defined on the basis of the maximum tilt angle and cant deficiency compensation factor. This combination of parameters is optimised via the P_{CT} factors approach to choose a basic operating condition. The procedure follows a minimisation approach of dynamic effects relative to tilt angles, roll velocities and lateral accelerations (Appendix B).

For the straight track case the 'rule-of-thumb' which is currently followed by designers *is to allow the degradation of the lateral ride quality of the tilting train by no more than a specified margin compared with the non-tilting vehicle, a typical value being* $7.5\% - 10\%$.

5. DESIGN METHODOLOGY

The control design is based upon the LQG method, which is well documented in Maciejowski (Maciejowski, 1989), and defines the following state-space plant model

$$\dot{x} = Ax + Bu + \Gamma w \quad (5)$$

$$y = Cx + v, \quad (6)$$

where w, v are *white uncorrelated* process and measurement noises which excite the system, and are characterised by covariance matrices W, V respectively. The *separation principle* can be then applied to first find the optimal control $u = -K_r x$ which minimises (7)

$$J = \lim_{T\to\infty} \frac{1}{T} E\left\{ \int_0^T [x^T Q x + u^T R u] d\tau \right\}, \quad (7)$$

or in the case of output, rather than state, regulation

$$J_o = \lim_{T\to\infty} \frac{1}{T} E\left\{ \int_0^T [y^T Q_o y + u^T R u] d\tau \right\} \quad (8)$$

where $K_r = R^{-1} B^T X$ and X is the positive semi-definite solution of the following Algebraic Riccati Equation (ARE)

$$\begin{bmatrix} X & -I \end{bmatrix} \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix} \begin{bmatrix} I \\ X \end{bmatrix} = 0 \quad (9)$$

Next find the optimal state estimate $\hat{x}$ of x where

$$\dot{\hat{x}} = A\hat{x} + Bu + K_f (y - C\hat{x}) \quad (10)$$

to minimise $E\left\{ [x - \hat{x}]^T [x - \hat{x}] \right\}$. The optimal Kalman gain is given by $K_f = YC^T V^{-1}$ and Y is the positive semi-definite solution of the following ARE

$$\begin{bmatrix} Y & -I \end{bmatrix} \begin{bmatrix} A^T & -C^T V^{-1} C \\ -\Gamma W \Gamma^T & -A \end{bmatrix} \begin{bmatrix} I \\ Y \end{bmatrix} = 0 \quad (11)$$

Weighting matrices $Q \geq 0$, $R > 0$ for control and $W \geq 0$, $V > 0$ for estimation can be tuned accordingly to provide the desired result.

5.1 Design Issues

In order to achieve the specific design requirements for tilt control the following were considered.

LQR: to guarantee set-point regulation on steady curve the system is augmented with an extra state, i.e. *the integral of the effective cant deficiency angle.*

$$\begin{pmatrix} \dot{x} \\ \dot{x}^i \end{pmatrix} = \begin{pmatrix} A & 0 \\ C^i & 0 \end{pmatrix} \begin{pmatrix} x \\ x^i \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} u \quad (12)$$

where $x^i = \int \theta'_{dm}$ and C^i is the selector matrix for integral action.

<u>Remarks on estimator design:</u> track information during the track curve section is associated with signals of the disturbance vector w. There is no prior knowledge for these signals and also is not practical to measure such track parameters. Thus, the system is augmented with the disturbance signals for *track elevation*, *track elevation rate* and *track curvature*, which do not affect the LQR design but are used in the Kalman Filter for correct estimation. For design purposes, small time constants were considered, related to the disturbance signals, in the augmented state matrix for full state observability.

The control concept is very similar to the early days of tilt nulling controllers, however it utilises LQG control to take account of the dynamic complexity and provide a superior performance compared to its classical counterpart. The values for the weighting factors Q, R, W, V were based upon practical considerations for a realistic design approach to the tilt problem, details of which can be found in (Zolotas, 2002).

Note that the LQR problem utilises output regulation, on the signals of body roll rate and integral of effective cant deficiency to directly manipulate the minimisation of transitional oscillations (provide a smooth rate of roll) and provide a fast speed of response respectively. The weighting matrices were set to $Q_o = diag(5^{-2}, 0.1^{-2})$ and $R = 0.215^{-2}$.

The Estimation problem utilises realistic process noises related to track geometry characteristics, rather than virtual ones. It also incorporates realistic measurement devices for the case of sensor noise information based upon industrial standards and practical design issues. Namely, measurements of *body lateral acceleration*, *body roll rates* and *body yaw motion*. The process noise W, related to the rate of track curvature and track cant acceleration, was set to $diag(10^{-5}, 8.5 \cdot 10^{-4})$. The sensor noise covariance was chosen based upon realistic noise levels as $diag(1.6 \cdot 10^{-3}, 1.88 \cdot 10^{-6}, 10^{-6})$.

5.2 Tilt Controller

The optimal control K_r and optimal estimation K_f gains were calculated based upon the previous specifications and the resultant LQG controller realisation is given by:

$$K_{lqg} \stackrel{s}{=} \left[\begin{array}{c|c} A - BK_r - K_f C & K_f \\ \hline -K_r & 0 \end{array} \right] \quad (13)$$

The overall LQG controller size is 13^{th} order, including the three extra states from the estimator system and the (one) extra integral state from the extended regulator part. A set of principal gain plots can be seen in Figure 5.

The controller assessment for the current design is presented in Table 1, while the time-domain result for passenger acceleration is shown in Figure 6. It can be seen that the response with LQG-nulling (depending solely on local measurements) is quite

Table 1. Tilt performance assessment @ 58(m/s)

Deterministic (curve transition)		LQG-nulling	Precedence	
Lateral accel.	- steady-state	9.53	9.53	(%g)
(actual vs ideal)	- R.M.S. deviation error	2.30	1.6	(%g)
	- peak value	14.0	12.2	(%g)
Roll velocity	- R.M.S. deviation	0.023	0.018	(rad/s)
	- peak value	0.1	0.105	(rad/s)
	- peak jerk level	6.9	6.8	(%g/s)
P_{CT}/P-factor	- standing	52.1	48.0	(% of passengers)
	- seated	15.1	13.5	(% of passengers)
Stochastic (straight track)		**LQG-nulling**	**Precedence**	
Passenger comfort	- R.M.S. passive (equiv.)	3.78	3.78	(%g)
	- R.M.S. active	3.66	3.31	(%g)
	- degradation	-3.18	-12.12	(%)

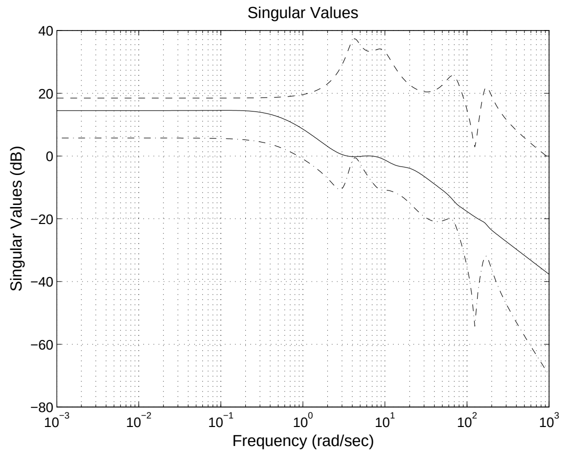

Fig. 5. Principal gains: G [dash], $K_{lqg} \times G$ [dash-dot], K_{lqg} [solid]

close to the one obtained from precedence control (preview information) with matching delay. The controller is fast enough to accommodate all stochastic long wavelengths (low frequency), thus the improvement in ride quality. However, faster controller designs will unavoidably degrade curve transition performance because of both increased jerk and roll velocity levels. Note that the body roll angle is inherently constrained by weighting its rate.

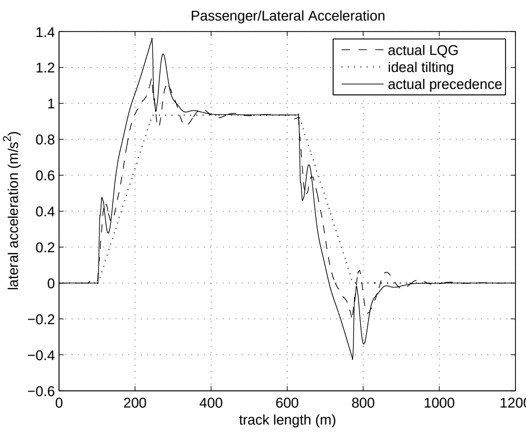

Fig. 6. Passenger acceleration (veh. speed 58m/s)

6. CONCLUSIONS AND FUTURE WORK

The paper presented a novel approach to the improvement of 'localised nulling-tilt' control via the use of an optimal LQG controller based upon practical design considerations. The controller comprises both proportional and integral parts for correct set-point regulation, while the interwoven Kalman Filter provides information on both states and track disturbance signals. Future work is concentrated on re-formulation of the scheme with other sensor/signals combinations to improve transition response, model/controller reduction and robustness analysis.

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Appendix A. AIRSPRING MODELLING

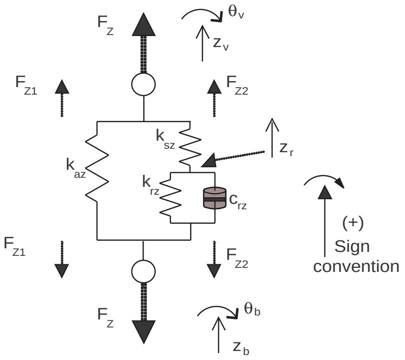

Fig. A.1. Schematic representation of an airspring

Disregarding vertical motions and substituting $d_1\theta_r$ for z_r:

$$F_z = -k_{az}(d_1\theta_v - d_1\theta_b) - k_{sz}(d_1\theta_v - d_1\theta_r) \tag{A.1}$$

$$\dot{\theta}_r = -\frac{(k_{sz}+k_{rz})}{c_{rz}}\theta_r + \frac{k_{sz}}{c_{rz}}\theta_v + \frac{k_{rz}}{c_{rz}}\theta_b + \dot{\theta}_b \tag{A.2}$$

Appendix B. ASSESSMENT APPROACH

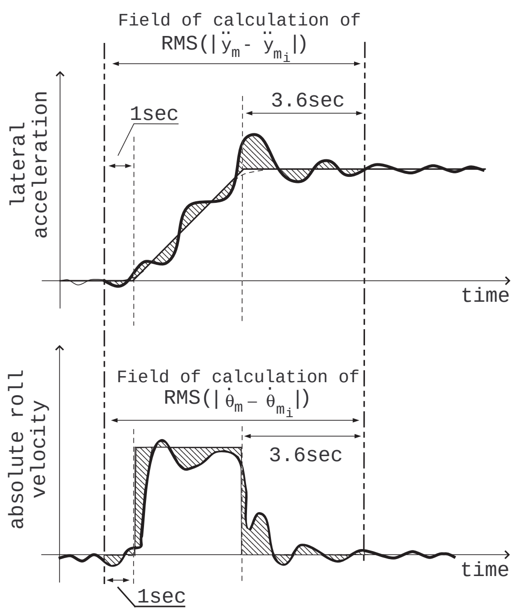

Fig. B.1. "Ideal Tilting"- Calculation of deviation of actual from ideal responses for acceleration and roll velocity

$|\ddot{y}_m - \ddot{y}_{m_i}|$, *the deviation of the actual lateral acceleration* $\ddot{y}_m$ *from the ideal lateral acceleration* $\ddot{y}_{m_i}$, *in the time interval between 1s before the start of the curve transition and 3.6s after the end of the transition.*

$\left|\dot{\theta}_m - \dot{\theta}_{m_i}\right|$, *the deviation of the actual absolute roll velocity* $\ddot{\theta}_m$ *from the ideal absolute roll velocity* $\ddot{\theta}_{m_i}$, *in the time interval between 1s before the start of the curve transition and 3.6s after the end of the transition.*

For a detailed analysis of the overall assessment and more information on the 'P_{CT} factors*' see (Goodall* et al.*, 2000).*

Appendix C. PARAMETER VALUES AND NOTATION

y_v, y_b, y_o	Lateral displacement of body, bogie and track
θ_v, θ_b, δ_a	Roll displacement of body, bogie and actuator
θ_o, R	Track cant, curve radius
θ_r	Airspring reservoir roll deflection
v	Vehicle forward speed
m_v	Half body mass, 19,000(kg)
i_{vr}	Half body roll inertia, 25,000(kgm^2)
m_b	Bogie mass, 2,500(kg)
i_{br}	Bogie roll inertia, 1,500(kgm^2)
g	gravitational acceleration, 9.81(ms^{-2})

Values per bogie side

k_{az}	Airspring area stiffness, 210,000($\frac{N}{m}$)
k_{sz}	Airspring series stiffness, 620,000($\frac{N}{m}$)
k_{rz}	Airspring reservoir stiffness, 244,000($\frac{N}{m}$)
c_{rz}	Airspring reservoir damping, 33,000($\frac{Ns}{m}$)
k_{sy}	Secondary lateral stiffness, 260,000($\frac{N}{m}$)
c_{sy}	Secondary lateral damping, 33,000($\frac{Ns}{m}$)
k_{vr}	Anti-roll bar stiffness/bogie, 2,000,000($\frac{Nm}{rad}$)
k_{pz}	Primary vertical stiffness, 2,000,000($\frac{N}{m}$)
c_{pz}	Primary vertical damping, 20,000($\frac{Ns}{m}$)
k_{py}	Primary lateral stiffness, 35,000,000($\frac{N}{m}$)
c_{py}	Primary lateral damping, 16,000($\frac{Ns}{m}$)
d_1	Airspring semi-spacing, 0.90(m)
d_2	Primary vertical suspension semi-spacing, 1.00(m)
h_1	2ndary lateral susp. height(body cog), 0.9(m)
h_2	2ndary lateral susp. height(bogie cog), 0.25(m)
h_3	Primary lateral susp. height(bogie cog), -0.09(m)
h_{g2}	Bogie cog height(rail level), 0.37(m)
h_{g1}	Body cog height(rail level), 1.52(m)