NEW CONGESTION CONTROL SCHEMES OVER WIRELESS NETWORKS: DELAY SENSITIVITY ANALYSIS AND SIMULATIONS [1]

Alessandro Abate * **Minghua Chen** *
Shankar Sastry *

* *Department of Electrical Engineering and Computer Sciences*
University of California, at Berkeley
Berkeley, CA 94720
{*aabate, minghua, sastry*}*@eecs.berkeley.edu*

Abstract: This paper proposes two new congestion control schemes for packet switched wireless networks. Starting from the seminal work of Kelly (Kelly *et al.*, Dec 1999), we consider the decentralized flow control model for a TCP-like scheme and extend it to the wireless scenario. Motivated by the presence of channel errors, we introduce updates in the part of the model representing the number of connections the user establishes with the network; this assumption has important physical interpretation. Specifically, we propose two updates: the first is static, while the second evolves dynamically. The global stability of both schemes has been proved; also, a stochastic stability study and the rate of convergence of the two algorithms have been investigated. This paper focuses on the delay sensitivity of both schemes. A stability condition on the parameters of the system is introduced and proved. Moreover, some deeper insight on the structure of the oscillations of the system is attained. To support the theoretical results, simulations are provided.

Keywords: Communication networks and protocols, flow control, large-scale systems, stability robustness, delay analysis.

1. INTRODUCTION

In these past years the control community has tried to systematically model Communication Networks in order to analyze their structure, properties and behavior in depth. Among the algorithms that have been employed for controlling the flow of information in a network, the Transmission-Control-Protocol(TCP) has been the most successful one (Jacobson, 1998). The seminal work of Kelly has introduced a rigorous mathematical model for TCP (Kelly, 2003)-(Kelly *et al.*, Dec 1999) based on an underlying optimization problem; other authors have studied and interpreted this or similar models (Alpcan and Basar, Dec 2003)-(Kunniyur and Srikant, Mar 2001)-(Kunniyur and Srikant, Oct 2003). The fundamental notion of stability has been considered in (Paganini *et al.*, to appear), while robustness, in particular with respect to delays, is the focus of (Johari and Tan, Dec 2001)-(Vinnicombe, 2001)-(Vinnicombe, 2002). All these efforts have been focused on a model for the wired case. The wireless scenario presents more subtleties then the wired one: here the packet loss is due both to congestion at the link, and to channel error. (Chen *et al.*, 2005*b*) proposed two schemes, a static and

[1] This work was supported by NSF grant ANI-9905799, the AFOSR contract F49620-00-1-0327 and the NSF CCR-0225610 grant.

a dynamic one, to fix the sub optimality of the equilibrium points of the network. In that work, global stability of the two schemes was proved, and delay sensitivity analysis studied. In another paper by the same authors (Chen *et al.*, 2005*a*), a stochastic stability analysis is derived, and the rate of convergence of these two schemes is computed.

In this paper we focus on the presence of delays. As commonly known, delay is one of the main causes of instability in otherwise stable dynamical models, as well as of oscillatory dynamics. We introduce a condition to ensure stability for the trajectories of the system. Furthermore, we shed some light on the structure of the oscillations induced by delays in the system. We present simulations to verify our theoretical results.

2. PROBLEM SETUP

A network is described via its J *resources*, its links, and its R *users* (sender-receiver pairs), which can also be conceived as the *routes*, subsets of J. Each link j has a finite capacity C_j. The connections of the network are described via a routing matrix $A = (a_{jr}, j \in J, r \in R)$, where $a_{jr} = 1$ if $j \in r$. Every user is endowed with a sending rate $x_r \geq 0$ and a utility function $U_r(x_r)$, assumed to be increasing, strictly concave and $\mathcal{C}^1$.

Kelly (Kelly *et al.*, Dec 1999) introduced the following fluid flow model, which is a more general, continuous-time version of the TCP-like additive increase, multiplicative decrease algorithm:

$$\frac{d}{dt}x_r(t) = k_r \left(w_r^o - x_r(t) \sum_{j \in r} \mu_j(t) \right), r \in R \quad (1)$$

with k_r being a positive scale factor affecting the adaptation rate. The term w_r^o can be thought as the number of connections the user has with the network. The congestion signal is generated at a link j as

$$\mu_j(t) = p_j \left(\sum_{s:j \in s} x_s(t) \right). \quad (2)$$

Here the $p_j(y)$'s are the prices at the links and are assumed to be non-negative, continuous and increasing functions; moreover, they depend on the aggregate rate passing through the link. Throughout this paper we use the following shape for the price function, the "packet loss rate",

$$p_j(y) = \frac{(y - C_j)^+}{y}. \quad (3)$$

The end-to-end packet loss rate for user r is $1 - \prod_{j \in r} p_j(\sum_{s:j \in s} x_s)$, which is approximately $\sum_{j \in r} p_j(\sum_{s:j \in s} x_s)$ if $p_j(\sum_{s:j \in s} x_s)$ is small. With this primal scheme (1)-(2), the unique, globally and asymptotically stable points of the entire network, denoted by $x^o = (x_r^o, r \in R)$ [2], are given by

$$x_r^o = \frac{w_r^o}{\sum_{j \in r} p_j \left(\sum_{s:j \in s} x_s^o \right)}, \quad r \in R. \quad (4)$$

As already stated, one of the main differences between the wired case and the wireless one is the presence of physical channel errors in this latter case; in the setting of our model these affect the packet loss rate, which in the wired case only depends on the congestion measure. We model this occurrence deterministically: assuming every link j is affected by the error ϵ_j, the new price function ν_j is given by:

$$\nu_j(t) = p_j \left(\sum_{s:j \in s} x_s(t) \right) + \left(1 - p_j \left(\sum_{s:j \in s} x_s(t) \right) \right) \epsilon_j$$
$$\triangleq q_j \left(\sum_{s:j \in s} x_s(t) \right) \geq p_j \left(\sum_{s:j \in s} x_s(t) \right). \quad (5)$$

The primal scheme (1) then adapts itself according to this new price functions q_j, which have the same structural properties as the old p_j; a close look shows that, compared to the wired case, the equilibrium point is sub optimal. We introduce the following two schemes to address this problem.

3. TWO NEW CONTROL SCHEMES

3.1 Static Update

Assume the term ω_r is time dependent, $w_r(t)$, and is adjusted according to the following law:

$$w_r(t) = w_r^o \frac{\sum_{j \in r} \nu_j(t)}{\sum_{j \in r} \mu_j(t)}. \quad (6)$$

Then, the source rate for user r is given by:

$$\frac{d}{dt}x_r(t) = k_r \left(w_r(t) - x_r(t) \sum_{j \in r} \nu_j(t) \right). \quad (7)$$

A rapid calculation shows that, under this change, the equilibrium of the system is again x^0. Intuitively, if the noise is large, i.e. $\nu_j(t) > \mu_j(t)$, an increase in $w_r(t)$ counteracts it.

3.2 Dynamic Update

Rather than an instantaneous adaptation rule, we advance a dynamic update for w_r as follows:

$$\frac{d}{dt}w_r(t) = c_r \left(w_r^o - w_r(t) \frac{\sum_{j \in r} p_j(\sum_{s:j \in s} x_s(t))}{\sum_{j \in r} q_j(\sum_{s:j \in s} x_s(t))} \right). \quad (8)$$

---

[2] In order to lighten the notation, throughout the whole paper users or links variables with no subscript will directly denote vectorial quantities.

The equilibrium points of the new, extended system are composed of a first part given by the vector x^o and a second part, for the new dynamics, given by $w_r^o \frac{\sum_{j\in r}\nu_j(t)}{\sum_{j\in r}\mu_j(t)}$. The system of coupled equations (1)-(2)-(8) is strongly nonlinear and asymmetric.

4. DELAY SENSITIVITY ANALYSIS

System delay is one of the main causes of oscillations. Since oscillations are one of the main concerns with TCP schemes, an analysis of the delay sensitivity is necessary. The setting we adhere to is that developed by (Johari and Tan, Dec 2001). The relations (1)-(2) for the primal scheme in the presence of delays can be expressed as:

$$\frac{d}{dt}x_r(t) = k_r w_r^o - k_r x_r(t-T_r)\sum_{j\in r}\mu_j(t-d_2(j,r)) \tag{9}$$

and

$$\mu_j(t) = p_j\left(\sum_{s:j\in s} x_s(t-d_1(j,s))\right), \tag{10}$$

where

$$d_1(j,r)+d_2(j,r) = T_r \quad \forall r\in R. \tag{11}$$

Here $d_1(j,r)$ is the forward delay from the sender of route r to link j, and $d_2(j,r)$ is the return delay from link j to the sender of route r. Hence T_r is the round trip time on route r, which is assumed to be fixed. In the following, we introduce two conditions for enforcing stability under delays (the proof for the first one can be found on (Chen *et al.*, 2005*b*)).

4.1 Static Update

Theorem 1. The system (6)-(7) is locally stable if $\forall r\in R$,

$$k_r\frac{\sum_{j\in r}q_j}{\sum_{j\in r}p_j}\left(\sum_{j\in r}p_j+\sum_{j\in r}p_j'\sum_{s:j\in s}x_s^o\right) < \frac{\pi}{2T_r}, \tag{12}$$

where p_j, q_j are the values of $p_j(\cdot)$ and $q_j(\cdot)$ evaluated at the equilibrium point; p_j' is the derivative of $p_j(\cdot)$ evaluated at the equilibrium point.

4.2 Dynamic Update

Lemma 1. Let the matrices $P = P^* \succ 0$, $Q = Q^* \succ 0$, $L = diag\{l_i\}$, $l_i \in \mathbb{C}$, $\forall i$. Then $\sigma(Q^{-1}LP) \subset \rho(Q^{-1}P)(Co(0\cup\{l_i\})\cup Co(0\cup\{-l_i\}))$.

Proof: Let v be a right eigenvector of P corresponding to the eigenvalue λ and such that the vector Pv is normalized. Then $Q^{-1}LPv = \lambda v$. Then $LPv = \lambda Qv \Rightarrow (Pv)^*L(Pv) = \lambda(Pv)^*Qv = \lambda v^*P^*(QP^{-1})Pv$. Therefore, naming $k = \rho(Q^{-1}P)((Pv)^*QP^{-1}(Pv))$ and observing that $|k|\geq 1$,

$$\lambda = \frac{(Pv)^*L(Pv)}{v^*P^*(QP^{-1})Pv} = \rho(Q^{-1}P)\big(\sum_i \frac{|Pv_i|^2}{k}(\pm l_i)+0\big).$$

Theorem 2. The system (1)-(2)-(5)-(8) is locally stable if the following two conditions hold, $\forall r\in R$:

$$k_r\left(\sum_{j\in r}q_j+\sum_{j\in r}q_j'\sum_{s:j\in s}x_s^o\right) < \frac{\pi}{2T_r}; \tag{13}$$

$$c_r\frac{\left(\sum_{j\in r}p_j+\sum_{j\in r}p_j'\sum_{s:j\in s}x_s^o\right)}{min_{r\in R}\sum_{j\in r}q_j} < \frac{\pi}{2T_r}. \tag{14}$$

Proof: As in the proof of global stability for the dynamic update case, we shall exploit the idea of the two times scale. The first condition indeed refers to the boundary layer. The linearization of the relation for the reduced system around its equilibrium and the manifold equation, comprehensive of the delays, are:

$$\begin{aligned}\dot{\omega_r}(t) &= c_r\omega_r^o - c_r x_r(t-T_r)\\ &\cdot\sum_{j\in r}p_j(\sum_{s:j\in s}x_s(t-d_1(j,s)-d_2(j,r)));\\ \omega_r(t-T_r) &= x_r(t-T_r)\\ &\cdot\sum_{j\in r}q_j(\sum_{s:j\in s}x_s(t-d_1(j,s)-d_2(j,r))).\end{aligned}$$

Taking the derivative of the second term, after a proper shift in time, gives:

$$\begin{aligned}\dot{\omega_r}(t) &= \dot{x}_r(t)\sum_{j\in r}q_j(\sum_{s:j\in s}\tilde{x}_s(j,r))\\ &+x_r(t)\sum_{j\in r}q_j'(\sum_{s:j\in s}\tilde{x}_s(j,r))\sum_{s:j\in s}\dot{\tilde{x}}_s(j,r),\end{aligned}$$

where $\tilde{x}_s(j,r) = x_s(t-d_1(j,s)-d_2(j,r)+T_r)$. Substituting this last relation within the first one, letting $x_r(t) = x_r^o + y_r(t), \dot{x}_r(t) = 0 + \dot{y}_r(t)$, and linearizing around these equilibrium points gives:

$$\begin{aligned}&\sum_{j\in r}q_j\dot{y}_r(t) = -c_r x_r^o\sum_{j\in r}p_j'\sum_{s:j\in s}y_s(t-d_1(j,s)\\ &-d_2(j,r)) - c_r y_r(t-T_r)\sum_{j\in r}p_j\\ &-x_r^o\sum_{j\in r}q_j'\sum_{s:j\in s}\dot{y}_s(t-d_1(j,s)-d_2(j,r)+T_r).\end{aligned}$$

Taking he Laplacian transform and simplifying out common matrix terms, we obtain:

$$\begin{aligned}&\Big(diag\{x_r^o\}^{-1}diag\big\{\sum_{j\in r}q_j\big\}+N(s)\Big)sY(s) = -diag\{c_r\}\cdot\\ &diag\{e^{-sT_r}\}\Big(diag\{x_r^o\}^{-1}diag\big\{\sum_{j\in r}p_j\big\}+M(s)\Big)Y(s),\end{aligned}$$

where $N(s)$ and $M(s)$ are composed of

$$N_{rq}(s) = \sum_{j \in r \cap q} q_j' exp(-s(d_1(j,q) - d_1(j,r)));$$

$$M_{rq}(s) = \sum_{j \in r \cap q} p_j' exp(-s(d_1(j,q) - d_1(j,r))).$$

Then, naming the quantities inside the two big parentheses $\mathbf{Q}(s)$ and $\mathbf{P}(s)$, we have

$$sY(s) = -\mathbf{Q}(s)^{-1} diag\{c_r\} diag\{e^{-sT_r}\}\mathbf{P}(s)Y(s). \quad (15)$$

We are interested in checking the stability of this interconnection, and in case pose conditions on the c_r terms to enforce it. Name $L = diag\Big\{c_r \frac{\pi}{2}\frac{exp(-j\omega T_r)}{j\omega T_r}\Big\}$, $P = diag\{\sqrt{x_r}\}\mathbf{P}(j\omega)diag\{\sqrt{x_r}\}$, $Q = diag\{\sqrt{x_r}\}\mathbf{Q}(j\omega)\cdot diag\{\sqrt{x_r}\}$. Then, employing the observation that the matrices $\mathbf{Q}(s)^{-1}diag\{c_r\}diag\{e^{-sT_r}\}\mathbf{P}(s)$ and $Q^{-1}LP$ are similar, we obtain

$$\sigma(Q^{-1}LP) \subset \rho(Q^{-1}P)\Big[Co\Big(0 \cup \Big\{c_r \frac{\pi}{2}\frac{exp(-j\omega T_r)}{j\omega T_r}\Big\}\Big) \cup Co\Big(0 \cup \Big\{-c_r \frac{\pi}{2}\frac{exp(-j\omega T_r)}{j\omega T_r}\Big\}\Big)\Big].$$

The necessary introduction of the additional negative terms in the convex hull does not change its structural property of excluding the -1 point in the complex plane. The problem then boils down to posing conditions on the term $\rho(Q^{-1}P)$; we know that $\rho(Q^{-1}P) \leq \rho(Q^{-1})\rho(P) \leq \frac{\rho(P)}{min_\lambda \sigma(Q)}$. Notice that $N(s) = A^T(-s)diag\sum_{j \in s} q_j' A(s)$ and that $N(s) = N(-s)$, $N(j\omega) \succ 0, \forall \omega$. From a linear algebra fact, given two matrices $A = A^* \succeq 0$ and $B = B^*$, then the eigenvalues of their sum, ranked increasingly, are correspondingly lower bounded by those of B. Therefore, focusing on the structure of matrix $Q = diag\{x_r^o\}^{-1}diag\{\sum_{j\in r} q_j\} + N(s)$, we claim that $min_r(\sum_{j \in r} q_j) \leq min_\lambda \sigma(Q)$ [3]. This translates into the condition that, $\forall r \in R$:

$$\rho(Q^{-1}P) < c_r \frac{\Big(\sum_{j\in r} p_j + \sum_{j \in r} p_j' \sum_{s:j\in s} x_s^o\Big)}{min_{r\in R}\sum_{j\in r} q_j} < \frac{\pi}{2T_r}.$$

We include a simulation, in figure (1), to show that the condition holds. The network topology we analyze is that of two users with one shared link (see figure (2) with $n = 2$). The link error rate is set to 2%. We assume the following parameters: $w_1^o = 60$, $w_2^o = 30$; $T_1 = 0.1s$, $T_2 = 0.16s$; $k_1 = 0.35$, $k_2 = 0.2$. The condition fixes the constants $c_1 = 0.004$, $c_2 = 0.004$.

5. THE STRUCTURE OF THE DELAYS

It is quite important to understand the properties of the oscillations present in a system. In our rather complex setting, it is only possible to get quantitative results locally, considering a linearized version of the model. Consider a multidimensional delay differential equation of the following kind

$$\dot{x}(t) = \sum_{i=1}^{n} A_i x(t - \tau_i). \quad (16)$$

[3] The reader should realize that, in a worst case scenario, the bound can be not so tight.

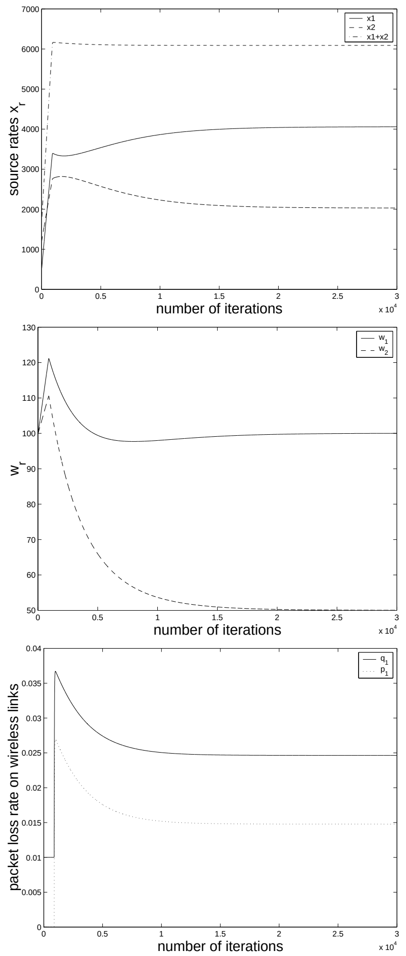

Fig. 1. Dynamic update system with delay: convergence of (a) rates $x_r(t), r = 1, 2$, (b) $w_r(t), r = 1, 2$, (c) packet loss rate $p_j(\cdot)$ and $q_j(\cdot), j = 1, 2$, with initial rate set to 0.

The above *delay differential equation* (DDE), which is defined for say $t \geq t_0$, needs to be specified also in the interval $[t_0 - T, t_0]$, where $T = max_{i=1,\dots,n}\tau_i$, through a *pre-shape function* $x(t) = \phi(t)$. The frequency of the oscillations of the different modes can be derived from the expression of the poles of the system, which are the solutions of its characteristic equation:

$$det(\lambda I - \sum_{i=1}^{n} A_i e^{-\lambda \tau_i}) = 0. \quad (17)$$

If we make the simplifying assumption, as already postulated in (Johari and Tan, Dec 2001), that the delay is the same for every user, then $n = R$, otherwise in general $n = J\binom{R}{2} + R$.

Unfortunately the roots of the characteristic equation (17), which is non linear and transcendental, cannot be expressed in a closed form. As a matter of fact, we need to resort to a class of functions $W(s)$ known as *Lambert* functions. A detailed description of them can be found in (Asl and Ulsoy, 2003). The general solution of the DDE can be expressed, similarly to the case of the solution in terms of the state transition matrix for the ODE, as:

$$x(t) = \sum_{k=-\infty}^{+\infty} e^{1/T W_k(-AT)t} C_k. \qquad (18)$$

W_k denote the k^{th} branch of the function. The coefficients C_k are vectors determined by the preshape functions. In our particular case, given that the initial condition for the state equations (rates) is less than the optimal rate [4], then the dynamics will experience no delay as long as all the users do not congest the network. In fact, in this situation both the price function and its derivative will be zero for all the links. As soon as one link gets congested, the users that rely on it will start experiencing a delay. This implies that the preshape functions in our setting are actually those trajectories that describe the transient of the sending rates. In some special cases (depending on the form of the preshape functions), it could happen that only few of the coefficients C_k are different from zero, which may simplify the analysis of the oscillation frequencies. Unfortunately, it should be clear from the reasoning how this does not happen in our quite involved case.

In the following, we shall focus on two very simple topologies; exploiting some approximations in the first case, we will try to describe how oscillations are characterized in the second, more general case. For the sake of simplicity, we shall focus on the dynamical equations with no update, which structure actually encompasses also the case of the static update. The first network topology we

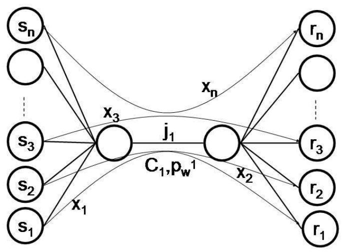

Fig. 2. Multi-user, one-link topology.

study is that of one user, with one single link (see figure(2), with $n = 1$). Considering the linearized expression

$$\dot{y}_1(t) = -k_1(p_1 + x_1^o p_1')y_1(t-T),$$

we can infer that the two terms into the parenthesis will have different weight depending on the operating point of the system. Assuming the classical shape for the price, $p_1(x_1) = \frac{(x_1 - C_1)^+}{x_1}$, then the equilibrium of the system will be at the point $x_1^o = C_1 + \omega_1^o$. In the case of light congestion, $\omega_1^o \ll C_1$, then $p_1 \ll x_1^o p_1'$; in the opposite case of heavy congestion, $\omega_1^o \gg C_1$, then $p_1 \gg x_1^o p_1'$. This fact can be extended and used for the more general structure of the one bottleneck link, with n users. It is represented in figure (2). We assume that the forward and backward delays for each user are the same: $d_i(1,j) = d_j(1,i), \forall i,j = 1,\dots,n$. Therefore, the round trip times are also the same, $T_i = T_j = T, \forall i,j = 1,\dots,n$. The linearized dynamical equations in this case are:

$$\underline{\dot{y}}(t) = \begin{pmatrix} p_1 + \hat{y}_1 p_1' & \hat{y}_1 p_1' & \cdots & \hat{y}_1 p_1' \\ \hat{y}_2 p_1' & p_1 + \hat{y}_2 p_1' & \vdots & \hat{y}_2 p_1' \\ \vdots & \cdots & \ddots & \vdots \\ \hat{y}_n p_1' & \hat{y}_n p_1' & \cdots & p_1 + \hat{y}_n p_1' \end{pmatrix} \underline{y}(t-T) (19)$$

where we named $\underline{y} = [y_1, \dots, y_n]^T$. It is possible to extend the reasoning we advanced in the single user case to the multiuser case: if the network works in a highly congested region, then the linearization can be approximated as:

$$\underline{\dot{y}}(t) = p_1 I_{n\times n} \underline{y}(t-T), \qquad (20)$$

The matrix has its n eigenvalues equal to p_1 and, as eigenvectors, the orthonormal basis. Therefore, denoting with Λ the diagonal matrix of the eigenvalues, we have that

$$W(A) = W(\Lambda).$$

For the principal branch, $k = 0$, we obtain

$$W_0(\Lambda) = diag\{W_0(\lambda_i)\} = diag\Big\{\sum_{n=1}^{\infty} \frac{(-n)^{n-1}}{n!} p_1^n\Big\}.$$

The solution of the DDE equation then can be expressed as

$$\underline{\dot{y}}(t) = e^{diag\left\{\sum_{n=1}^{\infty} \frac{(-n)^{n-1}}{n!} p_1^n\right\} Tt} \begin{bmatrix} C_0^1 \\ \vdots \\ C_0^n \end{bmatrix} + \text{conj. branches.}$$

If the network has the optimum in a lightly congested point, then the linearization can be approximated as:

$$\underline{\dot{y}}(t) = p_1' diag\{\hat{y}_i\} \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \vdots & 1 \\ \vdots & \cdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{pmatrix} \underline{y}(t-T) (21)$$

The matrix has a special structure, and it can be proved that it has zero $n-1$ eigenvalues equal to 0, plus one equal to $p_1' \sum_{i=1}^n \hat{y}_i$. In this case, if we name V the matrix with the corresponding eigenvectors on its columns, we have

$$W_0(-AT) = V^{-1} W(-AT) V =$$
$$V^{-1} \begin{bmatrix} 0 & \cdots & & 0 \\ \vdots & \ddots & & \vdots \\ 0 & \cdots & \sum_{m=1}^{\infty} \frac{(-m)^{m-1}}{m!} & \Big(p_1' \sum_{i=1}^{n} \hat{y}_i\Big)^m \end{bmatrix} VT.$$

[4] This is typically what happens when a user starts sending packets through the network.

At this point, it would be interesting to check for sufficient conditions to avoid oscillations in a system of DDE's. Unfortunately the literature provides only sufficient conditions for the existence of components with oscillating dynamics, or necessary and sufficient conditions for the oscillations of all the components of the solutions of the DDE. All of them hinge upon the following fact, (Gopalsamy, 1992)-(Gyory and Ladas, 1991): *Every solution of equation (16) oscillates componentwise if and only if its characteristic equation (17) has no real roots.*

From this, we derive that both in the case of highly congested network, as well as in that of lightly congested one, there will be some solutions with components that will not oscillate, but which will rather converge to the equilibrium exponentially.

6. DISCUSSION AND CONCLUSION

In this paper we have completed the analysis of the structural properties of two new schemes for flow control over wireless networks. Both algorithms modify the number of connections that a single user has with the network; the first scheme employs a static algorithm, while the second applies a dynamic scheme. In a first paper (Chen *et al.*, 2005*b*) we motivated the structure of both schemes and analyzed their global stability; moreover, for the static scheme, we suggested a condition on some of the coefficients to obtain stability in the presence of heterogeneous delays at the users. This paper extends the same idea to the more complex dynamic scheme; simulations for this case are provided. Furthermore, we attempted to get some insight on the structure of the oscillations in the presence of delays. This is motivated by the fact that, in real world TCP schemes, oscillations represent an important issue. We should however mention that in real TCP schemes oscillations might be as well due to other important causes, for instance the discretization of the schemes. Resorting to the theory of Delay Differential Equations, and posing some simplifying assumptions, we managed to get some early results on the problem, which has never been systematically considered in the literature before. Nevertheless, it is been increasingly clear how hard the topic is to be studied analytically. Future research will stand on these results to proceed further.

Another paper by the same authors (Chen *et al.*, 2005*a*) focused on the stochastic stability analysis, and the calculation of the rate of convergence of both schemes. The authors are already working on the application of this scheme to a real TCP setting, extending the theory and getting more insight from the simulations.

REFERENCES

Alpcan, Tansu and Tamer Basar (Dec 2003). Global stability analysis of end-to-end congestion control schemes for general topology networks with delay. *IEEE Conference on Decision and Control.*

Asl, Farshid and Galip Ulsoy (2003). Analysis of a system of linear delay differential equations. *Journal of Dynamic Systems, Measurements and Control* **125**, 215–223.

Chen, Minghua, Alessandro Abate and Shankar Sastry (2005*a*). New congestion control schemes over wireless networks: Stability analysis. *Proceedings of the* 16^{th} *IFAC World Congress, Prague.*

Chen, Minghua, Alessandro Abate, Avideh Zakhor and Shankar Sastry (2005*b*). Stability and delay considerations for flow control over wireless networks. *UCB EECS ERL Technical Report.*

Gopalsamy, K. (1992). *Stability and Oscillations in Delay Differential Equations of Population Dynamics.* Kluwer Academic Publisher.

Gyory, I. and G. Ladas (1991). *Oscillation Theory of Delay Differential Equations with Applications.* Oxford Science Publications.

Jacobson, V. (1998). Congestion avoidance and control. *Proceedings of ACM SIGCOMM* pp. 314–329.

Johari, Ramesh and David Tan (Dec 2001). End-to-end congestion control for the internet: delays and stability. *IEEE/ACM Trans. on Networking* **9, no.6**, 818–832.

Kelly, F. (2003). Fairness and stability of end-to-end congestion control. *European Journal of Control* **9**, 159–176.

Kelly, F. P., A. Maulloo and D. Tan (Dec 1999). Rate control for communication networks: shadow prices, proportional fairness, and stability. *Journal of the Operationl Research Society* **49**, 237–252.

Kunniyur, S. and R. Srikant (Mar 2001). A time scale decomposition approach to adaptive ecn marking. *Proceeding of IEEE Infocom.*

Kunniyur, S. and R. Srikant (Oct 2003). End-to-end congestion control: Utility functions, random losses and ecn marks. *IEEE/ACM Transactions on Networking.*

Paganini, F., Z. Wang, J. C. Doyle and S. H. Low (to appear). Congestion control for high performance, stability and fairness in general network. *ACM/IEEE Trans. on Networking.*

Vinnicombe, G. (2001). On the stability of end-to-end congestion control for the internet. *University of Cambridge Technical Report CUED/F-INFENG/TR.398.*

Vinnicombe, G. (2002). Robust congestion control for the internet. *University of Cambridge Technical Report.*