MULTI-LOOP FEEDBACK CONTROL OF OIL WELL DRILLSTRINGS

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Abstract: A drillstring mechanism used for oil or gas drilling behaves as a torsional pendulum. The lowest part of the drillstring exhibits self-excited vibrations induced by nonlinear friction at the rock-crushing tool. Since these vibrations can lead to premature degradation of highly expensive mechanical and electronic components, a control strategy is necessary to attenuate or even eliminate their adverse effects. A multi-loop control design is proposed, which is composed by an inner and an outer loop. The aim of the inner-loop is to add enough damping to suppress stick-slip vibrations, while the objective of the outer-loop is to achieve the desired drillstring velocity. *Copyright © 2005 IFAC*

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1. INTRODUCTION

Oil and gas production relies strongly on the drilling of wells whose deep typically ranges from 1 to 5 km. The well drilling involves the crushing of rock by a rotating drill bit in order to create a borehole. The bit is driven by a drive system at surface and a drillstring that transmits torque from the drive system to the bit. The interested reader is referred to (Rabia, 1985) for a detailed description of the oil drilling processes and standard drilling ring configurations. The drive system consists of a electric motor, a gearbox and a rotary table, which is basically a heavy flywheel connected to the top of the drillstring. The drillstring transmits torque from the drive system to the bit and consists mainly of slender tubes, called drill pipes. The lowest part of the drillstring is loaded in compression, and in order to avoid buckling it consists of thick-walled tubes, called drill collars. A typical length of a drillstring is between 0 and 5 km with a drill collar section of a few hundred meters. Typical borehole sizes range from 100 to 850 mm; standard drill pipes have an outside diameter up to 250 mm and a wall thickness of 9 mm. The drillstring is an extremely slender structure, and during drilling the string is twisted several turns because of torque-on-bit between 500 to $10,000$ Nm.

The drillstring displays a complex dynamic behavior consisting of axial, lateral and torsional vibrations. Rotation measurements at the surface and at the bit have shown that the drillstring often behaves as a rotary torsional pendulum. That is, the top of the drillstring rotates with a constant angular velocity, whereas the bit displays

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self-exciting torsional oscillations. Such oscillations are driven by the nonlinear friction, often referred to as stick-slip, at near-zero bit velocities (Armstrong-Helouvry *et al.*, 1994). This stick-slip torsional vibrations are detrimental to the bit and the drillstring, which can even culminate in a twist-off (Jansen and van den Steen, 1995; Serrarens *et al.*, 1998).

In literature, some solutions have been presented to counteract the effects of stick-slip oscillations in oil well drillstrings. By using feedback control design based on a local linearization around the operating velocity, (Jansen and van den Steen, 1995) proposed and active damping system to extend the working range for vibration-free rotation. (Serrarens *et al.*, 1998) proposed a H_∞ control design approach where the nonlinear stick-slip friction is interpreted as an exogenous bounded disturbance. In this way, the objective of the H_∞ controller is to minimize the effects of such disturbance on the bit rotation velocity. It should be noted that, from a rigorous standpoint, the stick-slip friction is not an exogenous disturbance; rather, it is actually a state-dependent nonlinear function with a feedback effect on the closed-loop system.

In this paper, a feedback control strategy is described that strongly reduces stick-slip oscillations and maintains the bit rotation velocity at a desired operating value. To this end, a multi-loop control design is proposed, which is composed by an inner- and an outer-loop. The aim of the inner-loop is to add enough damping to suppress stick-slip vibrations via a simple linear feedback of velocity. On the other hand, the objective of the outer-loop is to achieve the desired operating drillstring velocity and is composed essentially of an integral action on the regulation velocity error.

2. A DRILLSTRING SYSTEM MODEL

Throughout the paper, the following notations will be used:

c_1: bit and BHA damping parameter
c_2: rotary table damping parameter
g_1 to g_3: inner-loop gains
h: inner-loop high-frequency gain (Eq. (4))
J_1: bit and bottomhole assembly (BHA) inertia
J_2: rotary table inertia
k: stiffness parameter
K_I: outer-loop integral gain
L: inner-loop adjustable parameter
T_{tob}: torque-on-bit
u: applied torque
u_n: nominal applied torque
u_{in}: inner-loop applied torque
u_{out}: outer-loop applied torque
v_1: bit and BHA rotation velocity
v_2: rotary table rotation velocity
$\theta = \theta_2 - \theta_1$: rotary table to bit twist
θ_1: bit and BHA rotation angle
θ_2: rotary table rotation angle
$\omega_{r,1} = \sqrt{k/J_1}$: bit and BHA resonance frequency

A 'hat' on top of symbols denotes the equilibrium value (*e.g.*, $\widehat{v}$), a 'dash' on top denotes estimated parameter or signal (*e.g.*, $\overline{v}$), and 'ref' as a superindex denotes reference value (*e.g.*, v^{ref}).

If the fluctuations of the rotary table speed are much smaller than those of the bottomhole assembly (BHA), the dynamics of the drillstring system can be described with a lumped model (Serrarens *et al.*, 1998; van den Steen, 1997). Another simplification of the model is the negligence of the motor dynamics used to generate the applied torque. Essentially, the drillstring model comprises two damped inertias mechanically coupled by an elastic shaft (see Fig. 1). The inertia J_2 represents the inertia of the rotary table augmented with the inertias of the electric motor and transmission box which is driven by a DC electric motor. The drillstring is modeled as a linear torsional spring with stiffness k. One-third of the drillstring inertia is lumped into the inertia J_1. The rest of J_1 is determined by the inertia of the BHA. J_1 is damped by a lumped damping c_1, which can be seen as a model for increasing energy dissipation at higher bit speeds. The bit and the BHA experience the nonlinear torque T_{tob} (Torque On Bit), which is mainly generated by the torque required to crush and cut the rock, and by friction along the BHA. Two degrees-of-freedom are defined; namely, the drive system rotation angle θ_2 and the BHA rotation angle θ_1. Furthermore, the quantities $\theta = \theta_2 - \theta_1$, $v_1 = \dot{\theta}_1$ and $v_2 = \dot{\theta}_2$ are defined.

The equations of motion are readily derived from Fig. 1:

$$\begin{aligned} \dot{v}_1 &= -a_1 v_1 + a_2\theta - a_3 T_{tob}(v_1) \\ \dot{v}_2 &= -a_4 v_2 - a_5\theta + a_6 u \\ \dot{\theta} &= v_2 - v_1 \end{aligned} \tag{1}$$

where $a_1 = c_1/J_1$, $a_2 = k/J_1$, $a_3 = 1/J_1$, $a_4 = c_2/J_2$, $a_5 = k/J_2$ and $a_6 = 1/J_2$.

2.1 Stick-Slip Oscillations

The real relationship between the bit speed v_1 and $T_{tob}(v_1)$ is very complex. (Pavone and Desplans, 1994) measured and interpreted stick-slip with downhole measurement equipment, and derived a decaying exponential law for $T_{tob}(v_1)$. In this way, the following properties of the relationship $T_{tob}(v_1)$ will be assumed in the sequel:

Property 1 $v_1 T_{tob}(v_1) > 0$ for all $v_1 \neq 0$.

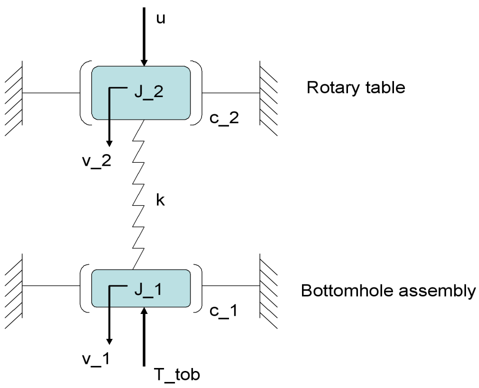

Fig. 1. Schematic diagram of the drillstring.

Property 2 $T_{tob}(v_1)$ is decreasing and continuously differentiable for all $v_1 \neq 0$. In general, due to the presence of the Coulomb friction, $T_{tob}(v_1)$ is discontinuous at $v_1 = 0$.

Property 3 $|T_{tob}(v_1)| \leq T_s$ and $|T_{tob}(v_1)| \to T_c$ as $|v_1| \to \infty$.

These properties are quite general and reflect the main characteristics of the stick-slip phenomena in oil well drillstring (Pavone and Desplans, 1994). Throughout this paper, it will be assumed that the actual $T_{tob}(v_1)$ is unknown, but that it satisfies Properties 1 to 3 above.

Example 1. A commonly used relationship satisfying properties 1 to 3 can be given as follows (Armstrong-Helouvry *et al.*, 1994):

$$T_{tob}(v_1) = \frac{T_s - T_c}{1 + \delta\,|v_1|} sign(v_1) + T_c \qquad (2)$$

where δ is a positive parameter. To illustrate the stick-slip phenomenon, the following set of parameters has been taken for numerical simulations (Serrarens *et al.*, 1998): $J_1 = 374\ kg\ m^2$, $J_2 = 2122\ kg\ m^2$, $c_1 = 0 - 50\ Nms/rad$, $c_2 = 425\ Nms/rad$, $k = 473\ Nm/rad$, $T_s = 5000\ Nm$, $T_c = 2000\ Nm$ and $\delta = 1.0$. Fig. 2 presents numerical simulations for $u = 6000\ Nm$ and initial conditions $v_1(0) = 0\ rad/\sec$, $v_2(0) = 5\ rad/\sec$ and $\theta(0) = 5\ rad$. Notice that the bit velocity changes are severe, ranging from $0\ rad/\sec$ to about $18\ rad/\sec$. If the bit is not initially at rest, for instance $v_1(0) = 5\ rad/\sec$, stick-slip oscillations are not present (see Fig. 3). Instead, an underdamped oscillatory behavior is displayed. This shows that stick-slip oscillations and underdamped oscillations coexist at the same operating conditions of the drillstring system. In principle, a unique attractor can be found if sufficiently large values of the applied torque are used. Fig. 4 shows the dynamic behavior of the drillstring system for the same initial conditions of Fig. 2 and $u = 8000\ Nm$. In this case, the stick-slip oscillations are not present and the drillstring system converges to a unique equilibrium velocity through underdamped oscillations. □

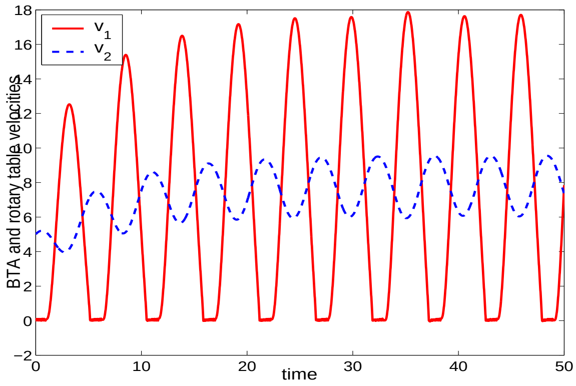

Fig. 2. Stick-slip phenomenon.

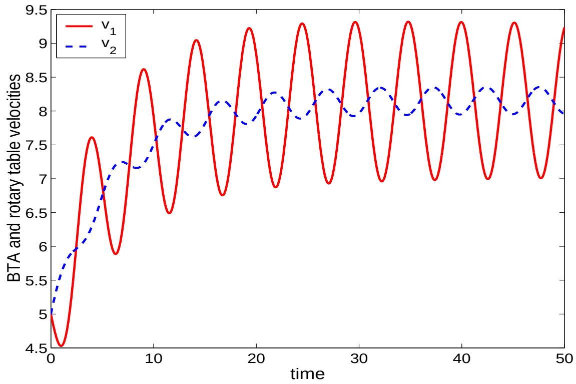

Fig. 3. Underdamped oscillations.

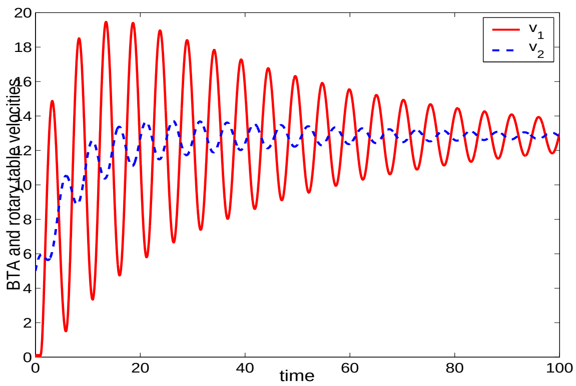

Fig. 4. Convergende to a unique equilibrium.

In practice, excessively large applied torques (*i.e.*, $u >> T_s$) are avoided because it leads to very large drillstring twist θ, which can culminate in a twist-off (Rabia, 1985). In this way, oil well drillstring system operate in the stick-slip regime where moderate applied torques can be used. Hence, from a practice standpoint, the design of feedback control strategies to reduce the effects of stick-slip oscillations and to regulate the bit velocity is of prime importance for an accurate and safe operation of oil well drillstrings (Serrarens *et al.*, 1998).

3. FEEDBACK CONTROL DESIGN

The control input u is divided into three parts:

$$u = u_n + u_{in} + u_{out} \tag{3}$$

where u_n is a nominal (constant) input, u_{in} and u_{out} are the inner- and outer-loop control inputs, respectively. In practice, u_n can be taken as an estimate of the steady-state input corresponding to the desired operating velocity v^{ref}. In a first step, the inner-loop control u_{in} is designed to add damping into the drillstring system. In a second step, the outer-loop control u_{out} is endowed with an integral action on the velocity regulation error to ensure that $v_1(t) \to v^{ref}$ asymptotically.

3.1 Inner-Loop Control

Let $p = [v_1, v_2, \theta]^T$ be the state vector. It is noted that the drillstring system (1) can be seen as a linear system perturbed by a bounded nonlinearity $T_{tob}(v_1)$:

$$\dot{p} = A_o p + Bu + ET_{tob}(v_1)$$

where

$$A_o = \begin{bmatrix} -a_1 & 0 & a_2 \\ 0 & -a_4 & -a_5 \\ -1 & 1 & 0 \end{bmatrix}$$

$B = [0, a_6, 0]^T$ and $E = [-a_3, 0, 0]^T$. Besides, it can be shown that the pair (A_o, B) is controllable. The idea that will be used for inner-loop control design is to exploit this structure of the drillstring system to make the linear system dynamics sufficiently fast to dominate the adverse effects of the nonlinearity $T_{tob}(v_1)$. To this end, let

$$h = \frac{k}{J_1 J_2} > 0 \tag{4}$$

which is the high-frequency gain of the map $u(t) \to v_1(t)$. For a given operating velocity v^{ref}, there exists a corresponding operating twist value θ^{ref}. However, since the torque-on-bit relationship $T_{tob}(v_1)$ is unknown, θ^{ref} is not exactly known. Let $\overline{\theta}^{ref}$ be an estimation of θ^{ref}. Consider the following state-feedback function:

$$\begin{aligned} u_{in} = & -h^{-1}[L^3 g_1(v_1 - v^{ref}) \\ & + L^2 g_2 \omega_{r,1}^2(\theta - \overline{\theta}^{ref}) \\ & + L g_3 \omega_{r,1}^2 (v_2 - v^{ref})] \end{aligned} \tag{5}$$

where $L > 0$ is a tuning parameter to be determined latter, and the gains $g_i's$ are positive constants such that the polynomial $s^3 + g_3 s^2 + g_2 s + g_1 = 0$ is Hurwitz. The rationale behind the inner-loop control structure (5) is the following. It can be shown that, as L is increased, the feedback function (5) adds arbitrary damping into the linear part of system (1) (*i.e.*, when $T_{tob}(v_1) = 0$). Since $T_{tob}(v_1)$ is globally bounded, the idea is to add sufficient damping into the linear part of the system (2) to counteract the adverse effects of the nonlinear function $T_{tob}(v_1)$. This is shown in the following proposition.

Proposition 1. Consider the closed-loop system formed by the drillstring system (1) and the control input (3) with u_{in} given by Eq. (5) and u_{out} taking values in an interval $D_{out} = \left[u_{out}^{\min}, u_{out}^{\max}\right]$. Then, there is a positive constant $L_2^{\min}$ such that the system has a unique globally asymptotically stable (GAS) equilibrium point $\overline{p}(L) = \left[\widehat{v}_1(L), \widehat{v}_2(L), \widehat{\theta}(L)\right]^T$ with $\widehat{v}_1(L) = \widehat{v}_2(L) > 0$, for all $L > L_2^{\min}$.

Proof. Firstly, let us study the equilibrium points of the closed-loop system. By recalling that $\widehat{v}_2 = \widehat{v}_1$ and considering that $\widehat{\theta} = a_2^{-1}\left[a_1\widehat{v}_1 + a_3 T_{tob}(\widehat{v}_1)\right]$, the $\widehat{v}_1$ component of the equilibrium $\widehat{p}$ is given by the solutions of the equation

$$\beta_1(L)\widehat{v}_1 - \beta_2(L) = -\beta_3(L) T_{tob}(\widehat{v}_1)$$

where

$$\begin{aligned} \beta_1(L) &= L^3 g_1 a_6 + L^2 g_2 \omega_{r,1}^2 a_1 a_2^{-1} a_6 \\ &\quad + L g_3 \omega_{r,1}^2 a_6 + h(a_1 a_2^{-1} a_5 + a_4) \\ \beta_2(L) &= L^3 g_1 a_6 v^{ref} + L^2 g_2 \omega_{r,1}^2 a_6 \overline{\theta}^{ref} \\ &\quad + L g_3 \omega_{r,1}^2 a_6 v^{ref} + h a_6 (u_n + u_{out}) \\ \beta_3(L) &= L^2 g_2 \omega_{r,1}^2 a_2^{-1} a_3 a_6 + h a_2^{-1} a_3 a_5 \end{aligned} \tag{6}$$

The $\beta_i(L)$'s are continuous functions of the parameter L. Since u_{out} takes values in the interval D_{out}, one can get the following approximation for sufficiently large L:

$$\begin{aligned} \beta_1(L) &\approx L^3 g_1 a_6 \\ \beta_2(L) &\approx L^3 g_1 a_6 v^{ref} \\ \beta_3(L) &\approx L^2 g_2 \omega_{r,1}^2 a_2^{-1} a_3 a_6 \end{aligned}$$

Notice that the constraint $u_{out} \in D_{out}$ is used to ensure uniformity (with respect to u_{out}) in the above approximation. This means that, for sufficiently large L, the $\widehat{v}_1$ component of the equilibria is given as the solution of the algebraic equation

$$-\frac{L g_1 a_2}{g_2 \omega_{r,1}^2 a_3}(\widehat{v}_1 - v_{ref}) = T_{tob}(\widehat{v}_1) \tag{7}$$

The left-hand side of (7) is a straight line with slope $m(L) = -\frac{L g_1 a_2}{g_2 \omega_{r,1}^2 a_3} < 0$ and intersection $i(L) = \frac{L g_1 a_2}{g_2 \omega_{r,1}^2 a_3}\widehat{v}_1 > 0$. Notice that both $|m(L)|$ and $i(L)$ are strictly increasing functions of L. Since $\widehat{v}_1 T_{tob}(\widehat{v}_1) > 0$ for $\widehat{v}_1 \neq 0$, one has that $\widehat{v}_1 > 0$, for sufficiently large L. Furthermore, since $T_{tob}(\widehat{v}_1)$ is a decreasing function for $\widehat{v}_1 \neq 0$, one can conclude that there is a positive constant $L_0^{\min}$ such that the equation (7) has a unique solution $\widehat{v}_1$ for all $L > L_0^{\min}$. The continuity of the $\beta_i(L)$'s with respect to L implies that there exists a positive constant $L_1^{\min} \geq L_0^{\min}$ such that the closed-loop system has a unique equilibrium point $\widehat{p}$, for all $L > L_1^{\min}$ and all $u_{out} \in D_{out}$. To establish the stability of the equilibrium point, introduce the change of coordinates

$$
\begin{aligned}
x_1 &= L^2(v_1 - \widehat{v}_1) \\
x_2 &= L\omega_{r,1}^2(\theta - \widehat{\theta}) \\
x_3 &= \omega_{r,1}^2(v_2 - \widehat{v}_2)
\end{aligned}
$$

By recalling that $a_2 = \omega_{r,1}^2$, the closed-loop system can be represented as

$$\dot{x} = LA_c x + \psi(x, L) \tag{8}$$

where

$$A_c = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -g_1 & -g_2 & -g_3 \end{bmatrix}$$

and $\psi(x, L) = [\psi_1(x, L), \psi_2(x, L), \psi_3(x, L)]^T$ with

$$
\begin{aligned}
\psi_1(x, L) &= -L^2 a_3 \left[T_{tob}(L^{-2}x_1 + \widehat{v}_1) - T_{tob}(\widehat{v}_1)\right] \\
\psi_2(x, L) &= -L^{-2}\omega_{r,1}^2 x_1 \\
\psi_3(x, L) &= -L^{-2}\omega_{r,2}^2 x_2 - a_4 x_3
\end{aligned}
$$

Since $\widehat{v}_1 \neq 0$, it is not hard to see that the function $\psi(x, L)$ satisfies the following properties for all $L > L_1^{\min}$: i) $\psi(0, L) = 0$, and ii) there exists a positive constant γ, independent of L, such that $\|\psi(x, L)\| \leq \gamma \|x\|$. On the other hand, since A_c is a Hurwitz matrix, there exists a positive-definite matrix P_c such that $P_c A_c + A_c^T P_c = -I$. Consider the quadratic function $V(x) = x^T P_c x$ and take its time-derivative along the trajectories of system (8):

$$
\begin{aligned}
\dot{V}_{(9)} &= -L\|x\|^2 + x^T P_c \psi(x, L) + \psi(x, L)^T P_c x \\
&\leq -L\|x\|^2 + 2\lambda_{\max}(P_c)\|x\| \|\psi(x, L)\| \\
&\leq -(L - 2\lambda_{\max}(P_c)\gamma)\|x\|^2
\end{aligned}
$$

where $\lambda_{\max}(P_c)$ denotes the maximum eigenvalue of P_c. Let $L_2^{\min} = \max\left\{L_1^{\min}, 2\lambda_{\max}(P_c)\gamma\right\}$. Consequently, $\dot{V}_{(9)} < 0$ for all $\|x\| \neq 0$ and all $L > L_2^{\min}$. This shows that the closed-loop system has a unique GAS equilibrium point $\widehat{p}(L)$, for all $L > L_2^{\min}$ and all u_{out} in the interval $D_{out} = \left[u_{out}^{\min}, u_{out}^{\max}\right]$. □

Remark 1. The above results states that, for all u_n, all $u_{out} \in D_{out}$ and all $L > L_1^{\min}$, there is only one equilibrium point $\widehat{p}$. It should be noticed that, in general, $\widehat{v}_1(L) \neq v^{ref}$, so that there is a steady-state offset in the velocity regulation. However, $\widehat{v}_1(L) \to v^{ref}$ as $L \to \infty$ (see Eq. (7)). That is, as in linear control systems, the desired velocity v^{ref} can be achieved with very high-gain feedback limit. Of course, excessive high-gain feedback control is undesirable in practice since it can require excessive control efforts and can induce high sensitivity to measurement noise and unmodeled (high-frequency) dynamics. In the next part of the paper, the use of excessive high-gains can be avoided by a suitable design of the outer-loop controller. □

Example 2. (Example 1, cont.). To illustrate the results in the above proposition, the following set of inner-loop control parameters have been taken: $u_n = 6000\ Nm$, $u_{out} = 0.0\ rad$, $v^{ref} = 4.5\ rad$, $g_1 = 1.0$, $g_2 = 3.0$, $g_3 = 3.0$, such that all the roots of the polynomial $s^3 + g_3 s^2 + g_2 s + g_1 = 0$ at -1. Besides, $\overline{\theta}^{ref} = 7.0\ rad$ has been taken as an estimate of the equilibrium twist θ^{ref}. The value $v^{ref} = 4.5\ rad$ has been taken since it corresponds to the equilibrium applied torque $u^{ref} = 5500\ Nm$, which is close to the backlash threshold $T_s = 5000\ Nm$. That is, the aim of the controller is to stabilize the drillstring system at operating conditions close to sticking conditions. Fig. 5 presents the behavior of the bit velocity $v_1(t)$ for $L = 1$ and initial conditions as in Figure 2. It is observed that $\widehat{v}_1(L)$ deviates slightly from v^{ref}. □

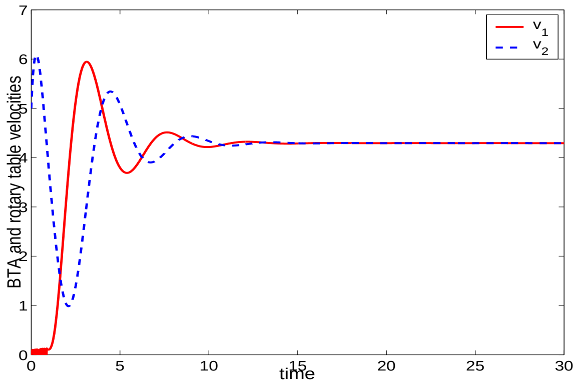

Fig. 5. Effect of the inner-loop control with $L = 1$.

3.2 Outer-Loop Control

The inner-loop controller (5) ensures global asymptotic stability about an equilibrium velocity $\widehat{v}_1(L)$, which can be different from the desired one v^{ref}. Proposition 1 implies the existence of a (steady-state) map $\widehat{v}_1 = \phi(u_{out})$, $u_{out} \in D_{out}$. That is, for each $u_{out} \in D_{out}$, there is a unique equilibrium velocity $\widehat{v}_1 > 0$. Assume that $v^{ref} \in image(\phi(u_{out}))$. In other words, assume that there is a constant input $u_{out}^{ref} \in D_{out}$ such that $v^{ref} = \phi(u_{out}^{ref})$. The aim of this part of the paper is to show that the integral compensator

$$u_{out} = Sat\left(K_I \int_0^t \left(v^{ref} - v_1(\sigma)\right) d\sigma\right), \tag{9}$$

where $K_I > 0$ is the outer-loop integral gain and Sat is a continuously differentiable saturation function with $u_{out}^{\min}$ and $u_{out}^{\min}$ as lower and upper limits, suffices to remove the steady-state offset in the bit velocity. The saturation function Sat is introduced to ensure that u_{out} computed from the feedback function (14) belongs to the interval D_{out}. The result is established in the following result.

Proposition 2. Consider the closed-loop system formed by the drillstring system (1), the multi-loop control input (3) with the inner-loop control u_{in} given by eqs. (5), and the outer-loop control u_{out} given by (9). Then, there exist two positive constants $L_4^{\min}$ and $K_I^{\max}$ such that all the states and signals of the closed system are bounded and $v_1(t) \to v^{ref}$ asymptotically.

Proof. Since Eq. (6) is a linear function of u_{out}, the map $\widehat{v}_1 = \phi(u_{out})$ is invertible. On the other hand, the following approximation can be obtained from (6):

$$L^3 g_1 a_6 \widehat{v}_1 - L^3 g_1 a_6 v^{ref} - h a_6 u_{out} = -L^2 g_2 \omega_{r,1}^2 a_2^{-1} a_3 a_6 T_{tob}(\widehat{v}_1)$$

Since $\widehat{v}_1 > 0$ for all $L > L_3^{\min}$, and $T_{tob}(\widehat{v}_1)$ is a differentiable function for all $\widehat{v}_1 \neq 0$, one has that

$$L^2 \left(L g_1 a_6 + g_2 \omega_{r,1}^2 a_2^{-1} a_3 a_6 T'_{tob}(\widehat{v}_1) \right) \frac{d\widehat{v}_1}{du_{out}} = h a_5,$$

where $T'_{tob}(\widehat{v}_1) = \frac{dT_{tob}(\widehat{v}_1)}{d\widehat{v}_1}$. One has that $T'_{tob}(\widehat{v}_1) \leq 0$ for all $\widehat{v}_1 > 0$. Besides, there is a positive constant δ such that $|T'_{tob}(\widehat{v}_1)| < \delta$, for all $\widehat{v}_1 > 0$. Therefore, there exists a positive constant $L_4^{\min}$ such that $L g_1 a_6 + g_2 \omega_{r,1}^2 a_2^{-1} a_3 a_6 T'_{tob}(\widehat{v}_1) > 0$ for all $\widehat{v}_1 > 0$ and $L > L_4^{\min}$. This implies that $\frac{d\widehat{v}_1}{du_{out}} > 0$ for all $\widehat{v}_1 > 0$ and $L > L_4^{\min}$. Essentially, the derivative $\frac{d\widehat{v}_1}{du_{out}}$ is the steady-state gain of the input-output map $u_{out} \to v_1$. The fact that $\frac{d\widehat{v}_1}{du_{out}} > 0$ implies that the integral gain K_I must be positive. From this point, the stability result can be established along the same lines as the proof of Theorem 1 in (Desoer and Lin, 1985). $\square$

Example 3. (Example 1, cont.). Proposition 2 ensures that an inner-loop controller with a sufficiently large gain and an outer-loop controller with a sufficiently small gain suffices to eliminate stick-slip oscillations and to regulate the bit velocity about a desired reference $v^{ref} > 0$. Fig. 6 shows the performance of the controller (3) with $K_I = 833$ and g_1, g_2, g_3, L as in Fig. 5. Note that $v_t \to v^{ref}$ indeed.

4. CONCLUSIONS

A multiloop control design approach for stick-slip suppression and bit velocity regulation for drillstring systems has been proposed in this paper. The inner-loop is a simple linear state-feedback controller that increases the damping in the face of torque-on-bit nonlinearities. The outer loop is a bit velocity regulator composed basically by an integral compensator, which provides asymptotic regulation of the bit velocity while maintaining ultimate boundedness of all trajectories in the domain of operation.

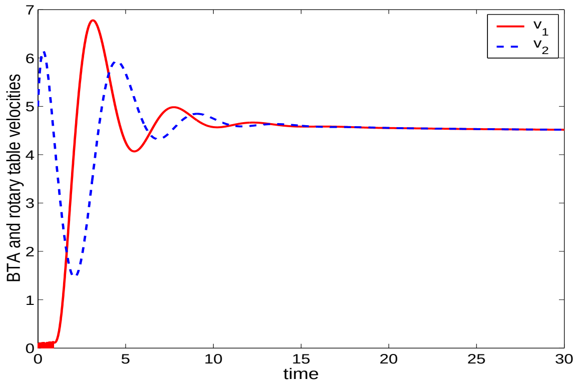

Fig. 6. Effect of the multi-loop control with $L = 1$.

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