

DECENTRALIZED NONLINEAR CONTROL DESIGN FOR MULTI-MACHINE POWER SYSTEMS

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Abstract: This paper is concerned with the control of power systems. We design a control approach for a class of nonlinear systems, closely related to sliding modes but without switching. Simulations results performed on the basis of two versions of the sliding-mode controller and applied to a multi-machine model have demonstrated better performances when compared to an Hamiltonian passive controller design. *Copyright ©2005 IFAC.*

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1. INTRODUCTION

The transient stability of an electrical power system (EPS) can be defined as the ability of an EPS to remain in synchronism after being subjected to a major system disturbance (see (De Mello, 1969; Pai, 1989)). The use of advanced control techniques for power system control has been one of the more promising application areas of automatic control, see (Chapman, 1993; Chow, 1995; Mielczarski, 1994). Compared with the use of traditional linear control theory where the operating domain of the controlled system is restricted to a small operation domain, or the structural properties of the system are lost or partially used, the design of excitation controls with nonlinear techniques allows the controlled system to face large disturbances and to recover a steady-state post-fault situation (see (Bergen, 1986)).

Nonlinear feedback linearizing controller design based on differential geometric techniques has been well investigated for power systems. The key feature of this controller design is that the control law can cancel the system nonlinear dynamics so that the resulting closed loop system behaves as an equivalent linear system. However, due to the presence of parametric uncertainties, unpredictable disturbances and faults occurring in the system, it follows that it is impossible to guarantee the exact cancellation of nonlinearities of the system. Recently, port-controlled Hamiltonian systems have been introduced (see (Maschke, 1998)). For this class of systems the Hamiltonian function is considered as the total energy and play the role of Lyapunov function for the system. The key of this technique is to express the electrical power system into a port-

controlled Hamiltonian system. This method was applied for improving the transient stability of a multi-machine power system by means of decentralized nonlinear excitation control (Xi, 2002). Another technique for improving robustness under parameter uncertainties and external disturbances is sliding-mode control design which has attracted a number of research (see (Utkin, 1986)). The implementation of discontinuous controllers yields the chattering phenomenon which can be avoided by approximating the discontinuous control law by a continuous one.

2. DYNAMICAL MODEL OF A MULTI-MACHINE POWER SYSTEM

Under some standard assumptions, the dynamics of n interconnected generators can be described by classical model with flux decay dynamics. The network has been reduced to internal bus representation. Furthermore, in practical power systems, line conductances G_{ij} can be neglected with respect to line susceptances B_{ij} ($G_{ij} \ll B_{ij}$). The dynamical model of the i -th machine is represented by (Bergen, 1986; Pai, 1989; Xi, 2002)

$$\begin{aligned}\dot{\delta}_i &= \omega_i - \omega_0 \\ \dot{\omega}_i &= \frac{1}{2H_i} (-D_i (\omega_i - \omega_0) + \omega_0 (P_{m_i} - P_{e_i})) \\ \dot{E}'_{q_i} &= \frac{1}{T'_{d_i}} (E_{f_i} - E_{q_i})\end{aligned}\quad (1)$$

where

$$\begin{aligned}P_{e_i} &= E'_{q_i} \sum_{j=1}^n E'_{q_j} B_{ij} \sin(\delta_i - \delta_j), \\ E_{q_i} &= E'_{q_i} - (X_{d_i} - X'_{d_i}) \sum_{j=1}^n E'_{q_j} B_{ij} \cos(\delta_i - \delta_j) \\ Q_{e_i} &= -E'_{q_i} \sum_{j=1}^n E'_{q_j} B_{ij} \cos(\delta_i - \delta_j), \\ I_{q_i} &= \sum_{j=1}^n E'_{q_j} B_{ij} \sin(\delta_i - \delta_j) \\ I_{d_i} &= - \sum_{j=1}^n E'_{q_j} B_{ij} \cos(\delta_i - \delta_j), \\ E_{q_i} &= E'_{q_i} + (X_{d_i} - X'_{d_i}) I_{q_i}, \\ E_{q_i} &= V_{t_i} + \frac{Q_{e_i} X_{d_i}}{V_{t_i}}\end{aligned}$$

and $\delta_i(t)$ is the power angle of the i -th generator, in p.u.; $\omega_i(t)$ represents the relative speed, in p.u.; $\omega_0 = 2\pi f_0$, is the synchronous machine speed, P_{m_i} the mechanical input power, in p.u.; $P_{e_i}(t)$ is the active power, in p.u.; D_i is the damping constant, in p.u.; H_i represents the inertia constant, in seconds; $E'_{q_i}(t)$ is the transient EMF in the quadrature axis, in p.u.; $E_{q_i}(t)$ is the EMF in the

quadrature axis, in p.u.; $E_{f_i}(t)$ is the equivalent EMF in the excitation coil, in p.u.; T'_{d_i} is the direct axis transient short circuit time constant, in seconds; X_{d_i} is the direct axis reactance of the i -th generator, in p.u.; X'_{d_i} is the direct axis transient reactance of the i -th generator, in p.u.; B_{ij} is the i -th row and j -th column element of the nodal susceptance matrix, symmetric matrix; at the internal nodes after eliminating all physical buses, in p.u.; $Q_{e_i}(t)$ the reactive power, in p.u.; $I_{q_i}(t)$ the direct axis current, in p.u.. We consider that $E_{f_i}(t)$ is the input of the system. Then, the state representation of the multi-machine power system is of the following form:

$$\begin{aligned}\dot{x}_{i1} &= x_{i2} \\ \dot{x}_{i2} &= -a_i x_{i2} + b_i - c_i x_{i3} \sum_{j=1}^n x_{j3} B_{ij} \sin(x_{i1} - x_{j1}) \\ \dot{x}_{i3} &= -e_i x_{i3} + d_i \sum_{j=1}^n x_{j3} B_{ij} \cos(x_{i1} - x_{j1}) + u_i\end{aligned}\quad (2)$$

where $a_i = D_i/2H_i$, $b_i = (\omega_0/2H_i)P_{m_i}$, $c_i = (\omega_0/2H_i)$, $d_i = (X_{d_i} - X'_{d_i})/T'_{d_i}$, $e_i = 1/T'_{d_i}$, are the systems parameters, $[x_{i1}, x_{i2}, x_{i3}]^T = [\delta_i(t), \omega_i(t), E'_{q_i}(t)]^T$ represents the state vector, and the control input is given by $u_i = (1/T'_{d_i})k_{ci}u_{fi}(t)$.

3. CONTINUOUS SLIDING-MODE CONTROLLER DESIGN

We consider the class of nonlinear systems described in the state space by

$$\dot{x} = f(x) + g(x)u, \quad x(t_0) = x_0, \quad (3)$$

where $t_0 \geq 0$, $x \in B_x \subset R^n$ is the *state vector*, $u \in R^r$ is the *control input vector*, f and g are assumed to be bounded with their components being smooth functions of x . B_x denotes a closed and bounded subset centered at the origin. We consider the following (r)-dimensional nonlinear surface defined by

$$\begin{aligned}\sigma(x - x^*) &= (\sigma_1(x - x^*), \dots, \sigma_r(x - x^*))^T \\ &= 0\end{aligned}\quad (4)$$

where x^* is equilibrium point of the system and each function $\sigma_i : B_x \times B_x \rightarrow R$, $i = 1, \dots, r$, is a C^1 function such that $\sigma_i(0) = 0$. The *equivalent control method* (see (Utkin, 1986)) is used to determine the system motion restricted to the sliding surface $\sigma(x - x^*) = 0$, which leads to the so-called *equivalent control*

$$u_e = - \left[\frac{\partial \sigma}{\partial x} g(x) \right]^{-1} \left[\frac{\partial \sigma}{\partial x} f(x) \right] \quad (5)$$

where the matrix $[\partial\sigma/\partial x]g(x)$ is assumed to be nonsingular for all $x, x^* \in B_x$. In order to complete the control design one sets

$$u = u_e + u_N \quad (6)$$

where u_e is the equivalent control (5), which acts when the system is restricted to $\sigma(x - x^*) = 0$, while u_N acts when $\sigma(x - x^*) \neq 0$. The control u_N is selected as

$$u_N = -\left[\frac{\partial\sigma}{\partial x}g(x)\right]^{-1}L(x)\sigma(x - x^*) \quad (7)$$

where $L(x)$ is an $r \times r$ positive definite matrix whose components are C^0 bounded nonlinear real functions of x , such that $\|L(x)\| \leq \rho, \forall x \in B_x$ with a constant $\rho > 0$. The equation that describes the projection of the system motion outside $\sigma(x - x^*) = 0$ is given by

$$\dot{\sigma}(x - x^*) = -L(x)\sigma(x - x^*). \quad (8)$$

Based on the sliding-mode control described above, the resulting the composite control is given by

$$u = -\left[\frac{\partial\sigma}{\partial x}g(x)\right]^{-1}\left[\frac{\partial\sigma}{\partial x}f(x) + L(x)\sigma(x - x^*)\right] \quad (9)$$

Stability of the subsystem (8) is not sufficient for guaranteeing asymptotic stability of the overall system since $(n - r)$ states have been rendered unobservable. The $(n - r)$ -dimensional zero dynamics has to be stable (to get a minimum phase nonlinear system). The problem of choosing σ in order to ensure that the system is minimum phase is the main design issue.

4. HAMILTONIAN CONTROLLER DESIGN

We consider the multi-variable nonlinear system Σ_{NL} expressed by the equations:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \quad (10)$$

where $x \in \mathbb{R}^n$ is the state vector of the system, $u \in \mathbb{R}^m$ is the control vector and $y \in \mathbb{R}^p$ is the output vector. In this paper we will be interested in the class of systems that can be equivalently represented in a Hamiltonian form with dissipative terms in the following way

$$\begin{aligned} \dot{x} &= (\mathcal{J}(x) - \mathcal{R}(x))\frac{\partial H^T}{\partial x} + g(x)u \\ y &= g^T(x)\frac{\partial H^T}{\partial x} \end{aligned} \quad (11)$$

where x, u, y are the energy variables, $H(x_1, \dots, x_n) : \mathbb{R}^n \rightarrow \mathbb{R}$ represents the total stored energy and

the interconnection structure is captured in the $n \times n$ matrix $\mathcal{J}(x)$ and the $n \times m$ matrix $g(x)$. The matrix $\mathcal{J}(x)$ is skew-symmetric, *i.e.*

$$\mathcal{J}(x) = -\mathcal{J}^T(x), \quad \forall x \in \mathbb{R}^n$$

and $\mathcal{R}(x)$ is a non-negative symmetric matrix depending on x , *i.e.*

$$\mathcal{R}(x) = \mathcal{R}^T(x) \geq 0, \quad \forall x \in \mathbb{R}^n.$$

If a system can be described by an Hamiltonian form, then the Hamiltonian function may be used to guarantee stability of the system. Moreover, from (11), we obtain the power-balance equation

$$\frac{dH}{dt} = -\frac{\partial H}{\partial x}\mathcal{R}(x)\frac{\partial H^T}{\partial x} + u^T y$$

with $u^T y$ the power externally supplied to the system and $-\frac{\partial H}{\partial x}\mathcal{R}(x)\frac{\partial H^T}{\partial x}$ representing the energy-dissipation due to the resistive elements. As it is well known (see (Maschke, 1998)), the above equality establishes the passivity properties of the system in the following sense.

Theorem 1: *We consider the class of systems defined by (11). We assume that the system is zero-state detectable and that the generalized Hamiltonian has a strict local minimum. Then it follows that x^* is a Lyapunov stable equilibrium point of the unforced dynamics. Moreover, it can be easily seen that in order to render the equilibrium point asymptotically stable, the following output feedback can be considered*

$$u = -y = -g^T(x)\frac{\partial H^T}{\partial x} \quad (12)$$

5. APPLICATION TO A MULTI-MACHINE SYSTEM

A three-machine system (see figure 1) is chosen to demonstrate the effectiveness of the proposed sliding mode controller. However, the here-proposed approaches can be easily extended to a n -machine system without restriction. In this case, the generator 3 is considered as an infinite bus, then the generator 3 is used as the reference, *i. e.* ($E'_{q3} = const = 1 \angle 0^\circ$).

5.1 Sliding-mode control design

Sliding-Mode Control 1

We consider the following nonlinear switching surface defined by

$$\sigma(x, x^*) = (\sigma_1(x, x^*), \sigma_2(x, x^*))^T = 0,$$

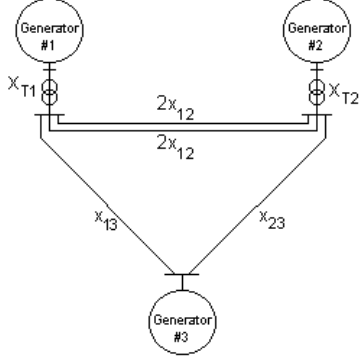


Fig. 1. a 3-machine power system.

where

$$\sigma_i(x, x^*) = s_{i1}(x_{i1} - x_{i1}^*) + s_{i2}(x_{i2} - x_{i2}^*) + s_{i3}(x_{i3} - x_{i3}^*), i = 1, 2,$$

and $x_i^* = (x_{i1}^*, x_{i2}^*, x_{i3}^*)$, for $i = 1, 2$, is an equilibrium point. The s_{ij} 's are chosen to guarantee that the zero dynamics associated to each output $y_i = \sigma_i$ are asymptotically stable. Then, the equivalent control is given by

$$\begin{aligned} u_{e_i} &= - \left[\frac{\partial \sigma_i}{\partial x_i} g_i(x) \right]^{-1} \left[\frac{\partial \sigma_i}{\partial x_i} f_i(x) \right] \\ &= - \frac{1}{s_{i3}} \{ s_{i1} x_{i2} + s_{i2} (-a_i x_{i2} + b_i - c_i x_{i3} I_{q_i}) \\ &\quad - c_i x_{i3} \sum_{j=1}^n x_{j3} B_{ij} \sin(x_{i1} - x_{j1}) \} \\ &\quad + s_{i3} (-e_i x_{i3} + d_i \sum_{j=1}^n x_{j3} B_{ij} \cos(x_{i1} - x_{j1})), \\ &\quad i = 1, 2 \end{aligned}$$

where $[\partial \sigma_i / \partial x_i] g_i = s_{i3}$ for all $x_i \in B_{x_i}$. On the other hand, the control u_{N_i} is selected as

$$\begin{aligned} u_{N_i} &= - \left[\frac{\partial \sigma_i}{\partial x_i} g_i(x) \right]^{-1} L_i(x) \sigma_i(x, x^*) \\ &= - \frac{L_i}{s_{i3}} \{ s_{i1}(x_{i1} - x_{i1}^*) + s_{i2}(x_{i2} - x_{i2}^*) \\ &\quad + s_{i3}(x_{i3} - x_{i3}^*) \}, i = 1, 2, \end{aligned}$$

where $L_i(x) = L_i = \text{constant}$. Finally, the overall control design is given by

$$\begin{aligned} u_i &= - \frac{1}{s_{i3}} \{ s_{i1} x_{i2} + s_{i2} (-a_i x_{i2} + b_i \\ &\quad - c_i x_{i3} \sum_{j=1}^{n=3} x_{j3} B_{ij} \sin(x_{i1} - x_{j1})) \\ &\quad + s_{i3} (-e_i x_{i3} + d_i \sum_{j=1}^{n=3} x_{j3} B_{ij} \cos(x_{i1} - x_{j1})) \\ &\quad + L_i (s_{i1}(x_{i1} - x_{i1}^*) + s_{i2}(x_{i2} - x_{i2}^*)) \} \end{aligned}$$

$$+ s_{i3}(x_{i3} - x_{i3}^*)) \}$$

for $i = 1, 2$. The multi-machine power system model can be stabilized around a prescribed operational equilibrium point by the following feedback

$$\begin{aligned} u_i &= - \frac{1}{s_{i3}} \{ s_{i1} x_{i2} + s_{i2} (-a_i x_{i2} + b_i - c_i x_{i3} I_{q_i}) \\ &\quad + s_{i3} (-e_i x_{i3} - d_i I_{d_i}) + L_i (s_{i1}(x_{i1} - x_{i1}^*) \\ &\quad + s_{i2} x_{i2} + s_{i3}(x_{i3} - x_{i3}^*)) \} \end{aligned}$$

or equivalently

$$\begin{aligned} u_i &= - \left\{ \frac{L_i}{s_{i3}} [s_{i1}(x_{i1} - x_{i1}^*) + s_{i2} x_{i2} \right. \\ &\quad + s_{i3}(x_{i3} - x_{i3}^*)] \\ &\quad + \frac{s_{i1}}{s_{i3}} x_{i2} + \frac{s_{i2}}{s_{i3}} (-a_i x_{i2} + b_i - c_i P_{e_i}) \\ &\quad \left. - e_i (V_{t_i} + \frac{Q_{e_i} X_{d_i}}{V_{t_i}}) \right\} \end{aligned}$$

where $x_{i2}^* = 0$, $P_{e_i} = x_{i3} \sum_{j=1, i \neq j}^n x_{j3} B_{ij} \sin(x_{i1} - x_{j1})$, $I_{d_i} = - \sum_{j=1, i \neq j}^n x_{j3} B_{ij} \cos(x_{i1} - x_{j1})$ and $I_{q_i} = \sum_{j=1, i \neq j}^n x_{j3} B_{ij} \sin(x_{i1} - x_{j1})$. We can notice that the controller is expressed only in terms of local measurable variables leading to a fully decentralized control scheme.

Sliding-Mode Control 2

We consider now the following nonlinear switching surface given by

$$\sigma_i(x, x^*) = s_{i1} \tilde{x}_{i1} + s_{i2} \dot{\tilde{x}}_{i1} + s_{i3} \ddot{\tilde{x}}_{i1}, i = 1, 2,$$

where $\tilde{x}_{i1} = x_{i1} - x_{i1}^*$.

The controller can be also expressed in terms of local measurable variables as

$$\begin{aligned} u_i &= \frac{1}{s_{i3} c_i I_{q_i}} \{ -L_i s_{i1} (x_{i1} - x_{i1}^*) \\ &\quad - L_i s_{i2} x_{i2} - L_i s_{i3} (-a_i x_{i2} + b_i - c_i P_{e_i}) \\ &\quad + s_{i1} + s_{i3} c_i Q_{e_i} + (s_{i2} - a_i) (-a_i x_{i2} + b_i - c_i P_{e_i}) \\ &\quad - s_{i3} e_i (V_{t_i} + \frac{Q_{e_i} X_{d_i}}{V_{t_i}}) \} \end{aligned}$$

for $i = 1, 2$ and for all $x_i \in B_{x_i}$.

5.2 Hamiltonian control design

In the Hamiltonian formulation for multi-machine power systems, the energy function of the overall system is the sum of the energy function of each

generators (Sun, 2001). We consider system (2) and the following energy function

$$H = \sum_{j=1}^{n=3} \left(\frac{1}{2c_i} x_{i2}^2 - \frac{b_i}{c_i} x_{i1} - \frac{1}{2} x_{i3} \sum_{j=1}^{n=3} x_{j3} B_{ij} \cos(x_{i1} - x_{j1}) \right) + \frac{e_i}{2d_i} x_{i3}^2. \quad (13)$$

It follows that the system dynamics can be written as a generalized Hamiltonian control system with dissipation according to what follows

$$\begin{pmatrix} \dot{x}_{i1} \\ \dot{x}_{i2} \\ \dot{x}_{i3} \end{pmatrix} = \begin{pmatrix} 0 & c_i & 0 \\ -c_i & -c_i a_i & 0 \\ 0 & 0 & d_i \end{pmatrix} \frac{\partial H}{\partial x_i} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u_i \quad (14)$$

where $x_i = \text{col}(x_{i1}, x_{i2}, x_{i3})$, $\mathcal{J}_i(x) = \begin{pmatrix} 0 & c_i & 0 \\ -c_i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$,

$\mathcal{R}_i(x) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -c_i a_i & 0 \\ 0 & 0 & d_i \end{pmatrix}$, $g_i(x) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$. Let $(x_{i1}^*, x_{i2}^*, x_{i3}^*)$ be the equilibrium point of (2), obtained from the following equations

$$\begin{aligned} x_{i2}^* &= 0 \\ -a_i x_{i2}^* + b_i - c_i x_{i3}^* \sum_{j=1}^{n=3} x_{j3}^* B_{ij} \sin(x_{i1}^* - x_{j1}^*) &= 0 \\ -e_i x_{i3}^* + d_i \sum_{j=1}^n x_{j3}^* B_{ij} \cos(x_{i1}^* - x_{j1}^*) + \tilde{u}_i &= 0 \end{aligned} \quad (15)$$

Defining the constant excitation control \tilde{u}_i , it follows that

$$\tilde{u}_i = e_i x_{i3}^* - d_i \sum_{j=1}^{n=3} x_{j3}^* B_{ij} \cos(x_{i1}^* - x_{j1}^*). \quad (16)$$

Now, defining the energy function which includes the equilibrium point of the following form

$$H_e = \sum_{j=1}^{n=3} \left(\frac{1}{2c_i} x_{i2}^2 - \frac{b_i}{c_i} (x_{i1} - x_{i1}^*) + \frac{e_i}{2d_i} (x_{i3} - x_{i3}^*)^2 \right) + \sum_{i=1}^{n=3} \left(x_{i3} \sum_{j=1}^n x_{j3} B_{ij} \cos(x_{i1} - x_{j1}) + x_{i3} \sum_{j=1}^n x_{j3}^* B_{ij} \cos(x_{i1}^* - x_{j1}^*) \right).$$

Then, system (14) can be represented by the Hamiltonian system with dissipation as

$$\begin{pmatrix} \dot{x}_{i1} \\ \dot{x}_{i2} \\ \dot{x}_{i3} \end{pmatrix} = \begin{pmatrix} 0 & c_i & 0 \\ -c_i & -c_i a_i & 0 \\ 0 & 0 & d_i \end{pmatrix} \frac{\partial H_e}{\partial x_i} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} v_i.$$

Since H_e is bounded from below, because of $x_{i1} \in [-\pi, \pi]$, and $\forall l > 0$ the set $\{x : H_e(x) \leq l\}$ is compact. Thus $H_e(x)$ has a strict local minimum at $(x_{i1}^*, x_{i2}^*, x_{i3}^*)$. Then, a control law which stabilizes the multi-machine power system is given by $u_i = \tilde{u}_i + v_i$, where

$$\begin{aligned} v_i &= -f_i g_i^T \frac{\partial H_e}{\partial x_i} \\ &= -f_i \left(- \sum_{j=1}^{n=3} B_{ij} [x_{j3} \cos(x_{i1} - x_{j1}) - x_{j3}^* \cos(x_{i1}^* - x_{j1}^*)] + \frac{e_i}{d_i} (x_{i3} - x_{i3}^*) \right) \\ &= -f_i \left\{ I_{d_i} + \frac{e_i}{d_i} x_{i3} \right. \\ &\quad \left. + \frac{1}{d_i} \left(d_i \sum_{j=1}^{n=3} B_{ij} x_{j3}^* \cos(x_{i1}^* - x_{j1}^*) - e_i x_{i3}^* \right) \right\} \\ &= -f_i \left\{ I_{d_i} + \frac{e_i}{d_i} x_{i3} - \frac{1}{d_i} \tilde{u}_i \right\} \end{aligned}$$

where $\tilde{u}_i = e_i x_{i3}^* - d_i \sum_{j=1}^{n=3} x_{j3}^* B_{ij} \cos(x_{i1}^* - x_{j1}^*)$. This control law is determined in advance in terms of the desired operational point. Now, using $E_{q_i} = E'_{q_i} + (X_{d_i} - X'_{d_i}) I_{q_i}$, and $d_i = (X_{d_i} - X'_{d_i}) / T'_{d_i}$, $e_i = 1 / T'_{d_i}$, it follows that $\frac{e_i}{d_i} = \frac{1}{(X_{d_i} - X'_{d_i})}$. Finally, the controller can be expressed only in terms of local measurable signals:

$$\begin{aligned} u &= \tilde{u}_i - f_i \left\{ \frac{1}{(X_{d_i} - X'_{d_i})} E_{q_i} - \frac{1}{d_i} \tilde{u}_i \right\} \\ &= \tilde{u}_i + \frac{f_i}{d_i} \tilde{u}_i - \frac{f_i}{(X_{d_i} - X'_{d_i})} (V_{t_i} + \frac{Q_{e_i} X_{d_i}}{V_{t_i}}) \end{aligned}$$

where $E_{q_i} = V_{t_i} + \frac{Q_{e_i} X_{d_i}}{V_{t_i}}$. Consequently, the resulting controller is a decentralized static output feedback.

6. SIMULATION RESULTS

The numerical values of the generator parameters (in per unit) were $D_1 = 5$, $D_2 = 3$, $X'_{d_1} = 0.252$, $X'_{d_2} = 0.319$, $X_{d_1} = 1.863$, $X_{d_2} = 2.36$, $H_1 = 1$, $H_2 = 2$, $T'_{d_1} = 6.9$, $T'_{d_2} = 7.96$, $E_{f_1} = 1.3$, $P_{m_1} = 0.35$, $P_{m_2} = 0.35$ and $\omega_s = 377$. $B_{12} = 0.56$, $B_{13} = 0.53$, $B_{23} = 0.6$. With this parameter choice, the stable equilibrium state of the generator is

$$\begin{aligned} x_{11}^* &= 0.6654, & x_{12}^* &= 0, & x_{13}^* &= 1.03 \\ x_{12}^* &= 0.6425, & x_{22}^* &= 0, & x_{23}^* &= 1.01. \end{aligned}$$

The initial value of the states variables were:

$$x_{11}(0) = 0.8, \quad x_{12}(0) = 0.3, \quad x_{13}(0) = 1.5,$$

$$x_{12}(0) = 0.5, \quad x_{22}(0) = 0.3, \quad x_{23}(0) = 0.5.$$

The controller parameters were chosen as: Control 1: $s_{11} = 1, s_{12} = 2, s_{13} = 10, L_1 = 2$. Control 2: $s_{21} = 1, s_{22} = 2, s_{23} = 10, L_2 = 2$. The closed-loop responses obtained for the internal states of generator 1 are shown in figures 2-4 (due to the lack of place, state behavior of generator 2 is not displayed, but the conclusions are similar). It can be seen that the sliding mode controller 2 can provide some better transient performances than sliding mode controller 1. The Hamiltonian controller of (Sun, 2001) provides the less performing responses, regarding both overshoot and response time.

7. CONCLUSIONS

A nonlinear control strategy based on a so-called continuous sliding mode design for a class of nonlinear systems has been developed and successfully applied to electrical power system control.

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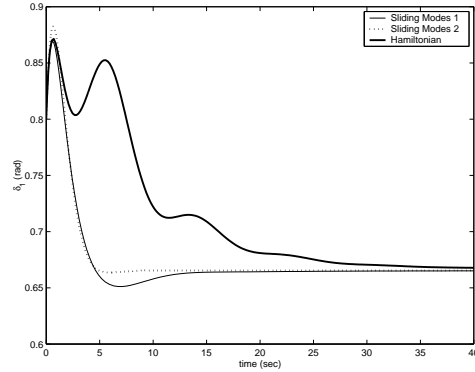


Fig. 2. Comparison of rotor angle responses of generator 1.

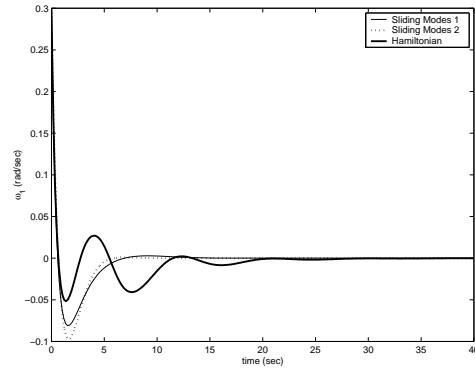


Fig. 3. Comparison of relative speed responses of generator 1.

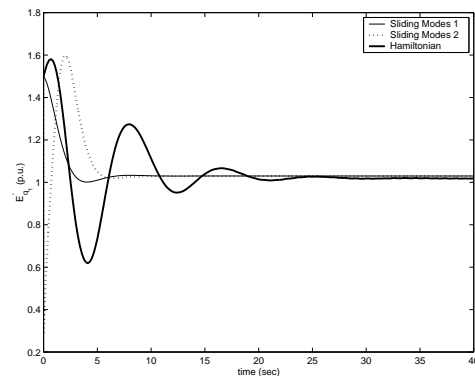


Fig. 4. Comparison of EMF responses of generator 1.