ADAPTIVE NARROW-BAND DISTURBANCE REJECTION FOR STABLE PLANTS UNDER ROBUST STABILIZATION FRAMEWORK

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Abstract: An adaptive control algorithm for rejecting narrow-band noise is proposed in this paper. This algorithm is developed under the framework of robust stabilization and the internal model principle. The internal model which contains the noise dynamic is updated using the estimated disturbance signal. The parameter adaptation is constrained to meet the condition of robust stabilization when the parameters are converged. Simulation examples are given to show the effectiveness of the control law under disturbance with unknown frequencies.

Keywords: Adaptive Control, Adaptive Digital Filters, Disturbance Rejection, Robust Stability, H-infinity Optimization.

1. INTRODUCTION

Internal Model Principle (IMP) has been used for controller design to track periodic trajectories (Tomizuka et al, 1989; Kempf et al, 1993) or reject narrow-band noises (Hu, 1996). For applications such as the active noise cancellation, IMP based controller has the advantage that the information of the noise source signal is not required, as compared with feedforward-type algorithms such as the Filter-X (Elliot and Nelson, 1993; Kuo and Morgan, 1996; Bjarnason, 1995). Therefore, when a coherent noise source is difficult to measure or the cost of sensor installation is high, IMP based controller becomes a good candidate for noise rejection.

The IMP based controller works well if the disturbance frequencies (or the fundamental period of a harmonic noise) are known. For applications like disk drive tracking (Moon et al, 1998), the condition is satisfied when the servo controller is synchronized with the disk rotation. In active noise cancellation (ANC), however, it may not be easy to locate these frequencies precisely. For example, narrow-band noise caused by rotational machinery may drift around some nominal frequencies due to small variations of the machine's speed. Earlier research efforts to identify the frequencies and adjust the control law on-line were reported (Hu, 1992) but only limited to harmonic cases. For digital implementation, if the fundamental period of the harmonic noise is not an integer-multiple of the sampling period, one also has to consider using techniques such as the fractional delay filter for the internal model (Hu and Yu, 2001). Another approach (Elliot and Goodwin. 1984; Palaniswami and Goodwin, 1987; Feng and Palaniswami 1992; Palaniswami, 1993) is to consider the non-minimal representation of the plant which includes the disturbance model. The disturbance model is then extracted (or factored) from the identified plant parameters. An indirect adaptive controller is implemented based on the pole-placement algorithm. Although it is not required to know the plant's parameters, extracting the disturbance model from the composite estimated polynomial may not be easy (Palaniswami 1992), especially when the number of disturbance frequencies is high. There were other adaptive IMP control schemes (Datta and Lei, 1998, 1999; Silva and Datta, 1999; Muramatsu and Watanabe, 2003; Watanabe and Muramatsu, 2003) but were aimed at identification of plants instead of the disturbance model.
In this paper, an adaptive internal model control algorithm is investigated under the framework of robust stabilization. The plant is assumed stable and

a nominal model is known. The output disturbance is narrow-band whose frequencies are unknown or changing slowly. The control objective is to reject the disturbance without affecting the close loop stability under the model uncertainty. Simulation results are shown to demonstrate the effectiveness of the control law..

2. FEEDBACK CONTROLLER BASED ON OUTPUT DISTURBANCE OBSERVATION

A feedback controller which utilizes the observation of the output disturbance can be shown as Figure 1.1. The idea is simple: if the estimation of the plant dynamic is accurate enough and the controller $K(z^{-1})$ provides a proper inversion of the plant within the bandwidth of the disturbance, the influence of the disturbance to the output can be minimized. Other than the uncertainty of the plant dynamic which may degrade the accuracy of disturbance observation, non-minimal phase zeros of the nominal plant also limit the performance of rejection.

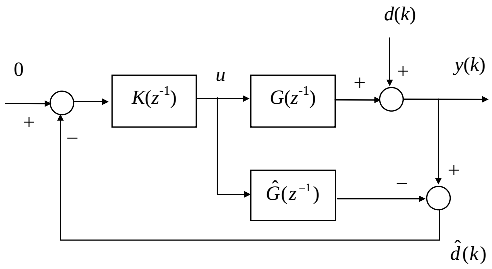

Figure 2.1 Feedback control system based on output disturbance observation ($\hat{G}(z^{-1})$ represents the nominal plant)

For stable plants, Figure 2.1 can be viewed as an Youla Parameterization where a stable controller $K(z^{-1})$ guarantees the closed loop stablility. If the plant is accurately identified (i.e., $G(z^{-1}) = \hat{G}(z^{-1})$), the disturbance to the output satisfies the following equation:

$$Y(z^{-1}) = D(z^{-1}) + G(z^{-1})K(z^{-1})D(z^{-1}) \tag{2.1}$$

Using FIR filter as the structure of $K(z^{-1})$, Eq.(2.1) becomes a standard Filter-X adaptive control setting if a signal correlated to $d(k)$ can be obtained, i.e.,

$$\begin{aligned} y(k) &= d(k) - G(q^{-1})K(q^{-1})d(k) \\ &= d(k) - \theta_d^T \phi_d(k) \end{aligned} \tag{2.2a}$$

where

$$K(z^{-1}) = a_0 + a_1 z^{-1} + a_2 z^{-1} + \cdots + a_N z^{-N} \tag{2.2b}$$

$$\theta_d = [a_0 \; a_1 \cdots a_N]^T \quad ,$$

$$\phi_d(k) = [d_g(k) \; d_g(k-1) \cdots d_g(k-N)]^T \tag{2.2c}$$

$$d_g(k) = G(q^{-1})d(k) \tag{2.2d}$$

Normally, $d(k)$ is replaced by the observed disturbance $\hat{d}(k)$ (Figure 2.1).

This method has found to be quite useful in rejecting narrow-band noises (Rafaely and Elliott, 1996). However, under the plant uncertainty, the $\mathbf{H}_\infty$ norm of $K(z^{-1})$ must be constrained in order to guarantee robustness. Secondly, when the disturbance is rich in frequency, and the plant is non-minimal phase, the length of the FIR filter has to be long enough. This would result in slow convergence. Consider an alternative representation of the control system in Figure 1 (see Figure 2.2 where $G(z^{-1})$ is the nominal plant hereafter).

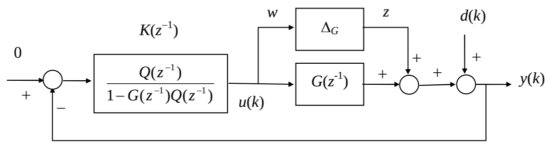

Figure 2.2 A unity feedback configuration of the control system based on output disturbance observation (Δ_G: plant uncertainty)

Suppose the disturbance model is $W_d(z^{-1})$. To design $Q(z^{-1})$ which satisfies the robustness and disturbance rejection, we can solve the following problem:

$$\underset{Q \in \mathbf{RH}_\infty}{Min} \left\| (1 - QG)W_d \right\| \text{ subject to } \left\| Q\Delta_G \right\| < 1$$

Another way to design the controller is to solve $K(z^{-1})$ directly using the $\mathbf{H}_\infty$ controller synthesis methods. Since all stablizing controller can be represented as shown in Figure 2.2 if the plant is stable (Doyle and Francis, 1992), we can obtain $Q(z^{-1})$ as:

$$Q(z^{-1}) = \frac{K(z^{-1})}{1 + K(z^{-1})G(z^{-1})} \tag{2.3}$$

In other words, we can obtain a "nominal" controller based on the undertstanding of the plant and the disturbance. To enhance the performance using adaptive methods, an additional adaptive FIR filter can be added to the controller as shown in Figure 2.3.

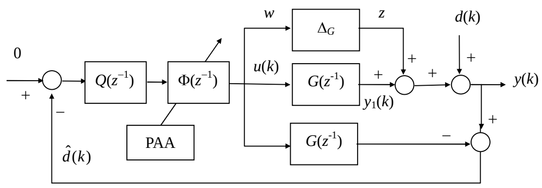

Figure 2.3 Output disturbance rejection with a nomial controller $Q(z^{-1})$ and an adaptive FIR filter $\Phi(z^{-1})$. (PAA: parameter adaptation algorithm)

From Figure 2.3, if $Q(z^{-1})$ is computed to satisfy the robustness constraint, the constraint for $\Phi(z^{-1})$ to maintain robusness at steady state (i.e., when parameters converge) is,

$$\left\| \Phi(z^{-1}) \right\| < 1 \tag{2.4}$$

It is obvious that adding the nominal controller $Q(z^{-1})$ provides a normalization across the frequency for the constraint in parameter adaptation.

3. CONSTRAINED ADAPTIVE CONTROL ALGORITHM

When the FIR filter $\Phi(z^{-1})$ is adjusted for each sampling period, the output of the system in Figure 2.3 can be derived as,

$$y(k) = d(k) - [G(q^{-1}) + \Delta_G]\Phi(k,q^{-1})Q(q^{-1})\hat{d}(k)$$
$$= d(k) - G(q^{-1})\Phi(k,q^{-1})\hat{d}_1(k) + z(k)$$
$$= d(k) + y_1(k) + v(k) \quad (3.1)$$

where $\hat{d}_1(k) = Q(q^{-1})\hat{d}(k)$ and $z(k)$ represents the uncertainty signal. Denote the predicted disturbance signal as,

$$\zeta(k) = y(k) - y_1(k) = d(k) + z(k) \quad (3.2)$$

Our goal is to adjust $\Phi(k,z^{-1})$ so that $y_1(k)$ matches $d(k)$. In other words, given a solution of $\Phi(k,z^{-1})$ that satisfies the goal (i.e., when parameters converge), the dynamics of $y_1(k)$ shall be able to represent $d(k)$. Let the filter $\Phi(k,z^{-1})$ be

$$\Phi(z^{-1}) = f_0(k) + f_1(k)z^{-1} + \cdots + f_L(k)z^{-L},$$

If $[A_d, B_d, C_d, D_d]$ are the state space matrices of the nominal plant, Eq.(3.1) and (3.2) can be represented in state space as,

$$\eta(k+1) = F(k)\eta(k) \quad (3.3a)$$
$$\zeta(k) = H(k)\eta(k) + z(k) \quad (3.3b)$$

where

$$F(k) = \begin{bmatrix} I_L & 0 \\ B_d\phi^T(k) & A_d \end{bmatrix}, \quad (3.3c)$$
$$H(k) = [D_d\phi^T(k) \quad C_d], \quad (3.3d)$$
$$\eta(k) = \begin{bmatrix} \theta(k) \\ x(k) \end{bmatrix}, \quad (3.3e)$$
$$\theta(k) = [f_0(k) \quad f_1(k) \quad \cdots \quad f_L(k)], \quad (3.3f)$$
$$\phi(k) = [\hat{d}_1(k) \quad \hat{d}_1(k-1) \quad \cdots \quad \hat{d}_L(k-L)] \quad (3.3g)$$

$x(k)$ is the state vector of the plant and I_L is an $L \times L$ unity matrix.

The technique of $\mathbf{H}_\infty$ suboptimal causal filtering (Sayyarrodsari et al, 1998) can be applied to the formulation above. We wish to find an optimal estimator such that,

$$\sup_{v,\eta_0} \frac{\sum_{k=0}^{M}\left[\zeta(k) - \hat{\zeta}(k|k)\right]^2}{\eta_0^T \Pi \eta_0 + \sum_{k=0}^{M} z(k)^2} \le \gamma^2 \quad (3.4)$$

where $\hat{\zeta}(k|k)$ is the posteriori prediction of $\zeta(k)$, η_0 the initial guess of the state vector and $\prod$ a positive definite matrix. Eq.(3.4) is valid as long as the the uncertainty signal z(k) is bounded. To show this, it is necessary that the $\mathbf{H}_\infty$ norm of the time varying filter $\Phi(k,z^{-1})$ is less than 1 (Eq.(2.4)). A sufficient condition to guarantee it is,

$$\theta^T(k)\theta(k) = \sum_{i=0}^{L} f_1(k)^2 \le 1, \forall k \quad (3.5)$$

Let Γ be the subspace of $\theta(k)$ (Eq.(3.3e)) such that $\theta^T(k)\theta(k) = 1$. Combining the constraint of Eq.(3.5) and the objective function, we can derive the parameter update law as (Sayyarrodsari et al, 1998),

$$s = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \quad (3.6a)$$
$$= F(k)\hat{\eta}(k) + K_1(k)(\zeta(k) - H(k)\hat{\eta}(k))$$

If $s_1^T s_1 > 1$, $\hat{\eta}(k+1) = \begin{bmatrix} \Pr\{s_1\} \\ s_2 \end{bmatrix}$, else $\hat{\eta}(k+1) = s$

(3.6b)

$$\hat{\zeta}(k|k) = H(k)\hat{\eta}(k) + H(k)P(k)H(k)^T \times R_H(k)^{-1}(\zeta(k) - H(k)\hat{\eta}(k)) \quad (3.6c)$$
$$K_1(k) = F(k)P(k)H(k)^T R_H(k)^{-1} \quad (3.6d)$$
$$R_H(k) = 1 + H(k)P(k)H(k)^T \quad (3.6e)$$
$$P(k+1) = F(k)P(k)F(k)^T, \; P(0) = \Pi \quad (3.6f)$$

where Pr{.} denotes the projection onto the space Γ.

Remark 1: The value of $\zeta(k)$ is computed by $\zeta(k) = y(k) - y_1(k)$ where $y_1(k)$ can be obtained by running a simulated nominal plant as $y_1(k) = G(q^{-1})u(k)$. But this is impossible if the nominal plant is proper but not strictly proper, i.e., $D_d \ne 0$.

The following list summarizes the procedure:
At every k-th sampling instant,

Step 1: compute the nominal plant output $y_1(k) = G(q^{-1})u(k)$ and from the measurement of $y(k)$ compute $\zeta(k) = y(k) - y_1(k)$

Step 2: compute the estimated disturbance $\hat{d}(k) = y(k) - \zeta(k)$.

Step 3: compute $\hat{d}_1(k) = Q(q^{-1})\hat{d}(k)$ and form the vector $\phi(k)$.

Step 4: compute the parameter and state update $\hat{\eta}(k+1)$ from Eq.(3.6a) – (3.6f)

Step 5: compute the control $u(k) = -\hat{\theta}(k+1)^T\phi(k)$.

4. DESIGN EXAMPLE AND SIMULATION

To illustrate the adaptive control algorithm, we consider the plant of an active controlled headset shown. The nominal plant represents the case when the headset is in a normal position which covers the full ear and the actual plant is when it covers ear partially. Table 4.1 shows the corresponding parameters.

Actual plant		Nominal plant	
Poles	Zeros	Poles	Zeros
0.96 $0.18 \pm 0.80j$ $0.57 \pm .50j$	-1.58 1.02 $0.37 \pm 0.56j$	0.96 $0.16 \pm 0.78j$ $0.58 \pm 0.51j$	-1.55 1.01 $0.41 \pm 0.56j$
Gain = -0.60441		Gain = -0.65292	

Table 4.1 Poles, zeros and gain of the nominal and actual plant

4.1 Design of the Central Controller

The procedure described in (Stoorvogel, 1992) is followed to design the central controller $K(z^{-1})$ of Figure 2.2 and the corresponding $Q(z^{-1})$ is computed. Figure 4.1 shows the structure of the controller design setting where w_1 and w_2 are uncertainty inputs, z the uncertainty output, u the input and y the measured output.

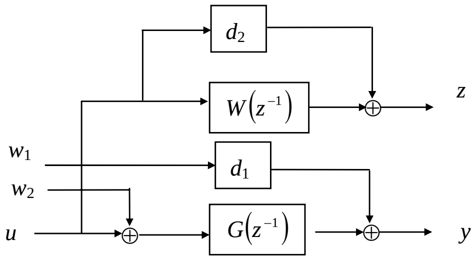

Figure 4.1 The robustness control setting to design the central controller ($G(z^{-1})$: the nominal plant; $W(z^{-1})$: the weighting function)

Denote the uncertainty signal z as $w_1 = \Delta_1 z$ and $w_2 = \Delta_2 z$, the I/O relationship is,

$$\frac{Y(z^{-1})}{U(z^{-1})} = G(z^{-1})\left(1 + \Delta_1 (W(z^{-1}) + d_2)\right) + \Delta_2 \left(d_2 + W(z^{-1})\right) d_1 \qquad (4.1)$$

The plant uncertainty is defined as Δ_G as shown in Figure 2.2. Therefore, we have,

$$G(z^{-1}) + \Delta_G = G(z^{-1})\left(1 + \Delta_1 (W(z^{-1}) + d_2)\right)$$

Or,

$$\frac{\Delta_G}{G(z^{-1})} = \Delta_1 (W(z^{-1}) + d_2) \qquad (4.2)$$

As a result, by properly choosing the weighting function $W(z^{-1})$ such that

$$\left\| W(e^{-j\omega}) + d_2 \right\| > \left\| \frac{\Delta_G}{G(e^{-j\omega})} \right\|, \forall \omega \in [0, \pi] \qquad (4.3)$$

we can scale the plant uncertainty to be less than 1 to design the robust controller as well as give the unity constraint of the adaptive FIR filter (Eq.(2.4)). The controller $K(z^{-1})$ can be solved by using *Matlab* and the controller is transformed to the internal model-based control structure by Eq.(2.3).

4.2 Simulation results

Figure 4.2 shows the simulation result when the noise is a single tone at 600 Hz. The length of the FIR filter is 6.

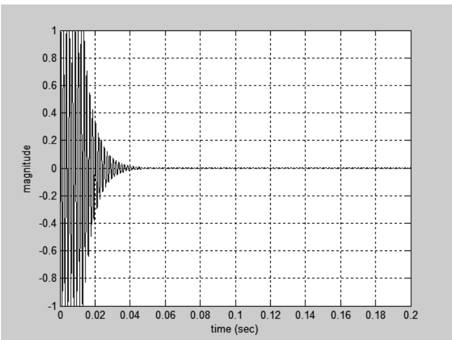

Figure 4.2 Simulation result of rejecting a tonal noise at 600 Hz

To see the rejection capability when the noise's frequency is changing, the following noise is applied.

$$d(k) = \sin(2\pi f_1(k)k / f_s)$$

where $f_1(k) = 600 + 10\sin(2\pi k f_m / f_s)$ and f_m = 3. It means the frequency swings between 590 and 610 Hz in a speed of 3 Hz. Figure 4.3 shows that the controller is able to adapt the changes. For comparison, another simulation is performed using the feedforward-type of algorithm where the observed noise $\hat{d}(k)$ (Figure 2.3) is replaced by a 600 Hz tone. Figure 4.4 shows that the modulation of the frequency remains at the output and has the tendency of growing. The length of the FIR filter in both cases is 60..

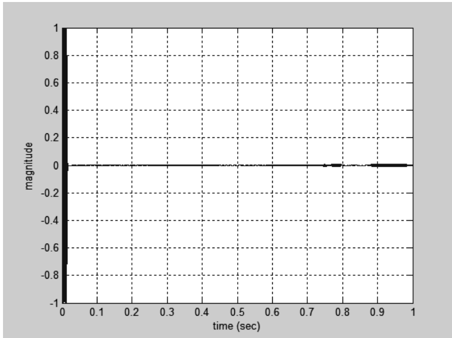

Figure 4.3 Simulation result for single tone noise with a slight frequency modulation

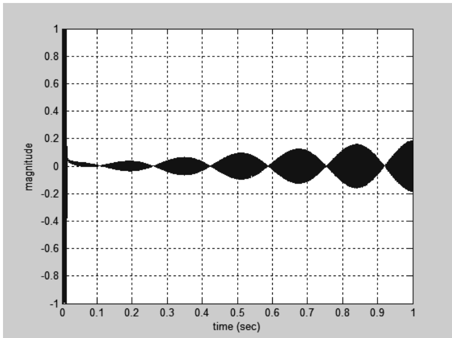

Figure 4.4 Simulation result for the feedforward type of algorithm without considering the change of noise frequency

Lastly, a noise data measured from a V8 internal combustion (IC) engine at idle speed is used for the disturbance. IC engine noise contains many narrow-band features due to the periodic nature of its operation. Figure 4.5 shows the noise and the noise reduction after applying the controller. The reduction is more clearly seen from the spectrum in Figure 4.6. The narrow-band noises at lower frequencies are reduced at the expense of slight increases at higher frequencies.

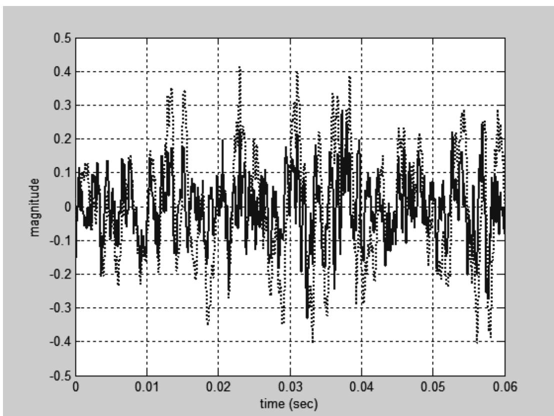

Figure 4.5 the simulation result when the disturbance is an engine noise (dot: original noise; solid: reduced noise)

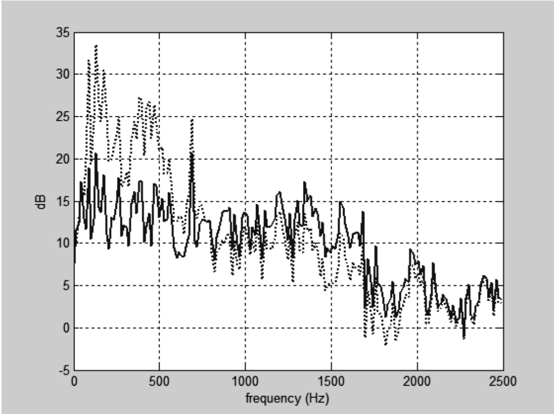

Figure 4.6 the spectrum of the noise and noise reduction (dot: original noise; solid: reduced noise)

5. CONCLUSION

An adaptive internal model-based control algorithm is presented in this paper. This algorithm utilizes the robustness control technique to scale the plant uncertainty and constrains the adaptive filter to meet the robustness requirement. An observer-based adaptive law with the H_∞ optimal filtering criterion is proposed. Simulation shows that this algorithm is able to reject narrow-band disturbances with unknown or slow-varying frequencies.

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