FINITE-TIME OUTPUT FEEDBACK CONTROL OF DISCRETE-TIME SYSTEMS

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Abstract: In this paper we deal with some finite-time control problems for discrete-time, time-varying linear systems. First we provide necessary and sufficient conditions for finite-time stability; these conditions require either the computation of the state transition matrix of the system or the solution of a certain difference Lyapunov inequality. Then we address the design problem. The proposed conditions allow to find state feedback and output feedback controllers which stabilizes the closed loop system in the finite-time sense.

Keywords: discrete-time systems, linear systems, finite-time stability, output feedback.

1. INTRODUCTION

When dealing with the stability of a system, a distinction should be made between *classical Lyapunov stability* and *finite-time stability* (FTS) (or *short-time stability*). The concept of Lyapunov asymptotic stability is largely known to the control community; conversely a system is said to be finite-time stable if, once we fix a time-interval, its state does not exceed some bounds during this time-interval. Often asymptotic stability is enough for practical applications, but there are some cases where *large* values of the state are not acceptable, for instance in the presence of saturations. In these cases, we need to check that these unacceptable values are not attained by the state; for these purposes FTS could be used.

Most of the results in the literature are focused on Lyapunov stability. Some early results on FTS can be found in (Dorato, 1961), (Weiss and Infante, 1967) and (D'Angelo, 1970); more recently the concept of FTS has been revisited in the light of recent results coming from Linear Matrix Inequalities (LMIs) theory (Boyd *et al.*, 1994), which has allowed to find less conservative conditions guaranteeing FTS and finite time stabilization of uncertain, linear continuous-time systems (see (Abdallah *et al.*, 2002), (Amato *et al.*, 2001)).

In this paper we consider time-varying discrete-time systems. In (Amato *et al.*, 2004) some conditions for finite-time stability have been provided. The main theorem of (Amato *et al.*, 2004) guarantees FTS *if and only if* either a certain inequality involving the state transition matrix is satisfied, or a symmetric matrix function solving a certain Lyapunov difference inequality exists.

The condition involving the state transition matrix cannot be used as the starting point to solve the synthesis problem. Therefore, in view of the design problem, we focus on the condition involving the Lyapunov inequality. However this condition can become computationally hard to apply, since it requires to study the feasibility of N difference inequalities, if $[1, N]$ is the time interval in which FTS is studied. For this reason a *sufficient* condition for FTS which requires to check the feasibility of *only one* inequality is used to address the problem of designing state feedback and output feedback controllers guaranteeing some finite-time performance.

The paper is organized as follows: in Section 2 the definition of finite-time stability is recalled and specialized to the discrete-time case, the conditions for finite-time stability provided in (Amato *et al.*, 2004) are recalled and the problem we want to solve is formally stated. In Section 3 we address the FTS synthesis problems, namely some sufficient conditions for the existence of an output feedback controller guaranteeing finite-time stabilization of the closed loop system are provided. Our conclusions are drawn in Section 4.

2. PROBLEM STATEMENT AND PRELIMINARIES

In this paper we consider the following discrete-time time-varying linear system

$$x(k+1) = A(k)x(k) + B(k)u(k) \tag{1a}$$
$$y(k) = C(k)x(k) \tag{1b}$$

where $A(k)$, $B(k)$ and $C(k)$ take value in $\mathbb{R}^{n\times n}$, $\mathbb{R}^{n\times m}$ and $\mathbb{R}^{p\times n}$, respectively.

The general idea of *finite-time stability* concerns the boundedness of the state of the system over a finite time interval for some given initial conditions; this concept can be formalized through the following definition, which is an extension to discrete-time systems of the one given in (Dorato, 1961).

Definition 1. (Finite-time stability). The discrete-time linear system

$$x(k+1) = A(k)x(k) \qquad k \in \mathbb{N}_0 \tag{2}$$

is said to be finite-time stable with respect to (δ, ϵ, R, N), where R is a positive definite matrix, $0 < \delta < \epsilon$, and $N \in \mathbb{N}_0$, if

$$x^T(0)Rx(0) \le \delta^2 \Rightarrow x^T(k)Rx(k) < \epsilon^2 \qquad \forall k \in \{1, \dots, N\}$$

△

Remark 2. Lyapunov Asymptotic Stability (LAS) and FTS are independent concepts: a system which is FTS may be not LAS; conversely a LAS system could be not FTS if, during the transients, its state exceeds the prescribed bounds. ◇

The following theorem is fundamental for the subsequent development.

Theorem 1. (Nec. and suff. conditions FTS). The following statements are equivalent:

i) System (2) is FTS with respect to (δ, ϵ, R, N).

ii) $\Phi(k, 0)^T R\Phi(k, 0) < \frac{\epsilon^2}{\delta^2} R$ for all $k \in \{1, \dots, N\}$, where $\Phi(\cdot, \cdot)$ denotes the state transition matrix.

iii) For each $k \in \{1, \dots, N\}$ let

$$P_k(k) = R$$
$$P_k(h) = A(k)^T P_k(h+1)A(k) \qquad h \in \{0, 1, \dots, k-1\}.$$

then $P_k(0) < \frac{\epsilon^2}{\delta^2} R$.

iv) For each $k \in \{1, \dots, N\}$ there exists a symmetric matrix-valued function $P_k(\cdot) : h \in \{0, 1, \dots, k\} \mapsto P_k(h) \in \mathbb{R}^{n\times n}$ such that

$$A(k)^T P_k(h+1)A(k) - P_k(h) < 0 \quad h \in \{0, 1, \dots, k-1\} \tag{3a}$$
$$P_k(k) \ge R \tag{3b}$$
$$P_k(0) < \frac{\epsilon^2}{\delta^2} R\,. \tag{3c}$$

Moreover, each one of the above conditions are implied by the following.

v) There exists a symmetric matrix-valued function $P(\cdot) : k \in \{0, 1, \dots, N\} \mapsto P(k) \in \mathbb{R}^{n\times n}$ such that

$$A(k)^T P(k+1)A(k) - P(k) < 0\,, \quad k \in \{0, 1, \dots, N-1\} \tag{4a}$$
$$P(k) \ge R\,, \qquad k \in \{1, \dots, N\} \tag{4b}$$
$$P(0) < \frac{\epsilon^2}{\delta^2} R\,. \tag{4c}$$

vi) There exists a symmetric matrix-valued function $Q(\cdot) : k \in \{0, 1, \dots, N\} \mapsto Q(k) \in \mathbb{R}^{n\times n}$ such that

$$A(k)Q(k)A(k)^T - Q(k+1) < 0\,, \quad k \in \{0, 1, \dots, N-1\} \tag{5a}$$
$$Q(k) \le R^{-1}\,, \qquad k \in \{1, \dots, N\} \tag{5b}$$
$$Q(0) > \frac{\delta^2}{\epsilon^2} R^{-1}\,. \tag{5c}$$

PROOF. The equivalence between i)–iv), together with the fact that each one of i)–iv) are implied by v), can be proven by following the guidelines of (Amato *et al.*, 2004).

Now we prove that the conditions v) and vi) are equivalent. Indeed, by using Schur complements, we have that (4a) is equivalent to

$$\begin{pmatrix} -P(k) & A(k)^T P(k+1) \\ P(k+1)A(k) & -P(k+1) \end{pmatrix} < 0 . \qquad (6)$$

By pre- and post-multiplying the last inequality by

$$\begin{pmatrix} P^{-1}(k) & 0 \\ 0 & P^{-1}(k+1) \end{pmatrix} ,$$

we obtain that (6) can be equivalently rewritten

$$\begin{pmatrix} -P^{-1}(k) & P^{-1}(k)A(k)^T \\ A(k)P^{-1}(k) & -P^{-1}(k+1) \end{pmatrix} < 0 , \qquad (7)$$

which in turn is equivalent to

$$\begin{pmatrix} -P^{-1}(k+1) & A(k)P^{-1}(k) \\ P^{-1}(k)A(k)^T & -P^{-1}(k) \end{pmatrix} < 0 ; \qquad (8)$$

the equivalence follows by letting $Q(k) = P^{-1}(k)$ and using Schur complements again. □

We have some remarks about the use of the results contained in Theorem 1.

Remark 3. Statements ii) and iii) are very useful to test the FTS of a given system. However, as shown in (Amato *et al.*, 2004), they cannot be used for design purposes. In the same way condition iv) is not useful from a practical point of view, since it requires to study the feasibility of N difference inequalities, if $[1, N]$ is the time interval of interest.

Conversely the sufficient condition v) and vi) require to check only *one* difference inequality. ◇

Remark 4. Note that a matrix function $P(\cdot)$ satisfying condition v) in Theorem 1 can be found, if one exists, by solving recursively an LMIs feasibility problem through the LMI Toolbox (Gahinet *et al.*, 1995). ◇

Remark 5. Condition vi) will be used for the state feedback design. ◇

In the following example we use the results of Theorem 1 to show that finite-time stability and asymptotic stability are *independent* concepts.

Example 1. (FTS and LAS). Let us first consider the system

$$x(k+1) = \begin{pmatrix} 0.8026 & 1.0000 & 0.2392 \\ -0.1842 & 0.8026 & 0.2034 \\ 0 & 0 & 0.3333 \end{pmatrix} x(k) .$$

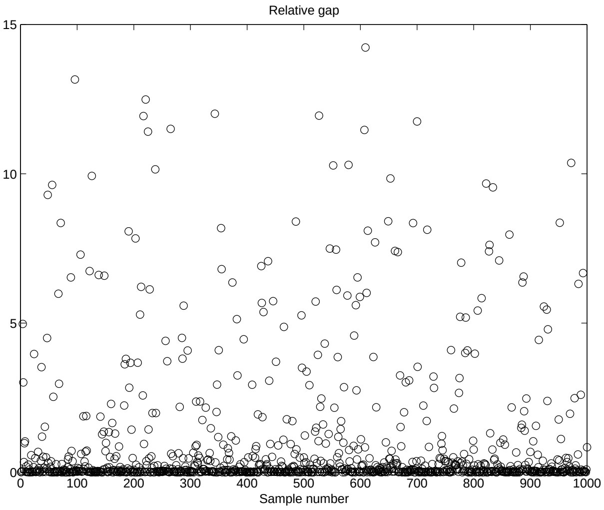

Fig. 1. Relative gap between the necessary and sufficient condition i)–iv) and the sufficient condition v) of Theorem 1

This system is asymptotically stable, since its eigenvalues are inside the unit circle. But it is not FTS with respect to (δ, ϵ, R, N) with $\delta = 1$, $\epsilon = 1.78$, $R = I$ and $N = 5$. Indeed condition ii), iii) and iv) of Theorem 1 fails for $k = 3$.

On the other hand, let us consider the system

$$x(k+1) = \begin{pmatrix} 0.3333 & 0.4189 & 0.0833 \\ 0 & 1.1053 & 0.4189 \\ 0 & 0 & 0.3333 \end{pmatrix} x(k) ,$$

which is unstable. Anyway, by applying condition ii) or iii) or iv) of Theorem 1 it is possible to show that this system is FTS with respect to (δ, ϵ, R, N) with $\delta = 1$, $\epsilon = 2.13$, $R = I$ and $N = 5$. △

Example 2. In order to compare the necessary and sufficient conditions i)–iv) stated in Theorem 1 with the sufficient condition v) we have randomly generated 1,000 discrete-time linear systems. For each sample we have computed the minimum ϵ such that the given system is FTS wrt (δ, ϵ, R, N) with $\delta = 1$, $N = 5$, $R = I$.

Figure 1 shows, for each generated system, the value of the following quantity

$$\text{err}_{\%} = 100(\epsilon_{\text{suff}} - \epsilon_{\text{true}})/\epsilon_{\text{true}} ,$$

where ϵ_{true} denotes the exact value of ϵ computed using conditions i)–iv) and ϵ_{suff} its estimated value obtained applying condition v).

Note that for most of the systems the value of $\text{err}_{\%}$ is close to zero. △

Now, given system

$$x(k+1) = A(k)x(k) + B(k)u(k) \qquad (9)$$

we consider the time-varying state feedback controller

$$u(k) = G(k)x(k) , \qquad (10)$$

where G takes value in $\mathbb{R}^{m\times n}$. One of the goal of this paper is to find some sufficient conditions which guarantee that the state of the system given by the interconnection of system (9) with the controller (10) is *stable over a finite-time interval.*

Problem 1. Given system (9), find a state feedback controller (10) such that the closed-loop system

$$x(k+1) = (A(k) + B(k)G(k))x(k) \qquad (11)$$

is finite-time stable with respect to (δ, ϵ, R, N). △

Then with respect to system (1), we consider the following dynamic output feedback controller.

$$x_c(k+1) = A_K(k)x_c(k) + B_K(k)y(k) \qquad (12a)$$
$$u(k) = C_K(k)x_c(k) + D_K(k)y(k)\,, \qquad (12b)$$

where the controller state vector $x_c(k)$ has the same dimension of $x(k)$.

The main goal of the paper is to find some sufficient conditions which guarantee the existence of an output feedback controller which finite-time stabilizes the overall closed loop system, as stated in the following problem.

Problem 2. Let us denote by R_K the weight of the controller state. Then, given system (1), find an output feedback controller (12) such that the corresponding closed-loop system is finite-time stable with respect to $(\delta, \epsilon, \mathrm{blockdiag}(R, R_K), N)$. △

Note that we assume, for the sake of simplicity, that the weighting matrix does not contain cross-coupling terms between the system state and the controller state. Moreover the definition of δ and ϵ in Problem 2 must take into account the augmented state of the closed loop system.

3. MAIN RESULTS

Let us start with the state feedback Problem 1. The solution of this problem is given by the following theorem.

Theorem 2. (FTS via state feedback). Problem 1 is solvable if there exists a positive definite matrix-valued function $Q(\cdot)$ and a matrix-valued function $L(\cdot)$ such that

$$\begin{pmatrix} -Q(k+1) & A(k)Q(k)+B(k)L(k) \\ Q(k)A(k)^T + L(k)^T B(k)^T & -Q(k) \end{pmatrix} < 0,$$
$$k \in \{0, 1, \ldots, N-1\} \qquad (13a)$$
$$Q(k) \le R^{-1}\,, \qquad k \in \{1, \ldots, N\} \qquad (13b)$$
$$Q(0) > \frac{\delta^2}{\epsilon^2} R^{-1}\,; \qquad (13c)$$

in this case the gain of a state feedback controller solving Problem 1 is given by $G(k) = L(k)Q(k)^{-1}$.

PROOF. First of all note that condition vi) in Theorem 1 can be equivalently rewritten as

$$\begin{pmatrix} -Q(k+1) & A(k)Q(k) \\ Q(k)A(k)^T & -Q(k) \end{pmatrix} < 0\,. \qquad (14)$$

Now we can apply the last inequality to system (11), by replacing $A(k)$ with $A(k)+B(k)G(k)$; in this way we find that the system is guaranteed to be FTS w.r.t. (δ, ϵ, R, N) if

$$\begin{pmatrix} -Q(k+1) & (A(k)+B(k)G(k))Q(k) \\ Q(k)(A(k)+B(k)G(k))^T & -Q(k) \end{pmatrix} < 0$$
$$k \in \{0, 1, \ldots, N-1\} \qquad (15a)$$
$$Q(k) \le R^{-1}\,, \qquad k \in \{1, \ldots, N\} \qquad (15b)$$
$$Q(0) > \frac{\delta^2}{\epsilon^2} R^{-1}\,. \qquad (15c)$$

The proof follows by letting $G(k)Q(k) = L(k)$. □

Remark 6. In order to find a numerical solution to Problem 1, i.e. to compute the matrix-valued functions $Q(\cdot)$ and $L(\cdot)$, a back-stepping algorithm can be used for conditions (13). In the first step inequalities (13a) and (13b) can be solved, obtaining the matrices $Q(N)$, $Q(N-1)$, $L(N-1)$. Given $Q(N-1)$, in the next step (13a) and (13b) can be solved for $k = N-2$, finding $Q(N-2)$, $L(N-2)$, and so on. The final step consists in solving (13a) and (13c) together for $k = 0$. In order to find the smallest value for ϵ, in the various steps a further condition can be added, which imposes the maximization of the smallest eigenvalue of $Q(k)$ at each step. ◇

Next, we move to finite-time stabilizability via output feedback. First we need the following technical lemma.

Lemma 1. ((Gahinet, 1996)). Given symmetric matrices $S \in \mathbb{R}^{n\times n}$ and $Q \in \mathbb{R}^{n\times n}$, the following statements are equivalent.

i) There exist a symmetric matrix $T \in \mathbb{R}^{n\times n}$ and matrices $M \in \mathbb{R}^{n\times n}$, $N \in \mathbb{R}^{n\times n}$ such that

$$P := \begin{pmatrix} S & M \\ M^T & T \end{pmatrix} > 0\,, \quad P^{-1} = \begin{pmatrix} Q & N \\ N^T & Z \end{pmatrix}. \tag{16}$$

ii)

$$\begin{pmatrix} Q & I \\ I & S \end{pmatrix} > 0\,. \tag{17}$$

Theorem 3. Problem 2 is solvable if there exist positive definite matrix-valued functions $Q(\cdot)$, $S(\cdot)$, an invertible matrix $N(\cdot)$, matrix-valued functions $\hat{A}_K(\cdot)$, $\hat{B}_K(\cdot)$, $\hat{C}_K(\cdot)$ and $D_K(\cdot)$ such that

$$\begin{pmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{12}^T & \Theta_{22} \end{pmatrix} < 0 \tag{18a}$$

$$\begin{pmatrix} Q(k) & \Psi_{12}(k) & \Psi_{13}(k) & \Psi_{14}(k) \\ \Psi_{12}^T(k) & \Psi_{22}(k) & 0 & 0 \\ \Psi_{13}^T(k) & 0 & I & 0 \\ \Psi_{14}^T(k) & 0 & 0 & I \end{pmatrix} \geq 0 \tag{18b}$$

$$\begin{pmatrix} Q(0) & I \\ I & S(0) \end{pmatrix} \leq \frac{\epsilon^2}{\delta^2} \begin{pmatrix} \Delta_{11} & Q(0)R \\ RQ(0) & Q(0) \end{pmatrix}, \tag{18c}$$

where

$$\Theta_{11}(k) = -\begin{pmatrix} Q(k) & I \\ I & S(k) \end{pmatrix} \tag{19a}$$

$$\Theta_{12}(k) = \begin{pmatrix} Q(k)A(k)^T + \hat{C}_K^T(k)B(k)^T & \hat{A}_K^T(k) \\ A(k)^T + C(k)^T D_K^T(k)B(k)^T & A^T S(k+1) + C^T \hat{B}_K^T(k) \end{pmatrix} \tag{19b}$$

$$\Theta_{22}(k) = -\begin{pmatrix} Q(k+1) & I \\ I & S(k+1) \end{pmatrix} \tag{19c}$$

$$\Psi_{12}(k) = I - Q(k)R \tag{19d}$$

$$\Psi_{13}(k) = Q(k)R^{1/2} \tag{19e}$$

$$\Psi_{14}(k) = N(k)R_K^{1/2} \tag{19f}$$

$$\Psi_{22}(k) = S(k) - R \tag{19g}$$

$$\Delta_{11} = Q(0)N(0)Q(0) + N(0)R_K N^T(0)\,. \tag{19h}$$

PROOF. From v) in Theorem 1 it follows that Problem 1 is solvable if there exist a positive definite matrix function $P(\cdot)$ and matrices $A_K(\cdot)$, $B_K(\cdot)$, $C_K(\cdot)$ and $D_K(\cdot)$ such that

$$\begin{pmatrix} -P(k) & A_{CL}^T(k)P(k+1) \\ P(k+1)A_{CL}(k) & -P(k+1) \end{pmatrix} < 0$$

$$k \in \{0, 1, \ldots, N-1\} \tag{20a}$$

$$P(k) \geq \text{blockdiag}(R, R_K)\,, \quad k \in \{1, 2, \ldots, N\} \tag{20b}$$

$$P(0) \leq \frac{\epsilon^2}{\delta^2}\,\text{blockdiag}(R, R_K)\,, \tag{20c}$$

where

$$A_{CL}(k) = \begin{pmatrix} A(k) + B(k)D_K(k)C(k) & B(k)C_K(k) \\ B_K(k)C(k) & A_K(k) \end{pmatrix} \tag{21}$$

is the closed loop system matrix.

Now, according to (Gahinet, 1996), let us define

$$P(k) = \begin{pmatrix} S(k) & M(k) \\ M^T(k) & \star \end{pmatrix},$$

$$P^{-1}(k) = \begin{pmatrix} Q(k) & N(k) \\ N^T(k) & \star \end{pmatrix},$$

where $\star$ denotes a ‘don’t care’ block, and

$$\Pi_1(k) = \begin{pmatrix} Q(k) & I \\ N^T(k) & 0 \end{pmatrix} \quad \Pi_2(k) = \begin{pmatrix} I & S(k) \\ 0 & M^T(k) \end{pmatrix}.$$

Note that by definition

$$P(K)\Pi_1(k) = \Pi_2(k)\,. \tag{22}$$

By pre- and post-multiplying inequality (20a) by blockdiag$(\Pi_1^T(k), \Pi_1^T(k+1))$ and blockdiag$(\Pi_1(k)$, $\Pi_1(k+1))$ respectively, pre- and post-multiplying (20b) and (20c) by $\Pi_1^T(k)$ and $\Pi_1(k)$ respectively, taking into account (22) and Lemma 1 the proof follows once we let

$$\hat{B}_K(k) = M(k+1)B_K(k) + S(k+1)B(k)D_K(k) \tag{23a}$$

$$\hat{C}_K(k) = C_K(k)N^T(k) + D_K(k)C(k)Q(k) \tag{23b}$$

$$\begin{aligned} \hat{A}_K(k) &= M(k+1)A_K(k)N^T(k) \\ &\quad + S(k+1)B(k)C_K(k)N^T(k) \\ &\quad + M(k+1)B_K(k)C(k)Q(k) \\ &\quad + S(k+1)\big(A(k) + B(k)D_K(k)C(k)\big)Q(k)\,. \end{aligned} \tag{23c}$$

Note that (18a) implies that, at each time instant k,

$$\begin{pmatrix} Q(k) & I \\ I & S(k) \end{pmatrix} > 0 \tag{24}$$

which, according to Lemma 1, guarantees the reconstruction of $P(k)$ starting from the knowledge of $S(k)$, $Q(k)$ and $N(k)$. □

Remark 7. (Controller design). Assume now that the hypothesis of Theorem 3 are satisfied; in order to design the controller the following steps have to be followed:

i) Find $Q(\cdot)$, $S(\cdot)$, $N(\cdot)$, $\hat{A}_K(\cdot)$, $\hat{B}_K(\cdot)$, $\hat{C}_K(\cdot)$ and $D_K(\cdot)$ such that (18) are satisfied.
ii) Find matrix function $M(\cdot)$ such that $M(k) = (I - S(k)Q(k))N^{-T}(k)$.
iii) Obtain $A_K(\cdot)$, $B_K(\cdot)$, $C_K(\cdot)$ and $D_K(\cdot)$ by inverting (23).

△

Note that (18a) and (18b) are linear difference matrix inequalities. Concerning the initial condition (18c), it has to be a posteriori checked; alternatively it can be taken into accout within the design cycle by solving a quadratic optimization problem for $k = 0$ over $Q(0)$, $S(0)$ and $N(0)$.

Moreover, in order to compute $M(\cdot)$, the matrix-valued function $N(\cdot)$ has to be invertible. To this end we can force the condition $N(k) > 0$ for all $k \in \{1, 2, \dots, N\}$. According to (Amato *et al.*, 2003) this does not cause any loss of generality, since the existence of a nonsingular $N(\cdot)$ satisfying (18b) and (18c) implies the existence of a positive definite $N(\cdot)$ satisfying the same inequalities.

3.1 An Example

In order to illustrate the application of the proposed controller design procedure, let us consider system (1) where

$$A = \begin{pmatrix} -0.2 & 0 & 1 \\ 0.2 & 0.4 & -0.6 \\ 0.2 & 0 & -0.6 \end{pmatrix} \quad B = \begin{pmatrix} 0.2844 & 0.9883 \\ 0.4692 & 0.5828 \\ 0.0648 & 0.4235 \end{pmatrix}$$
$$C = \begin{pmatrix} 0.5155 & 0.4329 & 0.5798 \\ 0.3340 & 0.2259 & 0.7604 \end{pmatrix}.$$

Our goal is to solving Problem 2 with $\delta = 1$, $\epsilon = 2$, $N = 5$, $R = R_K = I$.

First we solve (18b) for $k = 5$ and $k = 4$ together with (18a) for $k = 4$; we find $Q(4)$, $Q(5)$, $S(4)$, $S(5)$, $N(4)$, $N(5)$, $\hat{A}_K(4)$, $\hat{B}_K(4)$, $\hat{C}_K(4)$, $D_K(4)$. Then, according to Remark 7, point ii), we compute $M(4)$ and $M(5)$.

Then, by using (23), we compute the controller matrices:

$$\begin{aligned} B_K(4) &= M^{-1}(5)\hat{B}_K(4) - M^{-1}(5)S(5)B(4)D_K(4) \\ &= \begin{pmatrix} -0.6365 & 0.8893 \\ 0.2006 & -0.2804 \\ 1.2811 & -1.7899 \end{pmatrix} \\ C_K(4) &= \hat{C}_K(4)N^{-T}(4) - D_K(4)C(4)Q(4)N^{-T}(4) \\ &= \begin{pmatrix} 0.3327 & -0.2352 & -0.1029 \\ 0.0323 & -0.0937 & 0.0684 \end{pmatrix} \\ A_K(4) &= M^{-1}(5)\big\{\hat{A}_K(4) - S(5)B(4)C_K(4)N^T(4) \\ &\quad - M(5)B_K(4)C(4)Q(4) \\ &\quad - S(5)\left[A(4) + B(4)D_K(4)C(4)\right]Q(4)\big\}N^{-T}(4) \\ &= \begin{pmatrix} -0.0766 & 0.0830 & -0.0082 \\ 0.0285 & -0.0255 & -0.0114 \\ 0.1372 & -0.1682 & 0.0677 \end{pmatrix}. \end{aligned}$$

Then we solve (18a) and (18b) for $k = 3$ using the matrices computed at $k = 4$; the procedure is iterated until $k = 0$, where the satisfaction of (18c) is checked.

4. CONCLUSIONS

In this paper we have dealt with the finite-time control of linear time-varying systems. Starting from some conditions guaranteeing finite-time stability we have provided sufficient conditions for the solution of state and output feedback problems. The proposed design conditions are expressed in terms of linear difference matrix inequalities.

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