CONTROL OF NONHOLONOMIC SYSTEMS: A SIMPLE STABILIZING TIME-SWITCHING STRATEGY [1]

Daniele Casagrande [*] **Alessandro Astolfi** [**]
Thomas Parisini [*]

[*] *Dept. of Electrical, Electronic and Computer Engineering, University of Trieste, Via Valerio, 10 - 34127 Trieste, ITALY*
[**] *Department of Electrical and Electronic Engineering Imperial College, Exhibition Road, London SW7 2BT, UK*

Abstract: The problem of asymptotic stabilization for a class of nonholonomic systems is studied and solved by means of a hybrid control law which makes use of a (deterministic) finite state machine. It is shown that, by using a simple switching control scheme, the origin is a globally asymptotically stable equilibrium in the sense of Lyapunov. The control law can take into account the presence of input saturation. Simulation results are reported showing the performance of the proposed control scheme. *Copyright © 2005 IFAC*

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1. INTRODUCTION

Switching control has recently attracted considerable attention in the research community. One of the main reasons behind this interest lies on the fact that, in several situations, switching control laws may outperform control laws whose structures do not change over time (see, for instance, (Narendra and Balakrishnan, 1997; Hespanha and Morse, 2002; Hespanha *et al.*, 2003*b*; Hespanha *et al.*, 2003*a*)).

The stability analysis of switched control systems is, of course, a crucial task that has been thoroughly investigated by many authors, starting from the works (Michel and Ye, 1998) and (Branicky, 1998). Various approaches can be found dealing with the stability properties of switched systems in which the system model is unique, but several controllers are available (see, for instance, the review papers (Decarlo *et al.*, 2000; Michel, 1999) and the recent book (Liberzon, 2003)).

In this work, we focus on a specific class of nonlinear systems, namely *nonholonomic systems*, for which, as shown by Brockett theorem (Brockett, 1983), no continuous time-invariant stabilizer exists (as a matter of fact, as shown in (Ryan, 1994), the same conclusion holds even if a class of discontinuous controller is allowed). For such systems several different tecniques have been recently proposed (see, among others, (Astolfi, 1996; Hespanha, 1996; Bloch and Drakunov, 1996; Kolmanovsky and McClamroch, 1996; Murray and Sastry, 1993) and the survey paper (Kolmanovsky and McClamroch, 1995)).

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In the present paper we consider the problem of stabilization of a class of nonholonomic systems by means of a hybrid control law, *i.e.* a control law which makes use of a discrete state which is updated by a finite state machine. The approach is similar to the one in (Hespanha, 1996), however, unlike the results therein we are able to explicitly compute a Lyapunov function for the closed-loop system. Finally, the proposed approach is applicable to more general classes of nonholonomic systems and can be exploited in an optimal control analysis.

The paper is organized as follows. In Section 2 we present the model of the system and we describe the hybrid control law and the strategy that is used to update the discrete state. In Section 3 we show that the closed–loop system is globally asymptotically stable in the sense of Lyapunov. In Section 4 we briefly discuss how control saturations can be incorporated in the proposed approach. Finally, in Section 5 we present a few simulations and in Section 6 we draw conclusions and discuss open problems and possible extensions.

2. DEFINITION OF THE CONTROL STRATEGY

Consider the following three–dimensional chained system (see (Murray and Sastry, 1993)):

$$\begin{cases} \dot{x}_1 = u_1 \\ \dot{x}_2 = u_2 \\ \dot{x}_3 = x_2 u_1 \,. \end{cases} \tag{1}$$

In what follows we consider a control law which is constructed by switching, with a strategy to be defined, between three controllers. Note that this strategy can be regarded as a time varying control strategy, hence the obstruction due to Brockett theorem does not enter into force.

To describe in compact form the control law we introduce a finite state machine and denote with K the discrete variable associated with its current state and with $\mathcal{K} = \{K_1, K_2, K_3\}$ the set of the possible values of K. Moreover, we define the functions

$$\mathrm{sg}(x) \triangleq \begin{cases} 1 & \text{if } x \geqslant 0 \\ -1 & \text{otherwise} \end{cases}$$

and

$$\mathrm{sat}(x) \triangleq \begin{cases} 1 & \text{if } x > 1 \\ x & \text{if } |x| \leqslant 1 \\ -1 & \text{if } x < -1 \,. \end{cases}$$

The control law is defined as follows

$$u^K \triangleq \begin{bmatrix} u_1^K(x) \\ u_2^K(x) \end{bmatrix} \tag{2}$$

with

$$u_1^K(x) \triangleq \begin{cases} -x_2^3 x_3 & \text{if } K = K_1 \\ -(x_1^3 + x_2 x_3) & \text{if } K = K_2 \\ -\mathrm{sat}(x_{03}^2)\mathrm{sg}(\phi(x_0)) & \text{if } K = K_3 \end{cases} \tag{3}$$

$$u_2^K(x) \triangleq \begin{cases} x_1^3 x_3 & \text{if } K = K_1 \\ -x_2 & \text{if } K = K_2 \\ \mathrm{sat}(x_{03})\mathrm{sg}(\phi(x_0)) & \text{if } K = K_3 \end{cases} \tag{4}$$

where $x_0 = (x_{01}, x_{02}, x_{03})^\top$ represents the state of the system (1) at the instant in which the discrete variable *switches to* the value K_3, and

$$\phi(x_0) \triangleq x_{01}^3 \mathrm{sat}(x_{03}^2) - x_{02}^3 \mathrm{sat}(x_{03}) + x_{02} x_{03} \mathrm{sat}(x_{03}^2).$$

If the finite state machine is in the state K_1 or K_2 then the resulting control signals vary continuously with time, whereas if the machine is in the state K_3 the control signals are evaluated when the controller becomes active and remain constant as long as the machine remains in the state K_3.

The control law defined by equations (3)–(4) is zero at zero, *i.e.* independently from the switching strategy, the zero equilibrium is preserved.

To define the switching strategy we introduce a generic smooth function $V(x)$, which will be specified in the next section, and, denoting with K_τ and x_τ the discrete state of the machine and the continuos state of system (1) at the instant $t = \tau$, we define the time [2]

$$T_{\min}(K_\tau, x_\tau)$$

as described hereafter. Let $x_{(K_\tau, x_\tau)}(t)$ be the corresponding forward trajectory of the closed loop system starting at $x = x_\tau$ for $t = \tau$. Then

$$T_{\min}(K_\tau, x_\tau) \triangleq \sup_{t \geqslant \tau} \{t \mid \dot{V}(x(\sigma)) \leqslant 0 \,, \forall \, \sigma \in [\tau, t]\} - \tau. \tag{5}$$

Note that

- $T_{\min}(K_\tau, x_\tau)$ may be zero. This is the case if $\dot{V}(x(t))|_{t=\tau} = 0$ and $\exists \, \epsilon > \tau \mid \dot{V}(x(t)) > 0$, $\forall t \in (\tau, \epsilon)$,
- $T_{\min}(K_\tau, x_\tau)$ may be infinite. This is the case if $\dot{V}(x(t)) \leqslant 0$, $\forall t \geqslant \tau$,
- $T_{\min}(K_\tau, x_\tau)$ may be undefined. This is the case if $\dot{V}(x(t))|_{t=\tau} > 0$. In this case we will set $T_{\min}(K_\tau, x_\tau) = 0$,
- in all the other cases, $T_{\min}(K_\tau, x_\tau)$ is finite and is the first instant in which the function V has a local minimum.

Finally, let $T_D > 0$ be given and consider the following switching strategy: every T_D time units the value of the discrete variable K is updated to a new value K^+ according to the rule

$$K^+(K_i, x) = \begin{cases} K_{i+1} & \text{if } T_{\min}(K_{i+1}, x) > T_D \\ K_1 & \text{otherwise.} \end{cases} \tag{6}$$

[2] To be precise $T_{\min}$ is also a function of V.

where x is the continuous state at the switching instant and $K_i = K_{i-3}$ for all $i > 3$.

In order to better understand how the hybrid control law works, we can describe the *hybrid variable* K as the state of a finite state machine (of Mealy type (Mealy, 1955)) like the one depicted in Figure 1.

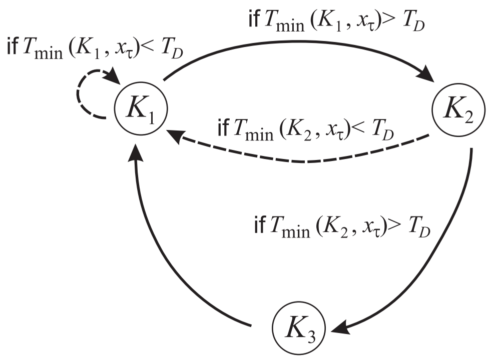

Fig. 1. The finite state machine describing the behaviour of the hybrid variable K.

3. STABILITY ANALYSIS

In this section we show that by selecting appropriately the function $V(x)$ and the constant T_D the hybrid control law (3), (4) and (6) globally asymptotically stabilizes the zero equilibrium of system (1).

To this end, we first establish a few preliminary facts.

Lemma 3.1. Consider the system (1) and the hybrid control law (3), (4) and (6). Let

$$V(x) \triangleq \frac{x_1^4}{4} + \frac{x_2^4}{4} + \frac{x_3^2}{2}, \qquad (7)$$

then

$$T_{\min}(K_1, x) = T_{\min}(K_2, x) = +\infty, \ \forall x \in \mathbb{R}^3.$$

Proof Consider the time derivative of the function $V(x)$ along the trajectories of the system (1) with [3] $u = u^{K_1}$ or $u = u^{K_2}$. Straightforward computations yield that [4]

$$\dot{V}_{K_1}(x) = -x_2^4 x_3^2 \leqslant 0, \qquad (8)$$

or

$$\dot{V}_{K_2}(x) = -x_2^4 - (x_1^3 + x_2 x_3)^2 \leqslant 0. \qquad (9)$$

This implies that, in both cases, $\dot{V}(x(t)) \leqslant 0$ for all $t \in \mathbb{R}$, hence the claim. ■

[3] With abuse of notation we use $u_i^{K_j}$ to denote $u_i^K(x)$ for $K = K_j$.

[4] To simplify the notation in the following we will write $\dot{V}_{K_i}(x)$ instead of $\dot{V}|_{K_i}(x)$ and $V_{K_i}(t)$ instead of $V|_{K_i}(x(t))$.

Unfortunately, it is not possible to prove a similar result when $u = u^{K_3}$. Nevertheless, the following weaker, yet useful, property holds.

Lemma 3.2. Let V be as in equation (7). Then

$$T_{\min}(K_3, x) \geqslant \sqrt{\frac{2}{5}}, \ \forall x \in Z, \qquad (10)$$

where

$$Z \triangleq \{(x_1, x_2, x_3)^\top \mid x_1 = x_2 = 0\}.$$

Proof Consider the solution of system (1) with [5] $u_1(t) = c_1 = \text{const}$ and $u_2(t) = c_2 = \text{const}$ and from the initial state $x_0 = (x_{01}, x_{02}, x_{03})^\top$, namely

$$\begin{aligned} x_1(t) &= x_{01} + c_1 t \\ x_2(t) &= x_{02} + c_2 t \\ x_3(t) &= x_{03} + x_{02} c_1 t + c_1 c_2 \frac{t^2}{2}. \end{aligned}$$

By some algebra it follows that the function $V_{K_3}(t)$ can be expressed as

$$V_{K_3}(t) = at^4 + bt^3 + ct^2 + dt + e, \qquad (11)$$

where

$$\begin{aligned} a &= \frac{c_1^4}{4} + \frac{c_2^4}{4} + \frac{c_1^2 c_2^2}{8} \\ b &= c_1^3 x_{01} + c_2^3 x_{02} + \frac{c_1^2 c_2 x_{02}}{2} \\ c &= \frac{3}{2} x_{01}^2 c_1^2 + \frac{3}{2} x_{02}^2 c_2^2 + \frac{x_{03} c_1 c_2}{2} + \frac{c_1^2 x_{02}^2}{2} \\ d &= x_{01}^3 c_1 + x_{02}^3 c_2 + c_1 x_{02} x_{03} \\ e &= \frac{x_{01}^4}{4} + \frac{x_{02}^4}{4} + \frac{x_{03}^2}{2}. \end{aligned}$$

Moreover, we have

$$\dot{V}_{K_3}(t)|_{t=0} = d = x_{01}^3 c_1 + x_{02}^3 c_2 + c_1 x_{02} x_{03}. \qquad (12)$$

and setting $c_1 = u_1^{K_3}$ and $c_2 = u_2^{K_3}$ yields

$$\dot{V}_{K_3}(t)|_{t=0} = -\phi(x_0)\mathrm{sg}\,(\phi(x_0)) \qquad (13)$$

and this implies

$$\dot{V}_{K_3}(t)|_{t=0} \leqslant 0, \quad \forall x_0 \in \mathbb{R}^3.$$

Consider now the second–order time–derivative at $t = 0$, namely

$$\ddot{V}_{K_3}(t)|_{t=0} = 2c =$$
$$3x_{01}^2 c_1^2 + 3x_{02}^2 c_2^2 + x_{03} c_1 c_2 + c_1^2 x_{02}^2.$$

Again, setting $c_1 = u_1^{K_3}$ and $c_2 = u_2^{K_3}$ and considering only points x_0 in the set Z, we have

$$\ddot{V}_{K_3}(t)|_{t=0} = -x_{03}\mathrm{sat}(x_{03})\mathrm{sat}(x_{03}^2) \qquad (14)$$

which is negative for all $x_{03} \neq 0$ and implies that $T_{\min}(K_3, x) > 0$, $\forall x \in Z$.

[5] Recall that when $u = u^{K_3}$ the corresponding control action turns out to be constant. In the proof the instant $t = 0$ is intended as the generic switching instant in which the machine switches to K_3.

If $T_{\min}(K_3, x) < +\infty$ we can determine it by computing the first positive time instant in which $\dot{V} = 0$. For, let $x = (0, 0, \bar{x})^\top$ be a point in Z and note that, for such a point, one has

$$\dot{V}_{K_3}(t) = 4at^3 + 2ct$$

where

$$a = \frac{1}{4}\text{sat}(\bar{x}^2)^4 + \frac{1}{4}\text{sat}(\bar{x})^4 + \frac{1}{8}\text{sat}(\bar{x}^2)^2\text{sat}(\bar{x})^2$$
$$c = -\frac{1}{2}\bar{x}\text{sat}(\bar{x})\text{sat}(\bar{x}^2)\,.$$

Consequently, $T_{\min}(K_3, x) = \sqrt{-c/2a}$, and we need to consider the following two cases

- if $|\bar{x}| > 1$, then $a = \dfrac{5}{8}$ and $c = -\dfrac{1}{2}|\bar{x}|$ which implies $T_{\min}(K_3, x) = \sqrt{\dfrac{2}{5}|\bar{x}|}$
- if $|\bar{x}| \leqslant 1$, then $a = \dfrac{\bar{x}^8}{4} + \dfrac{\bar{x}^4}{4} + \dfrac{\bar{x}^6}{8}$ and $c = -\dfrac{\bar{x}^4}{2}$ which implies $T_{\min}(K_3, x) = \sqrt{\dfrac{1}{\bar{x}^4 + 1 + \frac{1}{2}\bar{x}^2}}$.

In both cases $T_{\min}(K_3, x) \geqslant \sqrt{\frac{2}{5}}$ thus ending the proof. ■

We can now simplify the sketch of the finite state machine of Figure 1. In fact, by Lemma 3.1 and according to the strategy (6), $K^+(K_3, x) = K_1$ and $K^+(K_1, x) = K_2$ which means that if the machine is in K_3, at the following switching instant it will switch to K_1 and analogously from K_1 it switches to K_2. Therefore as depicted in Figure 2, only when switching from K_2 the test on $T_{\min}(K_3, x)$ is needed.

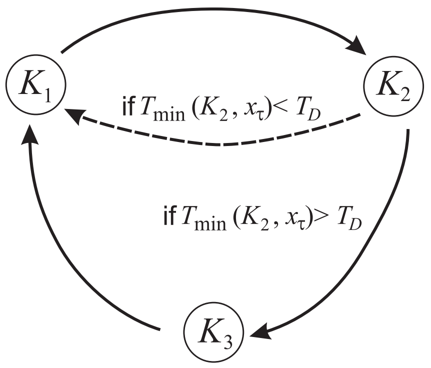

Fig. 2. A simplification of Figure 1. Only when switching from K_2 a test is required.

Remark 3.1. From the third differential equation in (1) and from (8) it is clear that, if $K = K_1$ and no switching occurs, $x_2 x_3$ tends to zero as $t \to \infty$. Analogously, from the second differential equations in (1) and from (9) it is clear that, if $K = K_2$ and no switching occurs, x_2 tends to zero as $t \to \infty$ which implies, given the expression of $u_1^K(x)$ for $K = K_2$, that also x_1 tends to zero. On the other hand, when the continuous state is in Z, the Lyapunov function does not decrease unless the machine switches to $K = K_3$ which always happens as stated in Lemma 3.2.

We are now ready to prove the main stability result of the paper.

Theorem 3.1. Let $T_D \in \left(0, \sqrt{\frac{2}{5}}\right)$ and $V(x)$ be as in (7). Then the hybrid control law (3), (4) and (6) globally asymptotically stabilizes the zero equilibrium of system (1).

Proof We show that the function (7) is a Lyapunov function for the hybrid closed loop system.

Clearly $V(0) = 0$, $V(x) > 0$, $\forall x \in \mathbb{R}^3 \setminus \{0\}$ and V is radially unbounded. Moreover, Lemmas 3.1 and 3.2 imply that $V(x(t))$ is always non-increasing *i.e.* $\forall\, x \in \mathbb{R}^3 \setminus \{0\}$,

$$\forall\, i \in \{1, 2, 3\}\,,\ \dot{V}_{K_i}(x) \leqslant 0$$

for all time for which the discrete state is K_i and this shows that zero equilibrium of the closed-loop system is stable (in the sense of Lyapunov).

Note now that $V(x(t))$ is continuous and bounded from below, hence it has a well defined limit $V_\infty \geqslant 0$ for $t \to \infty$.

The asymptotic stability is guaranteed as the following property holds: $\forall\, x \in \mathbb{R}^3 \setminus \{0\}$

$$\exists\, i \in \{1, 2, 3\} \text{ s.t. } \dot{V}|_{K_i}(x) < 0\,.$$

In fact, if $x \notin Z$ by equation (9) we can pick $u = u^{K_2}$. On the other hand, if $x \in Z$ one has $\dot{V}|_{K_1}(x) = \dot{V}|_{K_2}(x) = 0$ but the analysis carried out in the proof of Lemma 3.2 allows to conclude that

$$\forall\, x \in Z \setminus \{0\}\,,\ \dot{V}_{K_3}(t) < 0\,,\ \forall\, t \in (0, T_D)\,.$$

This implies $V_\infty = 0$, hence asymptotic stability.■

Remark 3.2. Note that the parameters a, b, c and d of equation (11) are continuous functions of x_0 and that the same holds for the time instants in which $\dot{V}$ is zero. This, in turn, means that if x approaches the set Z, $T_{\min}(K_3, x)$ tends to $\sqrt{\frac{2}{5}}$. As a consequence, the choice of $T_D < \sqrt{\frac{2}{5}}$ implies that the condition allowing a switching from K_2 to K_3 not only is true for the states $x \in Z$ but also holds for some points sufficiently close to Z.

4. ANALYSIS IN THE CASE OF BOUNDED CONTROLS

In the previous section no restriction has been imposed on the control signals but in most of the practical situations such signals are bounded.

Nevertheless, it is possible to prove that the stability result is still valid also in the case of bounded controls. In fact, suppose that the restrictions $-\alpha_1 < u_1 < \beta_1$ and $-\alpha_2 < u_2 < \beta_2$, with $\alpha_1, \alpha_2, \beta_1, \beta_2 > 0$, are imposed to the control signals.

Set $\gamma \triangleq \min\{\alpha_1, \alpha_2, \beta_1, \beta_2\}$ and define the control law $u^K \triangleq [u^K_{B1}(x)\, u^K_{B2}(x)]^\top$ with

$$u^K_{B1}(x) \triangleq \begin{cases} -\gamma \dfrac{x_2^3 x_3}{\sqrt{1 + x_2^6 x_3^2 + x_1^6 x_3^2}} & \text{if } K{=}K_1 \\ -\gamma \mathrm{sat}(x_1^3 + x_2 x_3) & \text{if } K{=}K_2 \\ -\gamma \mathrm{sat}(x_{03}^2)\mathrm{sign}(\phi(x_0)) & \text{if } K{=}K_3 \end{cases} \quad (15)$$

$$u^K_{B2}(x) \triangleq \begin{cases} \gamma \dfrac{x_1^3 x_3}{\sqrt{1 + x_2^6 x_3^2 + x_1^6 x_3^2}} & \text{if } K{=}K_1 \\ -\gamma \mathrm{sat}(x_2) & \text{if } K{=}K_2 \\ \gamma \mathrm{sat}(x_{03})\mathrm{sign}(\phi(x_0)) & \text{if } K{=}K_3 \end{cases} \quad (16)$$

where the index B has been introduced to denote the boundedness of the signals.

It is straightforward to see that $\forall K \in \mathcal{K}$

$$-\gamma \leqslant u^K_{B1} \leqslant \gamma \text{ and } -\gamma \leqslant u^K_{B2} \leqslant \gamma\,,\ \forall\, x \in \mathbb{R}^3\,.$$

With this inputs the time derivative of V when the machine is in the states K_1 and K_2 turns out to be

$$\dot{V}|_{K_1}(x) = \frac{-\gamma x_2^4 x_3^2}{\sqrt{1 + x_1^6 x_3^2 + x_2^6 x_3^2}} \leqslant 0$$

and

$$\dot{V}|_{K_2}(x) =$$
$$-\gamma x_2^3 \mathrm{sat}(x_2) - \gamma(x_1^3 + x_2 x_3)\mathrm{sat}(x_1^3 + x_2 x_3) \leqslant 0$$

Therefore Lemma 3.1 still holds. Moreover, equations (13) and (14) become

$$\dot{V}_{K_3}(t)|_{t=0} = -\gamma \phi(x_0) \mathrm{sg}\,(\phi(x_0))$$

and

$$\ddot{V}_{K_3}(t)|_{t=0} = -\gamma x_{03} \mathrm{sat}(x_{03}) \mathrm{sat}(x_{03}^2)$$

which yields that $T_{\min}(K_3, x) > 0\,, \forall x \in Z$. The last part of Lemma 3.2 can also be repeated, the only difference being that now

$$T_{\min}(K_3, x) \geqslant \frac{1}{\gamma}\sqrt{\frac{2}{5}}\,, \forall x \in Z\,.$$

Finally, the reasoning of Theorem 3.1 can then be repeated for $T_D \in \left(0, \frac{1}{\gamma}\sqrt{\frac{2}{5}}\right)$ and we can conclude with the following statement.

Corollary 4.1. Let $T_D \in \left(0, \frac{1}{\gamma}\sqrt{\frac{2}{5}}\right)$ and $V(x)$ be as in (7). Then the hybrid control law (15), (16) and (6) globally asymptotically stabilizes the zero equilibrium of system (1).

5. SIMULATION RESULTS

In order to test the validity of the results presented in Section 3, simulations have been carried out setting $T_D = \frac{1}{2}\sqrt{\frac{2}{5}}$ and choosing three initial condition sampling real situations, namely $(0, 0, -1)^\top$, $(-\pi, -1, 0)^\top$ and $(\frac{\pi}{2}, 1, 0)^\top$. The value of the Lyapunov function has been reported in Figure 3 for the first fifty seconds.

Moreover, for the first of these initial conditions,

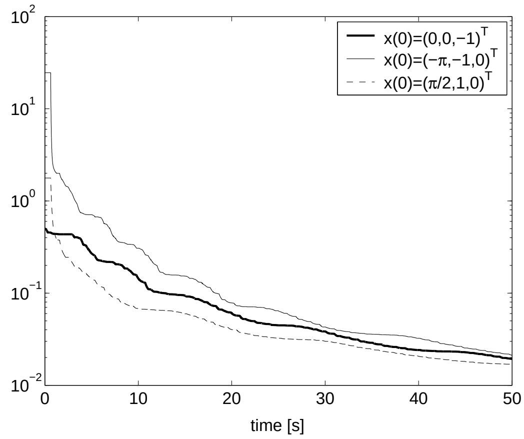

Fig. 3. Time history of the Lyapunov function for the three selected initial states.

the value of the three coordinates and the control signals have been plotted in Figures 4 and 5, respectively.

Finally, in order to prove the property enlightened

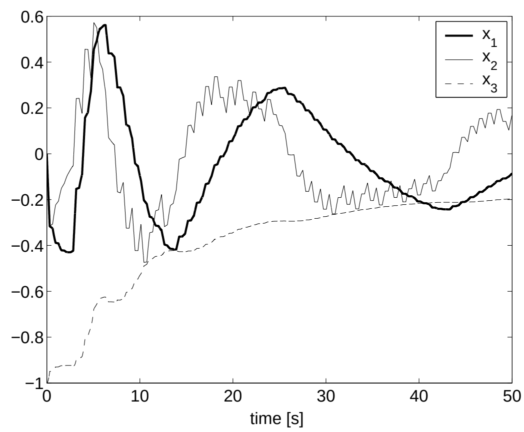

Fig. 4. Time history of x_1, x_2 and x_3 from the initial condition $x(0) = (0, 0, -1)^\top$.

in the previous section, the unsaturated and saturated control signals with $\gamma = 1$ have been applied from the initial condition $x(0) = (0, 0, -10)^\top$, and results are plotted in Figure 6.

6. CONCLUSIONS

A nonholonomic three dimensional system has been studied and a stabilizing switching strategy

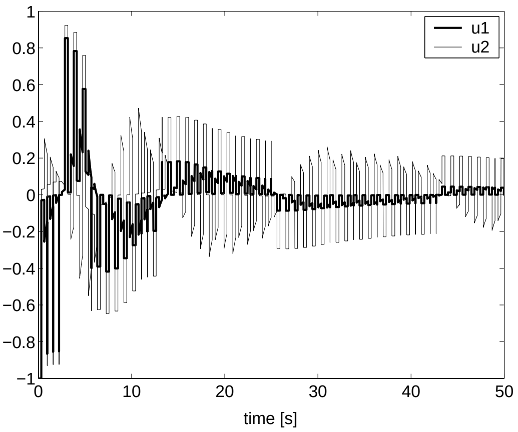

Fig. 5. Control signals from the initial condition $x(0) = (0, 0, -1)^\top$.

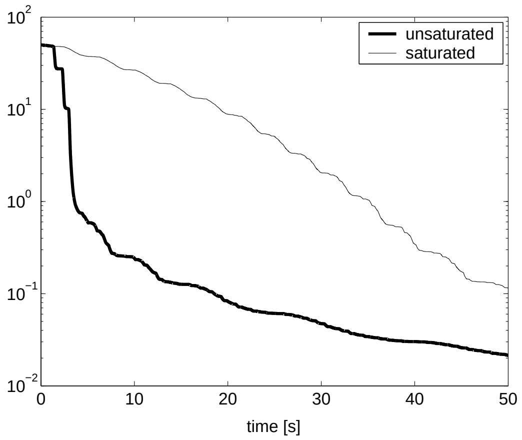

Fig. 6. Lyapunov function from the initial condition $x(0) = (0, 0, -10)^\top$ for the saturated and unsaturated controls.

has been designed and analized in detail. The asymptotic stability of the zero–equilibrium has been demonstrated theoretically and the result of some simulations have been presented in order to prove the validity of the design from a practical point of view. An extension of the stabilizability property to the case of bounded input signals has been developed. Future works will be devoted to analize the robustness of the paradigm and to generalize it to n–dimensional systems.

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