IDENTIFICATION OF FAST-RATE NONLINEAR OUTPUT ERROR MODELS FROM MULTI-RATE DATA

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Abstract: This work aims at the identification of a nonlinear fast rate model from multi-rate sampled data, which is corrupted with unmeasured disturbances and measurement noise. The model identification is carried out in two steps. In the first step, a MISO fast rate nonlinear output error (NOE) model with Weiner structure is identified from the multi-rate data. In the next step, a nonlinear auto regressive (NAR) model is developed, which whitens the residuals. The efficacy of the proposed modeling scheme is demonstrated by carrying out simulation studies on a CSTR system, which exhibits input multiplicities and change in the sign of the steady state gain in the desired operating region. The analysis of the simulation results reveals that the proposed multi rate models are able to capture the dynamics and the steady state behavior of the reactor reasonably accurately over a wide operating range.

Keywords: Multi-rate Systems, Fast Rate Models, Weiner structure, Nonlinear Output Error models, Input Multiplicity

1. INTRODUCTION

Most of the chemical processes like reactors, fermenters or high purity distillation columns, exhibit strongly nonlinear dynamics. Development of model based control schemes for such systems has been a major area of research over last two decades (Henson and Seborg, 1997). Most of these methods assume that all the measurements are available at a single fast sampling rate, which is identical to the rate at which manipulated input moves are made. However, in many of the practical situations, the primary variables of interest from control view point (such as concentrations) are available only at relatively slower rates. Thus, there has been considerable amount of interest in developing state estimation and control strategies for multi rate and nonlinear systems (Gudi and Shah, 1995; Bequette et al, 1991). These approaches typically assume the availability of a grey box model developed from first principles. In many situations such models are either not available or too difficult to develop. In such a situation, development of multi rate nonlinear model directly from the input output plant data is an attractive option.

There are many alternative nonlinear black box model structures available in the literature (Sjoberg et al. 1995). Nonlinear ARX (NARX) models are relatively easy to develop and appears to be the most favoured structure for model development (Herandez and Arkun, 1993; ; Oggunaike and Pearson, 1997). Determination of model structure for this type of models is not an easy task even for SISO case. Recently, Srinivasarao et al. (2004) have proposed a method for development of state space form of NARX models parameterized using generalized orthonormal basis filters (GOBF), which partially alleviates this difficulty. The bock oriented nonlinear models is another most frequently used class of nonlinear models. Number of researchers have proposed methods for identification and control based on models with Weiner and Hammerstein structure. Sentoni et al. (1998) used DAB Net (Decoupled A-B net) composed of a decoupled linear dynamic system followed by neural network to develop nonlinear output error (NOE) type models. The linear dynamic component is parameterized using Lauguerre filters, which are cascaded with a single hidden layer perceptron. They have used linear balancing technique on hidden layer of NN as part of identification procedure to reduce the dimensionality of perception inputs. This results in a large dimensional optimization problem. Also, they do not make nay attempt to characterize the unmeasured disturbance component. Recently, Gomez and Baeyens (2004) have proposed modeling scheme based on Hammerstein model and Wiener structures, which facilitates unmeasured disturbance modeling. In their approach the linear dynamic part is represented by GOBF and the static nonlinear map is represented by polynomial basis functions. While developing these models, they impose condition that the nonlinear output map should be invertible. The constraint on the invariability of the output map implies that the identified Wiener models cannot be used for capturing dynamics of systems exhibiting input multiplicity (Pearson and Pottmann, 2000). Saha (1999) and Saha et al. (2004) have proposed NOE type Lauguerre -Wiener models where the state-output map is constructed as a quadratic polynomial function. They also demonstrate that their modeling scheme can adequately represent dynamics of systems with input multiplicity.

The techniques for development of nonlinear time series modeling mentioned above require single rate input output data and their extension to deal with multi-rate systems is not obvious. In fact, identification of fast rate time series models from multi-rate data is a relatively new area of research even for linear systems (Li et al., 2001; Wang et al., 2004). The issue of identifying fast rate nonlinear models from multi-rate sampled data does not appear to have received much attention in the literature. This work aims at the identification of a nonlinear fast rate model from multi-rate sampled data, which is corrupted with unmeasured disturbances and measurement noise. The model identification is carried out in two steps. In the first step, a MISO fast rate NOE model with Weiner structure is identified from multi-rate data. In the next step, a nonlinear auto regressive (NAR) model is developed, which whitens the residuals. The linear dynamic component of the Weiner model in either case are parameterized using GOBF. The efficacy of the proposed modeling scheme is demonstrated by carrying out simulation studies on a CSTR system, which exhibits input multiplicities and change in the sign of the steady state gain in the desired operating region.

This paper is organized in four sections. The next section deals with the proposed model structure and formulation of parameter estimation problem. In the third section we present a simulation case study while the conclusions are presented in the final section.

2. DEVELOPMENT OF NOE+NAR MODEL

In this section, we propose a sequential approach to the development of models for asymptotically stable systems. In the first step, we develop a nonlinear output error model using the fast sampled inputs and the slowly sampled outputs.The deterministic component of the fast rate model is parameterized using OBF. The residuals generated at the slow rate are then used to develop a noise model, which is also parameterized using OBF.

2.1 Development fast rate state space model

While developing the fast-rate model for the multi-rate system under consideration it is assumed that

- Sampling rates for all measurements are integer multiples of some time period called 'shortest time unit' (T)
- All actuators are to be manipulated at frequency corresponding to the 'shortest time unit' (T).
- Some of the outputs are sampled at regular intervals such that the sampling period is an integer multiple of T
- The unmodelled disturbances are zero mean and their effect on the outputs is additive

Thus, the manipulated inputs are changed at $\{t_k = kT : k = 0, 1, 2, ...\}$ while the output measurements are assumed to be available only at sampling instants given by the sub-sequence $\{k_1, k_2, k_3, ...\}$ such that the difference $k_l - k_{l-1} = p\ (> 1)$ where p is an integer. Now, consider a Weiner type MISO fast rate model of the form

$$\mathbf{x}(k+1) = \Phi\mathbf{x}(k) + \Gamma\mathbf{u}(k) \quad (1)$$
$$y(k) = \Omega\left(\mathbf{x}(k)\right) + v(k) \quad (2)$$

where $\Omega(.) : R^n \to R$ represents nonlinear state-output map. In order to simplify identification problem, we can choose some canonical parameterizations of the (Φ, Γ) pair. For example, Φ can be chosen to be a diagonal matrix. We choose to parameterize (Φ, Γ) using GOBF (see Appendix). Also, the output state map is chosen as quadratic polynomial function. Thus, the resulting fast rate Weiner model can be expressed as

$$\mathbf{x}(k+1) = \Phi(\xi)\ \mathbf{x}(k) + \Gamma(\xi)\ \mathbf{u}(k) \quad (3)$$
$$\widehat{y}_u(k) = C^T\mathbf{x}(k) + \mathbf{x}(k)^T D\mathbf{x}(k) \quad (4)$$

where $\boldsymbol{\xi}$ represents a vector of GOBF poles and $\widehat{y}_u(k)$ represents output prediction at the fast rate. The measurement equation at the slow sampling instant is given as

$$y(k_l) = C^T\mathbf{x}(k_l) + \mathbf{x}(k_l)^T D\mathbf{x}(k_l) + v(k_l)$$

Here, $k = k_l$ represents the sampling instant and $v(k_l)$ represents un modelled disturbances. We define prediction error sequence at slow sampling instants as

$$\widehat{v}(k_l, \boldsymbol{\xi}, \boldsymbol{\theta}) = y(k_l) - \left[\mathbf{C}^T\mathbf{x}(k_l) + \mathbf{x}(k_l)^T\mathbf{D}\mathbf{x}(k_l)\right] \quad (5)$$

where

$$\boldsymbol{\theta} = \left[\ \mathbf{C}^T\ \mathbf{D}_{11}\ \mathbf{D}_{12}\ ....\ \mathbf{D}_{N_i,N_i}\ \right]^T$$

Note that $\mathbf{D}$ is a symmetric matrix and only $n \times (n+1)/2$ elements appear in $\boldsymbol{\theta}$ vector. Given vector of GOBF poles $\boldsymbol{\xi}$,the least square estimate of the parameter vector $\boldsymbol{\theta}$ can be obtained by solving the following minimization problem

$$\widehat{\boldsymbol{\theta}}_u(\boldsymbol{\xi}) = \arg\min_{\boldsymbol{\theta}_u} \frac{1}{N_s}\sum_{l=1}^{N_s}\widehat{v}(k_l, \boldsymbol{\theta})^2 \quad (6)$$

N_s represents total number of output samples available. Given a set of GOBF poles $\boldsymbol{\xi}$, the above minimization problem can be solved analytically using the following simple linear regression scheme

$$\widehat{\boldsymbol{\theta}}_u(\boldsymbol{\xi}) = (R)^{-1}\,\overline{E}\left(\mathbf{Z}(k_l)y(k_l)\right) \quad (7)$$
$$R = \left[\overline{E}\left(\mathbf{Z}(k_l)\mathbf{Z}(k_l)^T\right)\right] \quad (8)$$

where

$$\mathbf{Z}(k_l) = \left[\ (\mathbf{X}(k))^T\ \ (\mathbf{X}_q(k))^T\ \right]^T$$
$$\mathbf{X}_q(k) = \left[\ (\mathbf{X}_1(k))^2\ \ 2\mathbf{X}_1(k)\mathbf{X}_2(k)\ ....\ \right]^T$$

where $\overline{E}(.)$ represents expected value operator as defined in Ljung (1987). The parameter estimation procedure outlined above can be easily extended to a MIMO $(r \times m)$ system by formulating r MISO identification problems.

The next step is to estimate a model for the unmeasured disturbances from the estimated residual sequence $\widehat{v}(k_l, \boldsymbol{\xi}, \boldsymbol{\theta})$. A Nonlinear Auto Regressive (NAR) model that whitens the residual sequence can be developed as follows

$$\mathbf{x}_v(k_l + p) = \Psi_v(\xi_v)\,\mathbf{x}_v(k_l) + K_v(\xi_v)\,\widehat{v}(k_l) \quad (9)$$
$$\widehat{v}(k_l) = C_v^T\mathbf{x}_v(k_l) + \mathbf{x}_v(k_l)^T D_v\mathbf{x}_v(k_l) + e(k_l) \quad (10)$$

where (ξ_v) represents the vector of GOBF poles. Given GOBF pole vector ξ_v, parameter vector

$$\boldsymbol{\theta}_v = \left[\ \mathbf{C}_v^T\ \mathbf{D}_{v,11}\ \mathbf{D}_{v,12}\ ....\ \mathbf{D}_{n,N_i,N_i}\ \right]^T$$

can be estimated by linear regression similar to the deterministic component. The resulting state space model equation (9) can be rearranged as

$$\mathbf{x}_v(k_l + p) = \Xi\left[\mathbf{x}_v(k_l)\right] + K_v e(k_l) \quad (11)$$
$$\widehat{v}(k_l) = C_v^T\mathbf{x}_v(k_l) + \mathbf{x}_v(k_l)^T D_v\mathbf{x}_v(k_l) + e(k_l)$$

where

$$\Xi\left[\mathbf{x}_v(k_l)\right] = \left(\Psi_v + K_v C_v^T\right)\mathbf{x}_v(k_l) + K_v\mathbf{x}_v(k_l)^T D_v\mathbf{x}_v(k_l)$$

Using above state observer, the output predictions at the sampling instant $k = k_l$ can be given as

$$\widehat{y}(k_l) = \widehat{y}_u(k_l) + C_v^T\mathbf{x}_v(k_l) + \mathbf{x}_v(k_l)^T D_v\mathbf{x}_v(k_l)$$

The above formulation assumes that GOBF parameters can be specified based on some a-priori knowledge about the system. Alternatively, pole vector ξ can be estimated by formulated by formulating a nested optimization problem as suggested by Saha (1999). For example, given fast rate input sequence $\{\mathbf{u}(0), \mathbf{u}(1), ...\mathbf{u}(N)\}$ and infrequently sampled output $\{y(k_1), y(k_2), ...y(N_s)\}$, the least square estimate of the parameters of the output error model (3) can be obtained by solving the following nested minimization problem

$$\widehat{\xi} = \begin{matrix}\arg\min\\ \xi\end{matrix}\ \frac{1}{N_s}\sum_{l=1}^{N_s}\widehat{v}(k_l, \xi, \widehat{\boldsymbol{\theta}})^2 \quad (12)$$

subject to the constrain (9). In addition, the fact that GOBF poles are stable requires imposition of following additional constraint

$$|\xi_i| < 1\,;\quad i = 1, 2....n \quad (13)$$

The resulting nonlinear optimization problem is solved using MATLAB optimization toolbox.The optimum pole location for the NAR model is also be obtained in a similar manner.

3. SIMULATION CASE STUDY

The CSTR system under consideration consists of a reversible exothermic reaction $A \rightleftharpoons B$. The dynamic model for simulating the CSTR system is as follows

$$\frac{dC_a}{dt} = \frac{F_i}{h\,A_c}(C_{ai} - C_a) + K_1\,C_a - K_2\,C_b$$
$$\frac{dC_b}{dt} = -\frac{F_i}{h\,A_c}C_b + K_1\,C_a - K_2\,C_b$$
$$\frac{dT}{dt} = \frac{1}{h\,A_c}F_i(T_i - T) + \frac{-H_r}{\rho\,C_p}(K_1\,C_a - K_2\,C_b)$$
$$\frac{dh}{dt} = \frac{1}{A_c}(F_i - k\sqrt{h})$$
$$K_1 = k_f\,\exp(-E_f/T)\ ; K_2 = k_b exp(-E_b/T)$$

The nominal parameters and the operating steady state used in the simulation studies can be found in Patwardhan and Madhawan (1993). In the present work, the output concentration (C_b) and reactor temperature (T) in the CSTR are the two measured outputs of the system. The inlet flow rate and inlet temperature are used as manipulated variables and inlet concentration C_{ai} is treated as unmeasured disturbance. This system exhibits input multiplicity and change in the sign of steady state gain in the operating region. The difficulties associated with controlling such systems at the optimum operating point have been discussed in detail by Patwardhan and Madhawan (1993).

In the present study, shortest time unit (T) is chosen as 0.1 min i.e. the input moves are changed after every 0.1 minute. The inlet feed stream contains only A and its concentration is assumed to fluctuate according to following stochastic process

$$\delta C_{ai}(k) = \frac{0.05}{1 - 0.95z^{-1}}\,e(k) \tag{14}$$

where $e(k)$ is a white noise sequence with standard deviation 0.2. Note that $C_{ai}(t)$ is assumed to be a piecewise constant function during simulations. A Multilevel Pseudo Random Signals (MPRS), with standard deviations of 0.275 m^3/s and 19.766 K and switching times of 0.3 and 0.5 min, respectively. The MPRS signals were used to introduce simultaneous perturbations in both the inlet flow rate (F_{in}) and inlet temperature (T_i), respectively. Also, it was assumed that the raector concentration measuremements are available at the slow rate while temperature . In the present study two cases where considered. In **Case A** the reactor concentration (C_B) is sampled at 0.5 min inerval while in **Case B** it is sampled at every 1 min. interval. It is further assumed that concentration measurements are corrupted with measurement noise, which is a zero mean Gaussian white noise signal with standard deviation equal to 0.005. The inputs used to generate validation data and the unmeasured disturbance introduced in input concentration are given in figure(2) and figure(1) respectively.

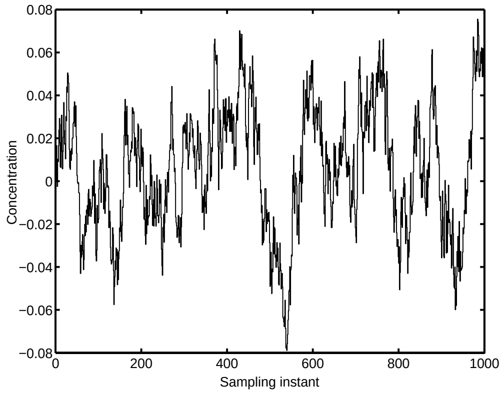

Fig. 1. The unmeasure disturbance introduced in input concentration

Model identification is carried out using data for generated for 800 minutes (8000 input samples). In case A the number of output samples (N_s) is 1600 while in Case B it is 800. The optimum set of poles obtained from the identification exercise in each case are reported in Table (1)

Table 1. Optimum values of GOBF poles for case-I and case-II

Ts out	u_1	u_2	e
case-I	[0.946 0.827]	[0.818 0.915]	[0.686]
case-II	[0.947 0.821]	[0.887 0.999]	[0.670]

The performance of the identified models is evaluated based on the following statistical criteria

- Percentage Prediction Error (PPE)

$$PPE = \frac{\sum\limits_{k_l=k_1}^{N} [y(k_l) - \hat{y}(k_l)]^2}{\sum\limits_{k_l=k_1}^{N_s} [y(k_l) - \overline{y}]^2} \times 100$$

  Note that, $\bar{y}$ in above definition represents the mean value of the slow sampled measured outputs data.

- Percentage Estimation Error (PEE)

$$PEE = \frac{\sum\limits_{k=1}^{N}[y(k) - \hat{y}(k)]^2}{\sum\limits_{k=1}^{N}[\widetilde{y}(k_l) - \overline{\widetilde{y}}]^2} \times 100$$

Note that $\widetilde{y}(k)$ in the above expression represents noise free outputs of the process obtained from simulations.This index cab computed only for simulated data.

The comparison validation data with p-step a ahead predictions and infinite horizon predictions (IHP) for Case A and Case B are given in Figures (3) and Figure(4), respctively. The corresponding

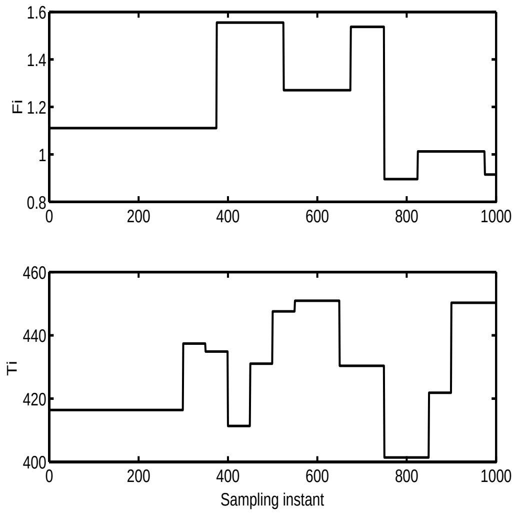

Fig. 2. Manipulated inputs used to generate validation data

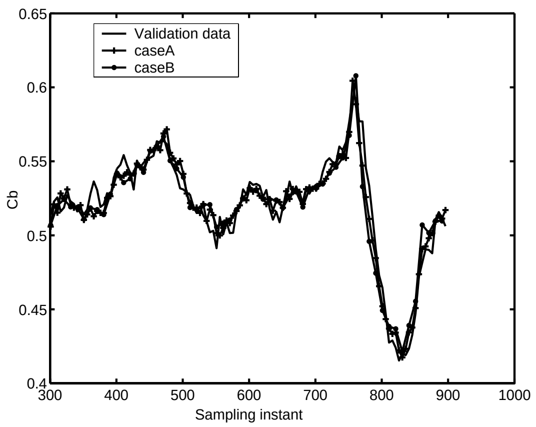

Fig. 3. Comparision validation data with pstep ahed predections for caseA and case B

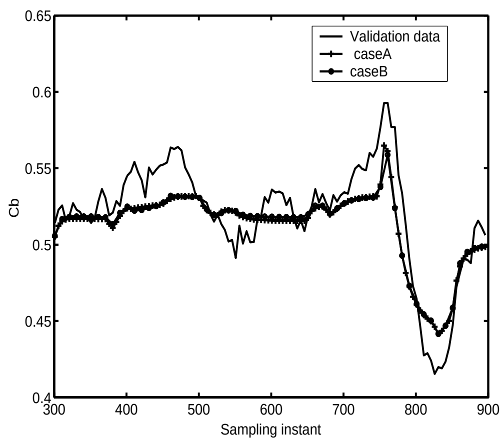

Fig. 4. Comparision infinite horizon predections for caseA and case B with validation data

PPE and PEE vlues are listed in table (2). Figure (4) and PEE values in Table (2) indicates that the identified fast rate model generates reasonably accurate fast rate predictions of concentration. From table(2), it can be observed that PPE values for p-step a head predictions are significantly less than those for the infinite horizon prediction in

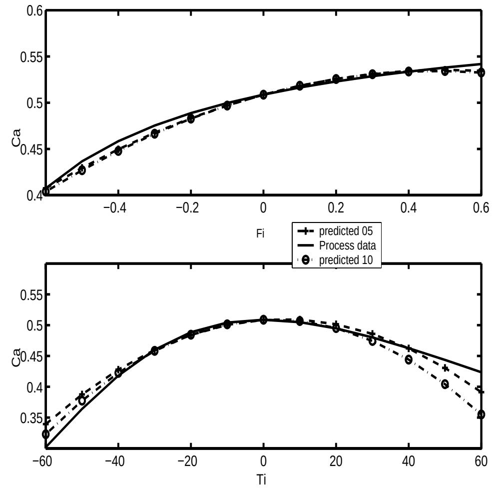

Fig. 5. Comparision of measured and predicted steady state concentration Cb in the reactor

Table 2. Comparison of PPE and PEE values

Variable		PPE	PPEE
Case A	P-Step	5.321	13.413
	Infinite	23.48	14.174
Case B	P-Step	9.201	16.012
	Infinite	25.21	16.131

both the cases. This indicates that the identified NAR model compensates for efect of unmeasured disturbances when measurement becomes available. However, insignificant variation in PEE values indicates that not much improvement is observed with respect to intersample predictions. Figure (5) presents the comparison of the steady state behavior of the process with that of the NOE models identified in each case. As can be seen from this Figure, the identified NOE models capture the steady state behavior of the system over a wide operating range around extremum operating point. Note that both the models capture the change in the sign of the steady state gain reasonably well and are able to model the input multiplicity behavior.

4. CONCLUSIONS

This work presents a method for development of a fast rate nonlinear model using multi-rate input-output data. The fast rate deterministic model is developed first from the Input Output data. The model identification is carried out in two steps. In the first step, a MISO fast rate NOE model with Weiner structure is identified from multi-rate data and residuals are evaluated. In the next step, a nonlinear auto regressive (NAR) model is developed, which whitens the residuals. The efficacy of the proposed modeling scheme is demonstrated by carrying out simulation studies on a CSTR system, which exhibits input multiplicities and change in the sign of the steady state gain in the desired operating region.The analysis of the simulation results reveals that the proposed multi rate models are able to capture the dynamics and the steady state behavior of the reactor reasonably accurately over a wide operating range.

Appendix: Parameterizations of Linear Dynamics using GOBF

Consider a SISO system represented by a strictly proper stable transfer function

$$\widehat{y}(z) = G(z)\, \upsilon(z)$$

where υ represents input and $\widehat{y}$ represents the model output. Let $\{F_k(z) : k = 0, 1, 2, ...\}$ represent an orthonormal basis for $\mathcal{H}_2$ (set of strictly proper stable transfer functions). Then, model that approximates $G(z)$ best in an $\mathcal{H}_2$ sense is given by (Van den Hof, 2000)

$$G_n(z) = \sum_{i=1}^{n} c_i F_i(z)$$

Ninness and Gustafsson (1997) have shown that

$$F_l(z, \xi) = \frac{\sqrt{(1 - |\xi_l|^2)}}{(z - \xi_l)} \prod_{i=1}^{l-1} \frac{(1 - \xi_i^* z)}{(z - \xi_i)}$$

forms a complete orthogonal set in $\mathcal{H}_2$, where $\{\xi_p : l = 1, 2, ...\}$ is an arbitrary sequence of poles inside the unit circle appearing in complex conjugate pairs. The GOBF can be used to parameterize the linear dynamic part of the Weiner type state space model (Srinivarrao et al., 2004)

$$\mathbf{x}(k+1) = \phi(\boldsymbol{\xi})\, \mathbf{x}(k) + \psi(\boldsymbol{\xi})\, \upsilon(k)$$
$$y(k) = \Omega(\,\mathbf{x}(k)) + \varepsilon(k)$$

where $\mathbf{x}(k) \in R^n$ is defined as

$$\mathbf{x}(k) = \left[\, F_1(z, \boldsymbol{\xi})\upsilon(k) \;\; ...... \;\; F_n(z, , \boldsymbol{\xi})\upsilon(k) \,\right]^T$$

Here, $\boldsymbol{\xi} \in R^l$ represents the vector of GOBF poles. The above state space model can be easily extended to represent a MISO model (Srinivarrao et al., 2004).

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