A DIFFERENTIAL HYSTERESIS MODEL

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Abstract: In this paper a new model of hysteresis is described. This new model allows to describe a wider class of rate independent hystereses than the previous Classic and nonlinear Preisach models. The broader area of applicability arises from the relaxation of the minor loops equal chord requirement by introducing a less stringent property SVEC(same-vertexes-equal-chords). The new model allows one to fit n-order transition curves, whereas the former models allow only for fitting of first and second-order transition curves. Due to this fact this new model improves upon existing results. The model structure has been developed to be easily implemented in inverse hysteresis control schemes, widely used for hysteretic systems regulation.

Keywords: Hysteresis, transition curve, inverse hysteresis

1. INTRODUCTION

Hysteresis phenomena are encountered in many different areas of science. Examples include magnetic materials (C. Natale, 2001), piezoelectric and piezoceramic actuators (Mrad and Hu, 2001), shape memory alloys (SMA, e.g. NiTiNol, Flexinol) sensors-actuators (Hughes and Wen, 1997), mechanical hysteresis, adsorption hysteresis, optical hysteresis, electron beam hysteresis and others.

The first mathematical model of hysteresis was introduced in (Preisach, 1935). Since then, many researchers have dedicated attention to the development of suitable mathematical models for the description of hystereses encountered in different physical materials. Hysteresis modeling is also relevant in control theory: in order to regulate hysteretic devices many authors use a inverse hysteresis scheme for both feed-forward control schemes (see (Alija-Garmn *et al.*, 2003), (Gobert *et al.*, 1998),) and feedback control schemes (Hasegawa and Majima, 1998). This paper deals with scalar rate independent hysteresis nonlinearities. Here the hysteretic system is assumed to be a hysteresis transducer $x(t)$ with input signal $h(t)$ and output signal $x(t)$ representing the state of the transducer (see figure 1). The hysteretic function $x(t)$ is assumed to be instantaneous; additional dynamics between $x(t)$ and $h(t)$ can be modeled separately. For example if $h(t)$ is the temperature of a Flexinol Wire (SMA) that causes hysteretic crystalline lattice transformation, and the real control variable is the electrical current [1], the dynamics of the actuators can be related to the dynamics between input current and temperature through a first order filter. A transducer is called hysteretic if its input-output relationship is

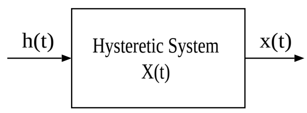

Fig. 1. Schematic representation of hysteretic systems.

[1] The temperature variation in the case of SMA wire actuators can be obtained by Joule effect.

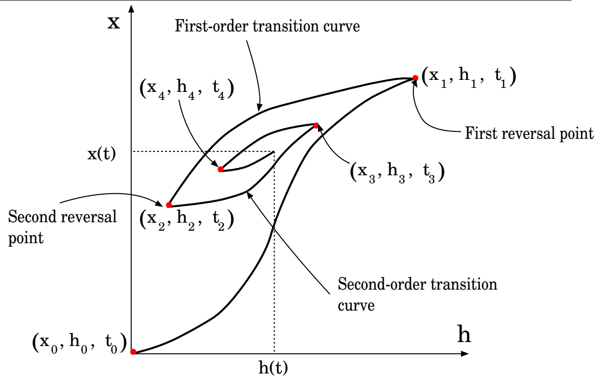

Fig. 2. An example of hysteresis multi-branching nonlinearity. Hysteresis branches are also called transition curves.

a multi-branch nonlinearity for which branch-to-branch transitions occur after input extrema (this definition has been introduced by I.D. Mayergoyz). The evolution of hysteretic systems doesn't uniquely depend on the instantaneous values of the system's input and output, but is also influenced by the system past history. Then, additional internal state variables are needed to complete the description of the system. This non unique relation between input and output is shown by systems that are not in thermodynamic equilibrium, in which the Gibbs free energy profile has many local minima and saddle points. The main idea of this model is based on a profile function to quantify the input variation related to the output variation. The paper is organized as follows. In Section 2 discuss the Preisach and Generalized Preisach models of the hysteresis. In Sections 3 and 4 we discuss the new model in two versions, direct and inverse, and it is shown a simulation example.

2. BACKGROUND

The most important hysteresis models have been introduced by F. Preisach (Preisach, 1935) and I.D. Mayergoyz (Mayergoyz, 1991). In these models, the system is seen as a collection of bistable units, called Preisach units, which are relay functions having different activation (α) and deactivation (β) thresholds parameters. The Preisach distribution function $\mu(\alpha, \beta)$, defined over the (α, β) plane, selects and weights the Preisach units, based on peculiar parameters, that constitute the system.

The number of elementary hysteresis operators (Preisach units) switched-on or -off by the input $h(t)$ are detected on the (α, β) plane by the function [2] $L(t)$ (see Fig. 3). The function $L(t)$ keeps memory of the past input extrema, according to the *wiping out* property, it divides the plane into

[2] $L(t)$ is a *cusp catastrophic function* that represents the bifurcation set on the (α, β) plane of the Preisach units.

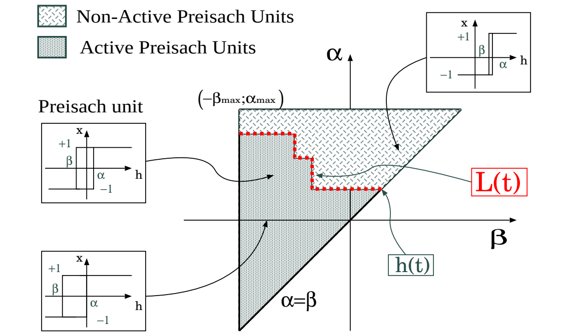

Fig. 3. Preisach units for different threshold parameters (α, β).

switched-on and switched-off units. The output $x(t)$ of the hysteretic transducer can be written as

$$x(t) = \iint_{\alpha \geq \beta} \mu(\alpha, \beta) \hat{\gamma}_{\alpha\beta} h(t) d\alpha d\beta \qquad (1)$$

The relay function $\hat{\gamma}_{\alpha\beta}$ is equal to 1 if the Preisach units with parameters (α, β) are switched-on; otherwise it is equal to -1. The output of the system is the sum of the outputs of all the elementary hysteresis units, weighted by the Preisach distribution $\mu(\alpha, \beta)$, which is obtained by interpolation of the experimentally measured first order transition curves of the system. For the Preisach model to be applicable, it is necessary that the system is rate independent and enjoys the wiping out and congruency properties. These conditions severely reduce the class of hysteresis functions reproduced by this model. The Generalized Preisach model introduced by I.D. Mayergoyz maintains the same structure: the system is seen as a collection of many bistable units and $L(t)$ is used to keep memory of the past history. The generalization stands in a new distribution function $\mu(\alpha, \beta, h(t))$, which allows to describe a wider class of hysteretic systems, by relaxing the congruency property and only requiring the equal chord one. This is due to the new dependence of $\mu(\cdot)$ on $h(t)$, which allows to take into account first and second order transition curves. The Prandtl-Ishlinskii(PI) model (used in (Alija-Garmn *et al.*, 2003)) is easy to implement and capable of describing rate-dependence phenomena, but it can't enlarge the class or the accuracy of hysteresis functions described by the above models and is not reliable for internal minor loops.

3. THE PROPOSED MODEL

The model that we propose here has a basically different structure from the previous models. In our model, starting from measurements of the system responses, a "profile function" $W(k, x)$ is created, that changes its shape to keep track of the past system history according to the past input.

In particular, the system history is embedded in $W(\cdot,\cdot)$ by way of suitable updating maps for $W(k,x)$, to be evaluated at each reversal point time t_k. All the definitions will be given in differential form, although the implementation of the model should follow standard discretization procedures. We will make use of the following assumption.

3.1 Formalization of key properties

Assumption 1. a) The input $h(t)$, $t \geq 0$ is a locally Lipschitz function. *b*) The hysteresis function is characterized by the input domain $[h_{min}, h_{max}]$ and by the output domain $[x_{min}, x_{max}]$. *c*) The hysteresis is initialized with $(h(0), x(0)) = (h_{min}, x_{min})$.

The following definition are necessary to build the mathematical basis of the proposed hysteresis model. For each of them, and as a whole in the next of this work, the above assumption 1 is assumed to be valid.

Definition 1. Consider a function $h(t)$ $t \geq 0$ and assume (for simplicity) that there exists a small enough time T such that $\dot{h}(t) \geq 0$ for almost all $t \in [0, T]$. The *positive reversal times* t_{2i+1}, and the *negative reversal times* t_{2i+2}, $i \geq 0$ are defined by setting $t_0 = 0$ and through the following definition: $t_{2i+1} := \max \bar{t}$ s.t. $\bar{t} > t_{2i}$ and $\dot{h}(t) \geq 0$ for almost all $t \in [t_{2i}, \bar{t}]$; $t_{2i+2} := \max \bar{t}$ s.t. $\bar{t} > t_{2i+1}$ and $\dot{h}(t) \leq 0$ for almost all $t \in [t_{2i+1}, \bar{t}]$.
¿From an intuitive viewpoint each reversal time identifies the time when the input $h(\cdot)$ changes the sign of its derivative.

Definition 2. The *positive input extrema* h_{2i+1} and the *negative input extrema* h_{2i+2} are defined, respectively, as $h_{2i+1} := h(t_{2i+1})$ and $h_{2i+2} := h(t_{2i+2})$, $i \geq 0$ (note that, by Assumption 1, $h_0 = h(t_0) = h_{min}$).

Definition 3. The *positive reversal points* x_{2i+1} and the *negative reversal points* x_{2i+2} are defined, respectively, as $x_{2i+1} := x(t_{2i+1})$ and $x_{2i+2} := x(t_{2i+2})$, $i \geq 0$ (note that, by Assumption 1, $x_0 = x(t_0) = x_{min}$).

Remark 1. Since t_{2i+1} and t_{2i+2} in the above definition are the positive and the negative reversal times, it is possible to associate to any given *reversal times sequence* $\{t_0, .., t_k\}$ the corresponding *input extremes sequence* $\{h_0, .., h_k\}$, satisfying $h_i = h(t_i)$ (see Fig. 2). The same association can be established between a *reversal times sequence*$\{t_0, .., t_k\}$ and the corresponding *reversal points sequence* $\{x_0, .., x_k\}$.

Is is possible now, based on the above definitions, to formalize the property of monotonicity assumed by the existing hysteresis's models.

Definition 4. A system with hysteresis joins the $\mathcal{C}^1$ *monotonicity* property if given any input function $h(\cdot)$ and the corresponding hysteresis response $x(t)$, the input extremes sequence $\{h_0, .., h_k\}$ and reversal points sequence $\{x_0, .., x_k\}$ correspond to the same reversal times sequence $\{t_0, .., t_k\}$. Moreover given any time interval (t_i, t_j) where $h(\cdot)$ is differentiable, $x(\cdot)$ is differentiable in (t_i, t_j) as well.

The above definition constraints the times at which the input and the output exhibit their extrema to be the same. According to Definitions 2 and 3, the time axis is partitioned in time intervals in which the input $h(\cdot)$ is alternatively not decreasing and not increasing. To suitably characterize these time intervals, we introduce next the concept of epochs, embedded with the concept of epoch's input and output domains.

Definition 5. The *positive epochs* $\mathcal{E}_{2i+1}$ and the *negative epochs* $\mathcal{E}_{2i+2}$, $i \geq 0$, correspond, respectively, to the following time intervals:

$$\mathcal{E}_{2i+1} := [t_{2i}, t_{2i+1}],$$
$$\mathcal{E}_{2i+2} := [t_{2i+1}, t_{2i+2}].$$

According to the previous definition, the time axis is partitioned in alternating positive (the input is non decreasing) and negative (non increasing) epochs $\mathcal{E}_k$, $k \geq 1$. The extremes t_k, $k \geq 1$ of the epochs correspond to the reversal times. Based on the next concept we can then formalize the rate independency property.

Definition 6. Two functions $h_a(\cdot), h_b(\cdot)$ (respectively, $x_a(\cdot), x_b(\cdot)$) are *extrema (respectively, reversal) equivalent* if they have the same extremes (respectively, reversal points) sequence: $h_{ai} = h_{bi}$ (respectively, $x_{ai} = x_{bi}$), $\forall i \geq 0$.

Definition 7. A system with hysteresis is *rate independent* if given any pair of extrema equivalent input functions $(h_a(\cdot), h_b(\cdot))$, the corresponding hysteresis response functions $(x_a(\cdot), x_b(\cdot))$ are reversal equivalent.

An intuitive interpretation of Definition 7 corresponds to the fact that given any input having extremes sequence $\{h_0, .., h_k\}$, the output reversal points sequence is independent of the reversal times selection. The following lemma (whose proof is omitted) is a key result necessary for the definition of our model.

Lemma 1. If the hysteretic system verifies the monotonicity and rate independency properties, then given any input function $h(\cdot)$ and its hysteresis response $x(t)$, $t \in \mathcal{E}_k$ in the $k-th$ epoch, there exists a unique $\mathcal{C}^1$ function $\gamma_k(\cdot) : \mathcal{X}_k \to \mathcal{H}_k$, invertible, s.t. $h(t) = \gamma_k(x(t))$, $\gamma_k^{-1}(h(t)) = x(t), \forall t \in \mathcal{E}_k$.

A last property that requires special attention to be suitably formalized is the so-called *wiping out* property. This property, which only makes sense under the rate independence assumption, corresponds to imposing that some input extremes (equivalently some epoches) are *wiping out* by the "memory" of the hysteretic system as they never would have been reached by the input. An implication if this property is that inner minor loops within the hysteresis are closed.

To find out which input extremes (and relative reversal times) of an input profile $h(\cdot)$ are wiped out, is sufficient follow the next rules: a maximum extrema $h(t_{2i+1})$ is wipable if exists one maximum $h(t_{2n+1})$ s.t. $h(t_{2n+1}) \geq h(t_{2i+1})$, respectively a minimum extrema $h(t_{2i+2})$ is wipable if exists one minimum $h(t_{2n+2})$ s.t. $h(t_{2n+2}) \leq h(t_{2i+2})$,$\forall i < n$. The ordered wiped $h(\cdot)$ input extremes are the extremes sequence of the wiped function $\bar{h}(\cdot)$ and its reversal times sequence give the new wiped epoches (see figure 4 for a graphical example of a wiped function $\bar{h}(\cdot)$ of $h(\cdot)$ up to time $\bar{t}$).

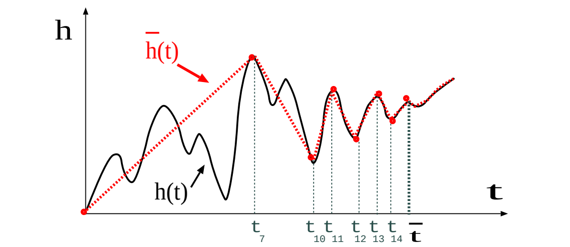

Fig. 4. An example of input $h(\cdot)$ and its wiped version $\bar{h}(\cdot)$ up to time $\bar{t}$.

Definition 8. A system with hysteresis joins the *wiping out* property if given any input function $h(\cdot)$ and any $\bar{t} \geq 0$, the wiped input $\bar{h}(\cdot)$ is such that $x(t) = \bar{x}(t)$ for all $t \geq \bar{t}$, where $x(\cdot)$ is the hysteresis response to $h(\cdot)$ and $\bar{x}(\cdot)$ is the hysteresis response to $\bar{h}(\cdot)$.

3.2 *The profile function*

The definitions introduced above allows us to embed the memory effects of the hysteresis phenomenon within the aforementioned profile function $W(k, x)$ which is updated at each reversal time t_k by a suitable update law and characterizes, within the epoch $\mathcal{E}_k$, the relationship between the output and input variations. In order to correctly characterize the function $W(\cdot, \cdot)$, we need to introduce some additional functions that, through an experimental phase, allow to capture the hysteresis behavior. Two version of the model, direct and inverse, in which the model output is $x(t)$ and $h(t)$ respectively, can be obtained just substituting the x with the h variable. For the upcoming derivation to be well defined, we need to rely on the following assumption.

Assumption 2. The hysteresis under consideration joins the $\mathcal{C}^1$-*monotonicity*, the *wiping out* and the *rate independency* properties.

Under Assumption 2, consider any input selection $h_0(t)$, $t \in [0, T]$, with $h_0(0) = h_{min}$ and $h_0(T) = h_{max}$, such that $\dot{h}_0(t) > 0$ for all $t \in [0, T]$. Given the corresponding hysteresis response $x(t)$, $t \in [0, T]$ (by Assumption 1, $x(0) = x_{min}$ and $x(T) = x_{max}$)), by Lemma 1, there exists a unique invertible function $\gamma_0(\cdot)$ such that $\gamma_0(x(t)) = h(t)$, for all $t \in [0, T]$, where $\gamma_0(x_{min}) = h_{min}$ and $\gamma_0(x_{max}) = h_{max}$. Then, we can define the continuous function of inverse model $w_{x0}(\cdot) : [x_{min}, x_{max}] \to \mathbb{R}_+$ and direct model $w_{h0}(\cdot) : [h_{min}, h_{max}] \to \mathbb{R}_+$ as

$$w_{x0}(x) := \frac{d\,\gamma_0(x)}{dx}; \qquad w_{h0}(h) := \frac{d\,\gamma_0(h)}{dh}$$

Intuitively, the function $w_{x0}(x)$ describes the slope of the external lower branch of the hysteresis function and should be identified by suitable experiments on the hysteretic system. The slope of the inner loops is captured by the functions introduced next. Consider now a generic inner loop starting at (x_{i-1}, h_{i-1}) and ending at (x_i, h_i), where $(x_{i-1},\ h_{i-1})$ and (h_i, x_i) both belong to the graph [3] of the hysteresis (see Figure 5).

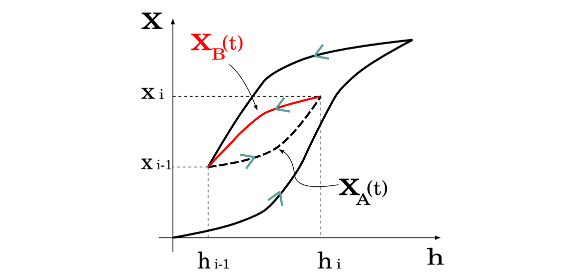

Fig. 5. Example of output responses to evaluate the function $w_x(x_{i-1}, x_i, h_i, x)$ (or $w_h(h_{i-1}, x_i, h_i, x)$).

Consider the response of the system cycling between these two points and note that by the wiping out property, the pair $(h(t), x(t))$ must form a closed loop on the input/output plane. In particular, assuming $x(t_{i-1}) = x_{i-1}$ select the function $h_A(t), t \in [t_{i-1}, t_i]$ with monotone derivative such that $h_A(t_i) = h_i$. By the rate independency, the corresponding hysteresis response $x_A(\cdot)$ satisfies $x_A(t_i) = x_i$. Then, by Lemma 1, there exists a unique invertible function $\gamma_A(\cdot)$ such that $\gamma_A(x_A(t)) = h_A(t)$, for all $t \in [t_{i-1}, t_i]$. In the same way the function $\gamma_B(\cdot)$ for the reverse input can be defined. Based on the unique functions $\gamma_A(\cdot)$ and $\gamma_B(\cdot)$ constructed based on the minor loop under consideration, we can define:

$$w_x(x_{i-1}, x_i, h_i, x) := \frac{d\,[\,\gamma_B(x) - \gamma_A(x)\,]}{dx}$$

if $x \in [x_{i-1}, x_i]$, $w_x(x_{i-1}, x_i, h_i, x) := 0$ otherwise (the same can be done for the direct model exchanging x with h). Despite the involved mathematical notation, the actual meaning of the functions $w_x(\cdot, \cdot, \cdot, \cdot)$ and $w_h(\cdot, \cdot, \cdot, \cdot)$ is quite intuitive and corresponds to the slope difference between the partial hysteresis response $x_A(\cdot), x_B(\cdot)$ shown in Fig. 5.

[3] The graph of the hysteresis denotes the set of all possible input/output pairs.

Remark 2. Note that, by construction, $w_x(x_{i-1}, x_i, h_i, \cdot)$ and $w_h(h_{i-1}, x_i, h_i, \cdot)$ verify $\int_{x_{i-1}}^{x_i} w_x(x_{i-1}, x_i, h_i, \xi)d\xi = 0$ and $\int_{h_{i-1}}^{h_i} w_h(h_{i-1}, x_i, h_i, \theta)d\theta = 0$. This embed wiping out property.

Based on the functions defined above it is finally possible to give a convenient representation of the direct and inverse input output relation of the hysteresis in a specific epoch $t \in \mathcal{E}_k$, as follows:

$$x(t) = x(t_{k-1}) + \int_{h(t_{k-1})}^{h(t)} (w_{h0}(\theta) + \sum_{i\in\mathcal{A}} w_h(h_{i-1}, x_i, h_i, \theta))d\theta \tag{2}$$

$$h(t) = h(t_{k-1}) + \int_{\hat{x}(t_{k-1})}^{\hat{x}(t)} (w_{x0}(\xi) + \sum_{i\in\mathcal{A}} w_x(\hat{x}_{i-1}, \hat{x}_i, h_i, \xi))d\xi \tag{3}$$

where $\mathcal{A}(t_{k-1})$ is an index set containing the indexes of the epochs of the wiped input $\bar{h}$ up to time $\bar{t} = t_{k-1}$ (namely all the indexes of the relevant epochs for the response during epoch $\mathcal{E}_k$) and $\hat{x}(\cdot)$ represent a desired output profile to be tracked by the inverse hysteresis model.

3.3 The proposed models

We are now ready to introduce the proposed direct and inverse hysteresis model, based on the developments of the previous sections and, in particular, on the relations (2) and (3). The model consists of an integral equation, relating input and output of the hysteresis during the current epoch, and of a discrete-time updating map, which is evaluated at each reversal time. The state of the hysteresis during the epoch $\mathcal{E}_k$ is represented by the variables $n(k)$ denoting the number of epochs in the wiped input $\bar{h}(t)$ at time t_{k-1}, the vectors $\mathbf{h}(k, i)$ and $\mathbf{x}(k, i)$, $i = 1, \ldots, n(k)$, representing the values of the hysteresis input and output, respectively, at the associated reversal times, and the profile functions $W_h(k-1, h)$ and $W_x(k-1, x)$ corresponding, respectively, to the integrand functions [4] in equations (2) and (3). Note that for both direct and inverse model the output at time $t \in \mathcal{E}_k$ is determined by $W_y(k-1, y)$ evaluated at time t_{k-1}. The direct model is initialized with $n(0) = 1$, $\mathbf{x}(0) = [x_{min}]$, $\mathbf{h}(0) = [h_{min}]$, $W_h(0, \theta) = w_{h0}(\theta)$ and the following updating law is evaluated at each reversal time t_k, $k = 1, 2, \ldots$:

(1) Evaluate the set $\bar{\mathcal{A}}(t_k)$ containing the indexes i of all wipable input extremes $h_i \in \mathbf{h}(n)$ (and then of all wipable reversal times t_i)up to time t_k s.t. $i < k$ and:
$$i : \left\{ \begin{array}{c} h(t_i) \leq h(t_k) \text{ if } i \text{ odd} \\ \text{or} \\ h(t_i) \geq h(t_k) \text{ if } i \text{ even} \end{array} \right\}$$

(2) Then is possible evaluate the new [5] $W_h(k, h)$:
$$\begin{aligned} n^+ &= n - |\bar{\mathcal{A}}(t_k)| + 1 \\ W_h(\theta)^+ &= W_h(\theta) - \sum_{i\in\bar{\mathcal{A}}(t_k)} w_h(h_{i-1}, x_i, h_i, \theta) + \\ &\quad + w_h(h_{n-1}, x(t_k), h(t_k), \theta) \\ \mathbf{x}^+ &= [\mathbf{x}_{1:n-1}^T, x(t_k)]^T \\ \mathbf{h}^+ &= [\mathbf{h}_{1:n-1}^T, h(t_k)]^T \end{aligned}$$

Note that h_i and $h(t_i)$ are the same thing. In a parallel way, the inverse model is initialized with $n(0) = 1$, $\mathbf{x}(0) = [x_{min}]$, $\mathbf{h}(0) = [h_{min}]$, $W_x(0, \xi) = w_{x0}(\xi)$ and the following updating law is evaluated at each reversal time t_k, $k = 1, 2, \ldots$ performing the same steps for the direct model substituting h with x.

Based on the above updating maps, the direct and inverse output of the Differential hysteresis model can be evaluated $\forall\, t \in \mathcal{E}_k$ respectively as:

$$x(t) = \mathbf{x}_n + \int_{\mathbf{h}_n}^{h(t)} W_h(\theta)d\theta \tag{4}$$

$$h(t) = \mathbf{h}_n + \int_{\mathbf{x}_n}^{x(t)} W_x(\theta)d\theta. \tag{5}$$

Note that, because by definition the $w_x(x_{i-1}, x_i, h_i)$ doesn't take into account the value of h_{i-1}, the branches of minor loops with one coinciding vertex (or triple $[x_{i-1}, x_i, h_i]$) have the same horizontal chord. We call this property *same-vertex-equal-chord* (SVEC).

Definition 9. The hysteretic system joins the *same-vertex-equal-chord* (**SVEC**) property when the minor loops branches, with one coincident vertex, have the same horizontal chord (see Fig. 6).

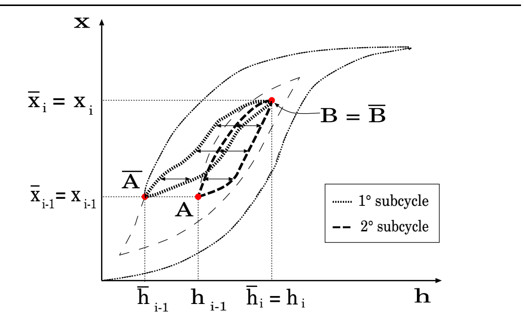

Fig. 6. **SVEC** property.

Remark 3. It is apparent how this property is much less conservative than equal chord and congruency properties, resulting in a broader area of applicability for the proposed model.

Theorem 1. **Representation Theorem:** the wiping out and SVEC properties constitute the necessary and sufficient conditions for the representation of the rate independent hysteresis nonlinearity by the Differential hysteresis model.

[4] Note that by this definition the actual time $t \in \mathcal{E}_k$, that means $t \in [t_{k-1}, t_k]$; t_k is unknown, it will be the next reversal time.

[5] To reduce the burden of notation, the dependence on k has been removed and the pushforward operator has been used to specify the update maps (e.g., $n(k) = f(n(k-1))$ is written as $n^+ = f(n)$).

The necessity proof of the theorem is trivial. Like for the sufficiency proof in the Mayergoyz's nonlinear model, the induction argument can be used combined with the SVEC property.

Remark 4. Due to the property pointed out in Remark 2, the profile functions $W_x(k,\cdot)$ and $W_h(k,\cdot)$ have constant integrals over their domain of definition for all Values of k.

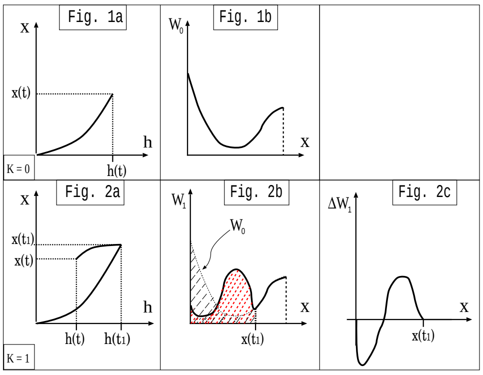

Fig. 7. Sample evolution of $W(k,\xi)$.

For illustration purposes, we show in Figures 7 a short example of the evolution of $W(\cdot,\xi)$ during two epochs in the implementation of an inverse model.

Figure 7.1 - the input increases monotonically (a), the first epoch is started and the initial profile $W(0,\xi) = w_{x0}(\xi)$ is active (b);

Figure 7.2 - at time t_1, the input starts decreasing, thus triggering a new epoch (a), the new $W(1,\xi)$ spans the same area as $W(0,\xi)$ by construction (b): the coarsely shaded area under $W(0,\xi)$ is equal to the finely shaded area under $W(1,\xi)$, $w_x(x(t_0), x(t_1), h(t_1), \xi)$ (with zero integral), only modifies the profile $W(\cdot,\xi)$ in the interval $(x_{i-1}, x_i) = (x_{min}, x(t_1))$ (c);

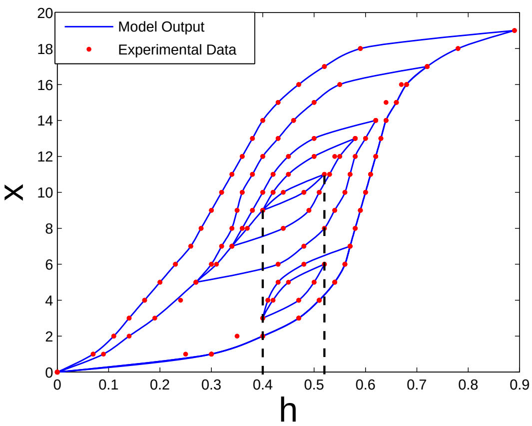

Fig. 8. Numerical example of the inverse model.

4. SIMULATION EXAMPLE

In this simulation example, a sample hysteresis function with $[x_{min}, x_{max}] = [0, 20]$ and $[h_{min}, h_{max}] = [0, 1]$ has been artificially created using a MATLAB code. The numerical implementation of the inverse model of Section 3 has been carried out by discretizing the output values in 20 equal intervals. A small subset of the functions $w_x(\cdot,\cdot,\cdot,\cdot)$ has been experimentally determined for the dots indicated in Figure 8. The model can exactly reproduce the measured hysteresis, in particular, for transition curve of arbitrarily order. A notable feature of the model appears from the input interval shown by the dashed vertical lines: minor loops not captured by the other models are correctly reproduced by the new proposed model.

5. CONCLUSIONS

A new hysteresis model has been proposed which allows to fit n-th order transitions curves and relaxes the equal chord requirement characterizing existing models. These features lead to extreme accuracy and widely extend the class of rate independent hysteretic systems that can be modeled. Moreover, the inverse model of hysteresis has been developed to be suitable implemented in inverse hysteresis control schema. Future work concerns the relaxation of rate independency constraint and modification to take into account "accomodation" process.

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