DATA-BASED CLOSED-LOOP SYSTEM SIMULATION

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Abstract: This paper provides a framework for control design and simulation of a closed-loop system with partial input/output data of a plant. Given the input/output crosscorrelation and output autocorrelation data of an open loop dynamic system, a simulation model implemented in fixed-point digital devices which matches these data is obtained using q-Markov Covariance Equivalent Realizations. These results allow the design of digital simulations with no error within the specified set of crosscorrelation and autocorrelation data. When a linear approximation of the plant is assumed, an LQG controller can be presented solely in terms of the input/output crosscorrelation data. This is the so-called the Markov data-based LQG control. With both the simulation model of the plant and the controller in hand, a closed-loop simulation can be constructed. This is yet another example showing that significant work can be done with very limited information of a plant.

Keywords: q-Markov Covariance Equivalent Realizations, Data-based optimal control, Finite wordlength.

1. INTRODUCTION

How much information of a plant do we need to design an LQG controller and simulate the closed-loop system? This paper shows that the input/output correlation data from *one* experiment will be enough for both identification and control design. Given a prespecified set of input/output crosscorrelation and output autocorrelation data of an open loop dynamic system excited by white noise signals, this paper produces all linear state space models which match these data with the presence of computational error in a fixed-point simulation environment. For an FDLTI (finite-dimensional linear time invariant) system, these input/output crosscorrelation and output autocorrelation data refer to Markov parameters and Covariance parameters respectively. Those methods developed in Yousuff et al. (1985) Skelton and Anderson (1986) and reference therein are called q-Markov COvariance Equivalent Realization (q-Markov COVERs, or QMC), and these methods guarantee stability and match the first q crosscorrelation and the first q output autocorrelation data. Especially such methods can guarantee to preserve the nonminimum-phase properties of the plant.

The QMC theory was originally developed for model reduction Yousuff et al. (1985), while the realization of all QMC from the input/output data of an unknown system is useful for identification Liu and Skelton (1992) Skelton and Shi (1996) Enqvist (2002). Unlike identification methods based on least squares, the q-Markov COVER gives a linear model that matches exactly the first q Markov parameters and the first q output covariance parameters Yousuff et al. (1985)Liu and Skelton (1992) Skelton and Shi (1996). As the Markov parameters and covariance parameters characterize respectively the transient and steady-state properties of a linear system, it is reasonable to use a QMC to approximate the real system.

However, a digital simulation of a QMC would not yield the correct values of the response data since the covariance parameters and Markov parameters will be distorted by the roundoff errors. Most simulation procedures available in the literature implicitly ignore the fact that the implementation of digital model imposes some fundamental limitations on the performance of the closed loop performance. In this scenario, a careful analysis of finite precision effects will be certainly required. In this paper we generalize the existing QMC theory to accommodate the finite wordlength effect. The so-called finite wordlength QMC (FWL-QMC) can *match* the Markov and Covariance parameters of the original model as if there are no roundoff errors. This consideration is indispensable in digital simulations using QMC theory.

While the model-based control theory promises good performance when the model is accurate, it can deliver much worse performance and even instability when the model upon which the controller based is not accurate. An alternative to model-based control is data-based control. Although most control methods (including LQG, H_∞, and MPC) are model-based, a complete state space model might be more than enough information for control design. It is shown that a finite horizon LQG control algorithm can be presented in terms of a set of Markov parameters Furuta et al. (1995) Shi and Skelton (2000). Furthermore, when a real plant is assumed to be a linear system, the input/output crosscorrelation data coincides with the Markov parameters. In this paper, the input/output crosscorrelation data is employed in deducing the LQG controller. Figure 1 illustrates the data-based closed-loop simulation.

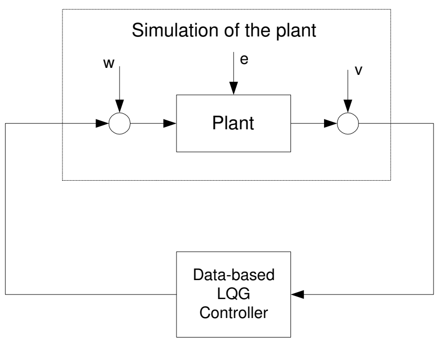

Fig. 1. Data-based closed-loop simulation

The outline of this paper is as follows: first, we show how to obtain the data, then a finite precision simulation model and an LQG controller are constructed based on the given data, followed by the closed-loop simulation framework.

2. OBTAINING THE DATA

Consider any nonlinear system whose inputs chosen for identification are white noise $u \in \mathbb{R}^{n_u}$ and corrupted by unknown zeros mean white noise $w(k) \in \mathbb{R}^{n_u}, k = 0, 1, 2, \ldots$ (with known covariance W). The outputs are denoted by the sequence $y(k) \in \mathbb{R}^{n_y}, k = 0, 1, 2, \ldots$; $v(k) \in \mathbb{R}^{n_y}$ is unknown zero mean, white noise with covariance V, corrupting the system measurement. It is assumed that $u(k), w(k), v(k)$ and the initial state of the plant are uncorrelated and the covariance of $u(k)$ is $U = I$. The input and output of the plant are $\bar{u}(k)$ and $\bar{y}(k)$, respectively. Hence, $\bar{u} = u + w$, $\bar{y} = y + v$.

Denote the output autocorrelation parameters by R_i, and the input/output crosscorrelation parameters (normalized by U) by $H_i, i = 0, 1, 2, ..., q-1$.

$$R_i \triangleq \lim_{N\to\infty} \frac{1}{N} \sum_{k=0}^{N-1} y(k+i)y^T(k). \qquad (1)$$

$$H_i \triangleq \lim_{N\to\infty} \frac{1}{N} \sum_{k=0}^{N-1} y(k+i)u^T(k). \qquad (2)$$

For a stochastic linear system, the output autocorrelation parameters R_i and the input/output crosscorrelation parameters H_i coincide with the covariance parameters and Markov parameters respectively Skelton and Shi (1996).

Define two Toeplitz matrices from the parameters (1) and (2):

$$\mathcal{R}_q \triangleq \begin{bmatrix} R_0 & R_1^T & \ldots & R_{q-1}^T \\ R_1 & R_0 & \ldots & R_{q-2}^T \\ \vdots & \vdots & \ddots & \vdots \\ R_{q-1} & R_{q-2} & \ldots & R_0 \end{bmatrix} \qquad (3)$$

$$\mathcal{H}_q \triangleq \begin{bmatrix} H_0 & 0 & \ldots & 0 \\ H_1 & H_0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ H_{q-1} & H_{q-2} & \ldots & H_0 \end{bmatrix} \qquad (4)$$

And define the block diagonal matrices for later use:

$$\mathcal{U}_q \triangleq I_q \otimes U, \quad \mathcal{W}_q \triangleq I_q \otimes W, \quad \mathcal{V}_q \triangleq I_q \otimes V$$

where $\mathcal{R}_q, \mathcal{V}_q \in \mathbb{R}^{n_y q \times n_y q}$; $\mathcal{H}_q \in \mathbb{R}^{n_y q \times n_u q}$; $\mathcal{U}_q, \mathcal{W}_q \in \mathbb{R}^{n_u q \times n_u q}$. In section 3, the data (1) (2) are utilized for identification of the plant. In section 4, the data (2) provides information for control design.

3. SIMULATION MODEL OF THE PLANT

3.1 The roundoff noise model and scaling condition

The conventional q-Markov COVER is a linear model in the form as follows

$$\begin{aligned}\bar{x}(k+1) &= A_r\bar{x}(k) + B_r\bar{u}(k) \\ \bar{y}(k) &= C_r\bar{x}(k) + D_r\bar{u}(k)\end{aligned} \quad (5)$$

where $\bar{x}(k) \in \mathbb{R}^{n_r}$. The existence condition and parameterizations of (5) can be found in Skelton and Shi (1996) Liu and Skelton (1992). In digital implementation, however, the effect of finite wordlength introduces defection which is realization dependent. When take the roundoff error into account, the simulation model becomes

$$\begin{aligned}\hat{x}(k+1) &= A_r(\hat{x}(k) + e_x(k)) + B_r(\bar{u}(k) + e_u(k)) \\ \hat{y}(k) &= C_r(\hat{x}(k) + e_x(k)) + D_r(\bar{u}(k) + e_u(k)) + e_y(k)\end{aligned} \quad (6)$$

where $\hat{x}_k \in \mathbb{R}^{n_r}$. $e_x(k)$ is the quantization error of the state signal, $e_u(k)$ is input error due to a possible A/D conversion and $e_y(k)$ is caused by roundoff at the outputs. It is known that neither the quantization error of the input e_u nor that of the output e_y depends on the realization, while the effect of the state roundoff error on the output is realization dependent Mullis and Roberts (1976). (6) is the simulation model of desirable dimension. It is our intention to find (A_r, B_r, C_r, D_r) such that up to q Markov and covariance parameters generated by (6) match those given data (1),(2). q is free to choose.

It has been shown that when overflow is rare, under sufficient excitation conditions, the fixed point computational error e_x, e_u and e_y can be modeled as zero-mean, uniform white noise sequences independent of other signals in the system. Since w and e_u, v and e_y are independent, introducing e_u and e_y does not add any difficulties. Thus, we shall not differentiate between w and e_u, nor v and e_y. In the sequel, we should focus on the roundoff error of the state signals. Each white noise sequence e_x, e_u and e_y has a diagonal covariance matrix E_j, where $j = x, u, y$.

$$[E_j]_{i,i} := \rho_{j_i} \qquad \rho_{j_i} = \frac{1}{12}2^{-2\beta_{j_i}} \quad (7)$$

where β_{j_i} is the fractional part of the wordlength (number of bits) used to store the ith variable in a digital device. To simplify the analysis, we assume that uniform wordlength are used among the states, that is, $E_x = \rho_x^2 I$, $E_u = \rho_u^2 I$ and $E_y = \rho_y^2 I$. Define $\mathcal{E}_q \triangleq I_q \otimes E_x$, $\mathcal{W}_q \triangleq I_q \otimes E_u$, $\mathcal{V}_q \triangleq I_q \otimes E_y$ for later use.

Since the computational errors are realization dependent, we shall use the variance oriented l_2-norm scaling constraint on the component of the transformed covariance matrix, that is, to impose the additional scaling constraint Liu et al. (1992).

$$[\hat{X}]_{(i,i)} \leq 1, \; i = 1, \ldots, n \quad (8)$$

A simplified scaling condition that is more tractable than (8) is

$$\hat{X} = I \quad (9)$$

which can be obtained from (8) by relaxation. It is clear that all inequalities in (8) hold whenever (9) holds. Scaling condition (9) is known as orthogonal filter structure. $\hat{X}$ is the state covariance matrix of computational model (6) and satisfies the Lyapunov equation

$$\hat{X} = A_r\hat{X}A_r^T + B_r(I + E_u)B_r^T + A_rE_xA_r^T \quad (10)$$

3.2 Parameterizing the simulation model

When the finite precision effects is considered, we need to answer the following question: "Does there exist an FDLTI in the form of (6) with finite wordlength (FWL) quantization errors which can match data $\{H_i, R_i | i = 0, 1, \ldots, q-1\}$? " If so, we shall call such a state space model FWL-QMC.

Define $\mathcal{D}_q \triangleq \mathcal{R}_q - \mathcal{H}_q(\mathcal{U}_q + \mathcal{W}_q)\mathcal{H}_q^T - \mathcal{V}_q$. $\mathcal{D}_q$ is referred to as *data matrix* since it contains all the known data. Define $S \in \mathbb{R}^{n_yq \times n_yq}$ as the lower shift matrix with ones on the first subdiagonal and zeros elsewhere, i.e. $\{S\}_{k,l} = \delta_{k-l-1}$. S will play an essential role in deducing FWL-QMC.

Assume there exists an FWL-QMC (6) which matches the data $\{H_i, R_i | i = 0, 1, \ldots, q-1\}$. Denote $\hat{u}(k) = \bar{u}(k) + e_u(k)$. The output sequence of (6) is given by

$$\hat{y}_q(k) = \mathcal{O}_q\hat{x}(k) + \hat{\mathcal{H}}_q\hat{u}_q(k) + \mathcal{N}_qe_{xq}(k) + e_{yq}(k) \quad (11)$$

where

$$\mathcal{O}_q \triangleq \begin{bmatrix} C_r \\ C_rA_r \\ \vdots \\ C_rA_r^{q-1} \end{bmatrix}$$

$$\hat{\mathcal{H}}_q \triangleq \begin{bmatrix} D_r & 0 & \ldots & 0 \\ C_rB_r & D_r & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C_rA_r^{q-2}B_r & \ldots & C_rB_r & D_r \end{bmatrix}$$

$$\mathcal{N}_q \triangleq \begin{bmatrix} C_r & 0 & \ldots & 0 \\ C_rA_r & C_r & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C_rA_r^{q-1} & \ldots & C_rA_r & C_r \end{bmatrix}$$

$$\hat{y}_q^T(k) \triangleq \begin{bmatrix}\hat{y}^T(k) & \hat{y}^T(k+1) & \ldots & \hat{y}^T(k+q-1)\end{bmatrix}$$

$$\hat{u}_q^T(k) \triangleq \begin{bmatrix}\hat{u}^T(k) & \hat{u}^T(k+1) & \ldots & \hat{u}^T(k+q-1)\end{bmatrix}$$

$$e_{xq}^T(k) \triangleq \begin{bmatrix}e_x^T(k) & e_x^T(k+1) & \ldots & e_x^T(k+q-1)\end{bmatrix}$$

$$e_{yq}^T(k) \triangleq \begin{bmatrix}e_y^T(k) & e_y^T(k+1) & \ldots & e_y^T(k+q-1)\end{bmatrix}$$

where $\mathcal{O}_q \in \mathbb{R}^{n_yq \times n_r}$, $\hat{\mathcal{H}}_q \in \mathbb{R}^{n_yq \times n_uq}$, $\mathcal{N}_q \in \mathbb{R}^{n_yq \times n_rq}$.

To match the data $\{H_i, R_i | i = 0, 1, \ldots, q-1\}$, we need $\hat{\mathcal{H}}_q = \mathcal{H}_q$. And the Toeplitz matrices (3) and (4) satisfy the following equation which is generated by taking the covariance of the vector $\hat{y}_q(k)$ in (11)

$$\mathcal{R}_q = \mathcal{O}_q \hat{X} \mathcal{O}_q^T + \mathcal{H}_q(\mathcal{U}_q + \mathcal{W}_q)\mathcal{H}_q^T + \mathcal{N}_q \mathcal{E}_q \mathcal{N}_q^T + \mathcal{V}_q \tag{12}$$

where $\hat{X}$ solves the Lyapunov equation (10) and satisfies the scaling condition (9). Any linear system in the form of (6) that can generate both Markov parameters and covariance parameters $\{H_i, R_i | i = 0, 1, \ldots, q-1\}$ must satisfy (12).

Note that the data H_i and R_i do not depend upon the choice of state space realization. Rewrite the scaling condition (9), (10) and the covariance equation (12)

$$A_r(I + \rho_x^2 I)A_r^T + B_r(I + \rho_u^2 I)B_r^T = I \tag{13}$$

$$\mathcal{D}_q = \mathcal{O}_q \mathcal{O}_q^T + \rho_x^2 \mathcal{N}_q \mathcal{N}_q^T \tag{14}$$

We shall find the parameters $\{A_r, B_r, C_r, D_r\}$ satisfying (13) and (14).

Theorem 1. Given the data $\{H_i, R_i | i = 0, 1, \ldots, q-1\}$ generated by a system with unit variance white noise excitation. Let the integer $q > 0$ be specified. Suppose $\mathcal{D}_q - \sum_{i=1}^{q-1} \frac{\rho_x^2}{(1+\rho_x^2)^i} S^{in_y} \mathcal{D}_q {S^{in_y}}^T \geq 0$, where $\mathcal{D}_q$ and S are defined as above. Then all stable linear models $\{A_r, B_r, C_r, D_r\}$ that match the given data are parameterized by

$$\begin{bmatrix} D_r & C_r \\ B_r & A_r \end{bmatrix} = \begin{bmatrix} I_{n_y} & 0 \\ 0 & \mathcal{O}_{q-1}^+ \end{bmatrix} [\mathcal{K}_q \quad \mathcal{O}_q] + \begin{bmatrix} 0 \\ V_b \hat{U} V_d^T \Lambda_{\rho_x} \end{bmatrix}$$

where $\mathcal{O}_q \mathcal{O}_q^T = \mathbb{D}$ is the minimal rank factorization of $\mathbb{D}$, and

$$\mathbb{D} \triangleq \frac{1}{(1+\rho_x^2)} \left[\mathcal{D}_q - \sum_{i=1}^{q-1} \frac{\rho_x^2}{(1+\rho_x^2)^i} S^{in_y} \mathcal{D}_q {S^{in_y}}^T \right]$$

$$\mathcal{O}_{q-1} = \begin{bmatrix} I_{n_y(q-1)} & 0 \end{bmatrix} \mathcal{O}_q$$

$$\mathcal{K}_{q-1} = \begin{bmatrix} 0 & I_{n_y(q-1)} \end{bmatrix} \mathcal{H}_q$$

$$\mathcal{J}_{q-1} = \begin{bmatrix} 0 & I_{n_y(q-1)} \end{bmatrix} \mathcal{O}_q$$

$\hat{U}$ is an arbitrary matrix of proper dimension satisfying $\hat{U}\hat{U}^T = I$.

$$\Lambda_{\rho_x} \triangleq \begin{bmatrix} (1+\rho_u^2)^{-\frac{1}{2}} I & 0 \\ 0 & (1+\rho_x^2)^{-\frac{1}{2}} I \end{bmatrix}$$

And V_b, V_d are given by the following SVD

$$\mathcal{O}_{q-1} = [U_a \quad U_b] \begin{bmatrix} \Sigma_a & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_a^T \\ V_b^T \end{bmatrix}$$

$$\begin{aligned} &\begin{bmatrix} (1+\rho_u^2)^{\frac{1}{2}} \mathcal{K}_{q-1} & (1+\rho_x^2)^{\frac{1}{2}} \mathcal{J}_{q-1} \end{bmatrix} \\ &= [U_a \quad U_b] \begin{bmatrix} \Sigma_a & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_c^T \\ V_d^T \end{bmatrix} \end{aligned}$$

Proof 1. See Li and Skelton (2004).

4. DATA-BASED LQG CONTROL DESIGN

Traditionally the LQG control design is based on a state space model, or a complete input/output description of an LTI system. It is shown in Shi and Skelton (2000) that the Markov parameters of a linear system are all the information needed to implement LQG control algorithms. In practice, the plant model is usually unknown. Hence, the Markov parameters are not available. In what follows we shall use the input/output crosscorrelation data to construct the LQG controller. Notice that those data reduces to Markov parameters when the plant model is linear. By doing so, we assume the unknown plant can be approximated by a linear system generating the same input/output crosscorrelation data. Next we shall briefly review the model based LQG control. Then it is shown all the control coefficients can be expressed in term of Markov parameters.

4.1 Model based LQG control

Consider a linear discrete-time system

$$\begin{cases} x(k+1) = Ax(k) + B(u(k) + w(k)) \\ y(k) = Cx(k) + v(k) \end{cases} \tag{15}$$

where $x(k), u(k), y(k)$ have dimension n, n_u, n_y respectively. $w(k), v(k)$ and the initial state $x(0)$ are zero mean, uncorrelated white noises.

The model-based LQG control problem is to find the functional

$$u_k = f(A, B, C, Q, R, W, V, u_{k-1}, y_{k-1}) \tag{16}$$

such that the quadratic cost

$$J = E\left[y_N^T Q y_N + \sum_{k=0}^{N-1} ({y_k}^T Q y_k + {u_k}^T R u_k) \right] \tag{17}$$

is minimized subject to system model (15) and known characteristics of the initial conditions and the disturbances (x_0, w_k, v_k), where Q and R are positive definite weighting matrices.

The solution of the model-based LQG control problem is well known. The optimal input is given by

$$u_k = -(R + B^T X_{k+1} B)^{-1} B^T X_{k+1} A \hat{x}_k$$
$$k = 0, 1, 2, \ldots, N-1$$

where X_{k+1} is the solution of the difference Riccati equation

$$\begin{aligned} X_k &= C^T Q C + A^T X_{k+1} A - A^T X_{k+1} B \\ &\quad \cdot (R + B^T X_{k+1} B)^{-1} B^T X_{k+1} A. \\ X_N &= C^T Q C \end{aligned} \tag{18}$$

The optimal state estimation, $\hat{x}$, can be obtained from

$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + L_k(y_k - C\hat{x}_k), \quad \hat{x}_0 = 0 \tag{19}$$

where the estimator gain L_k is given by

$$L_k = AY_kC^T(V + CY_kC^T)^{-1}$$

where Y_k is the solution of the following difference Riccati equation

$$\begin{aligned} Y_{k+1} =& BWB^T + AY_kA^T - AY_kC^T \\ &\cdot (V + CY_kC^T)^{-1}CY_kA^T. \end{aligned} \quad (20)$$

It is assumed that the initial conditions of the plant lie in the range space of the disturbance matrix B. That is, $x_0 = B\hat{w}_0$ for some $\hat{w}_0$ with known covariance $W_0 > 0$. Thus

$$Y_0 = E(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T = Ex_0{x_0}^T = BW_0B^T \quad (21)$$

4.2 Closed-form solutions to the Riccati equations

Next we seek to express the closed-form solution of the Riccati equations in terms of Markov parameters.

Lemma 1. The difference Riccati equation (18) is equivalent to the following closed-loop expression

$$\begin{aligned} \mathbf{X}_k =& \mathbf{C}_k^T(\mathbf{Q}_k^{-1} + \mathbf{S}_k\mathbf{R}_k^{-1}\mathbf{S}_k^T)^{-1}\mathbf{C}_k \\ & k = 2, 3, \ldots, N \end{aligned} \quad (22)$$

where

$$\mathbf{C}_k \triangleq \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{N-k} \end{bmatrix}$$

$$\mathbf{S}_k \triangleq \begin{bmatrix} 0 & & & \\ CB & 0 & & \\ CAB & CB & \ddots & \\ \vdots & \vdots & \ddots & \ddots \\ CA^{N-k-1}B & CA^{N-k-2}B & \ldots & CB & 0 \end{bmatrix}$$

$$\mathbf{S}_N = 0$$

$\mathbf{Q}_k = diag(Q, Q, \ldots, Q)$; $\mathbf{R}_k = diag(R, R, \ldots, R)$

$\mathbf{Q}_k$ and $\mathbf{R}_k$ contain $N - k + 1$ diagonal blocks respectively.

Proof 2. This lemma can be proved using backward induction and the matrix inversion lemma.

Similarly, we can express the solution to the estimation Riccati equation in terms of Markov parameters using a dual form of 22.

4.3 Data-based optimal estimation

Besides the solutions to Riccati equations, the LQG controller requires the optimal state estimation $\hat{x}_k$. Next an algorithm is given to compute the controller state recursively in terms of the Markov data sequences and the past observations. Note that in the model-based control, $\hat{x}_k$ is the optimal state estimation as well as the controller state vector. In our data-based control, we define the controller state vector as

$$\bar{x}_k^{N-k+1} \triangleq \mathbf{C}_k\hat{x}_k \quad (23)$$

where $\hat{x}_k$ is the optimal estimation of the plant states x_k, the superscript $N - k + 1$ indicates that, $\bar{x}_k^{N-k+1}$ has dimension of $(N - k + 1)n_y$, and the subscript k is the time index. It should be pointed out that $\hat{x}_k$ is computed using the state space model. In the following we will show that the data-based controller state vector $\bar{x}_k^{N-k+1}$ can be computed using only the Markov parameters.

Theorem 2. The data-based controller state equation is given in terms of the Markov parameters sequences as follows:

$$\begin{aligned} \bar{x}_k^{N-k+1} &= \mathbf{A}_k\bar{x}_k^{N-k+2} + \mathbf{B}_ku_{k-1} + \mathbf{F}_ky_{k-1} \\ \bar{x}_0^{N+1} &= 0 \end{aligned} \quad (24)$$

where $\mathbf{A}_k$, $\mathbf{B}_k$ and $\mathbf{F}_k$ are time varying gain matrices as follows

$$\mathbf{A}_k = [-\mathbf{F}_k, \mathbf{I}_{(N-k+1)n_y}]$$

$$\mathbf{F}_k = \mathbf{H}_k\mathbf{P}_k\mathbf{N}_k^T(V + \mathbf{N}_k\mathbf{P}_k\mathbf{N}_k^T)^{-1}$$

$$\mathbf{B}_k = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_{N-k+1} \end{bmatrix}$$

$$\mathbf{H}_k = \begin{bmatrix} H_2 & H_3 & \ldots & H_{k+1} \\ H_3 & H_4 & \ldots & H_{k+2} \\ \vdots & \vdots & \ddots & \vdots \\ H_{N-k+2} & H_{N-k+1} & \ldots & H_{N+1} \end{bmatrix}$$

$$\mathbf{P}_k = (\mathbf{W}_k^{-1} + \mathbf{T}_k^T\mathbf{V}_k^{-1}\mathbf{T}_k)^{-1}$$

$$\mathbf{N}_k = [H_1, H_2 \ldots H_k]$$

$$\mathbf{T}_k = \begin{bmatrix} 0 & H_0 & H_2 & \ldots & H_{k-1} \\ & 0 & H_1 & \ldots & H_{k-2} \\ & & \ddots & \ddots & \vdots \\ & & 0 & \ddots & H_1 \\ & & & & 0 \end{bmatrix}$$

$$\mathbf{W}_k = diag\{W, W, \ldots, W, W_0\}$$

$$\mathbf{V}_k = diag\{V, V, \ldots, V\}$$

where $\mathbf{W}_k$ and $\mathbf{V}_k$ contain k diagonal blocks respectively.

Proof 3. See Shi and Skelton (2000).

Next the control gain is presented in terms of the Markov parameters.

Theorem 3. The optimal data-based LQG control law associated with the cost function (17) is given by

$$u_k = \mathbf{G}_k \bar{x}_k^{N-k+1} \tag{25}$$

where $\mathbf{G}_k$ is referred to as the data-based control gain, $\bar{x}_k^{N-k+1}$ is the data-based controller state vector derived in Theorem 2, and

$$\mathbf{G}_k = -(R + \mathbf{B}_{k+1}^T(\mathbf{Q}_{k+1}^{-1} + \mathbf{S}_{k+1}\mathbf{R}_{k+1}^{-1})^{-1}\mathbf{B}_{k+1})^{-1} \cdot \mathbf{B}_{k+1}^T(\mathbf{Q}_{k+1}^{-1} + \mathbf{S}_{k+1}\mathbf{R}_{k+1}^{-1}\mathbf{S}_{k+1}^T)^{-1}[0_{n_y}, \mathbf{I}_{(N-k)n_y}]$$

$$\mathbf{B}_{k+1} = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_{N-k} \end{bmatrix}$$

$$\mathbf{S}_{k+1} = \begin{bmatrix} 0 & & & \\ H_1 & 0 & & \\ H_2 & H_1 & \ddots & \\ \vdots & \vdots & \ddots & \ddots \\ H_{N-k-1} & H_{N-k-2} & \dots & H_1 \; 0 \end{bmatrix}$$

$$\mathbf{Q}_{k+1} = diag\{Q, Q, \dots, Q\}$$
$$\mathbf{R}_{k+1} = diag\{R, R, \dots, R\}$$

and $\mathbf{Q}_{k+1}$ and $\mathbf{R}_{k+1}$ contain $N-k$ diagonal blocks respectively.

Proof 4. See Shi and Skelton (2000).

5. DATA-BASED CLOSED-LOOP SIMULATION SYNTHESIS

Theorem 2 and 3 provide the data-based LQG control design algorithm. Combining them leads to a recursive form as follows:

$$\bar{x}_k^{N-k+1} = \mathbf{A}_k \bar{x}_k^{N-k+2} + \mathbf{B}_k u_{k-1} + \mathbf{F}_k y_{k-1}$$
$$\bar{x}_0^{N+1} = 0$$
$$u_k = \mathbf{G}_k \bar{x}_k^{N-k+1}$$

where all the coefficient matrices can be expressed in terms of the given data. Meanwhile, an explicit simulation model of the plant is given in Theorem 1. Thus, a closed-loop simulation as in figure 1 can be constructed. Notice that the dimension of the simulation model depends on the number of data it has to match. A simulation model with dimension $q \cdot n_y$ can match the first q crosscorrelation and autocorrelation data. In data-based LQG control with horizon N, the first N crosscorrelation data is needed to obtain the controller parameters.

6. CONCLUSION

This paper integrates the q-Markov COVariance Equivalent Realization (q-Markov COVER) and Markov data-based LQG control. The q-Markov COVER gives an identification model while Markov data-based LQG control generates an LQG controller. Thus, a closed-loop system can be constructed. We extended this integration to closed-loop simulation where the roundoff error in digital simulations is accommodated with a finite wordlength q-Markov COVariance Equivalent Realization (FWL-QMC). Also we extended this integration to unknown dynamic systems where the Markov and Covariance parameters are related to the input/output crosscorrelation and output autocorrelation data of a plant with white noise excitations. Hence, a framework of data-based closed-loop simulation is developed with much less information than the plant model.

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