PROBABILISTIC ROBUST PARALLEL DESIGN OF THE SUBSYSTEMS CONSTITUTING A COMPLEX SYSTEM

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Abstract: The design of complex systems, consisting of several subsystems and with performance specifications from multiple disciplines, in parallel was addressed in a previous publication using a Robust Parallel Design (RPD) approach. In this paper, RPD is extended and a Probabilistic Robust Parallel Design (PRPD) approach is proposed to handle cases where the statistical properties of uncertainties are known. Monte Carlo simulation is used to determine the value of a subsystem objective, given the known statistical distributions of uncertainties. Random search techniques (e.g., Simulated Annealing) can then be used to minimize the subsystem objective. PRPD is illustrated using a passive suspension design example of a half-car model.

Keywords: Monte Carlo simulation, Optimization, Parallel processing, Probabilistic models, Random searches, Vehicle suspension.

1. INTRODUCTION

A complex system is often decomposed into smaller subsystems that can be designed in parallel. This requires that performance specifications, usually known at the system-level, be cascaded down to subsystem design targets. The process of target cascading should be performed in an "efficient" and "consistent" manner to avoid iterations at later stages of the design process and to ensure that once subsystem targets are met, the desired system-level specifications are achieved (Kim, 2001). Once the values of subsystem targets are determined, design teams work in parallel to achieve these targets.

Subsystem optimal design problems are solved in the presence of uncertainties. The Robust Parallel Design (RPD) approach introduced in a previous publication, (Mahmoud *et al.*, 2004b), deals with the worst-case values of the uncertainties present in a subsystem optimal design problem, i.e., the subsystem objective function is minimized assuming worst-case uncertainties. This results in a mini-max optimization problem that is computationally intractable.

In this paper, a new Probabilistic Robust Parallel Design (PRPD) approach is proposed to handle situations where the probability density functions (PDFs) of uncertainties are known. Rather than minimizing the value of a subsystem's objective function assuming worst-case uncertainties, a subsystem's objective function is minimized considering the PDFs of the present uncertainties.

2. LITERATURE REVIEW

The mathematical statement of an Optimal Design Problem (ODP) is (Krishnamachari and Papalambros, 1997):

$$\begin{aligned}&\underset{\mathbf{x}\in X}{\text{Minimize}}\; f(\mathbf{x},\mathbf{p},\mathbf{w})\\&\text{subject to } g(\mathbf{x},\mathbf{p}) \le \mathbf{0}\end{aligned} \qquad (1)$$

where

$\mathbf{x}$	vector of design variables
$\mathbf{p}$	vector of design parameters
X	set constraint
$g(\mathbf{x},\mathbf{p})$	inequality constraints
$\mathbf{w}$	vector of weights for different performance measures

This optimal design problem is NP-hard, i.e., solution times grow exponentially with the number of variables. Thus, it is desirable to decompose this problem into a set of smaller sub-problems where each sub-problem can be handled more efficiently. Other benefits of system decomposition include reducing a product's design-cycle time, by solving the subsystem design problems in parallel, and allowing companies to outsource some of their design tasks to their suppliers (Kim, 2001).

In the context of designing subsystems in parallel and having a subsystem design team account for the uncertainties in the values of the design variables of other subsystems, Chen and Lewis (1999) distinguish between two types of robust designs. A Type I robust design refers to the robustness of one subsystem to changes in the design variables of other subsystems, whereas a type II robust design refers to specifying values of design variables that can be allowed to vary within a range. Conceptual robustness and game theory are two approaches used to design subsystems that are robust with respect to the values of the design variables in other subsystems.

Conceptual robustness is based on the use of Taguchi's parameter design principles for minimizing the effects of noise factors on an engineering system (Chang and Ward, 1995; Chang *et al.*, 1994). Design variables of other subsystems are treated as "conceptual" noise factors. Taguchi's parameter design approach can then be used to design subsystems that are robust with respect to both physical noise factors and to the "conceptual" noise factors, i.e., the design variables of other subsystem. In their formulation, Chang and Ward (1995), assume that the design teams can communicate at certain intervals. A cost of delay is calculated to determine whether a design team should decide on the values of its design variables immediately or wait for more information from other design teams.

Although conceptual robustness is a promising approach for the parallel design of subsystems, there are several issues that need to be addressed. First, conceptual robustness assumes "reasonable independence" of the design variables (Chang and Ward, 1995). This allows a design team to optimize the subsystem objective with respect to each design variable independently, thereby greatly simplifying the optimization problem. However, the authors do not provide a measure of "reasonable independence" or how to formulate a problem so that the variables are independent. In the present work, the assumption of independence between the design variables within a subsystem is not required. Independence between the design variables of the different subsystems is achieved by solving a target cascading problem and specifying ranges for the values of the different design variables.

Chen and Lewis, (1999), used a game theoretic approach to solve multi-disciplinary optimization problems and design robust subsystems. A Stackelberg leader/follower protocol was used to achieve type II robustness and allow variables that are coupled between multiple players (disciplines) to vary within certain ranges. A multi-objective optimization problem was constructed to account for the needs of optimizing performance, minimizing performance deviations and maximizing flexibility at different priority levels. This approach is claimed to minimize the effects of decisions made by one discipline upon other disciplines, thereby saving iteration time. It also allows for better ability to make decisions concurrently. In general, game theoretic approaches are mathematically intractable and thus not suitable for large scale problems.

3. PROBABILISTIC ROBUST PARALLEL DESIGN (PRPD) APPROACH

In this paper a new Probabilistic Robust Parallel Design (PRPD) approach is proposed that enables subsystems to be designed in parallel while accounting for the uncertainties present in the subsystem design problems. First, the system is decomposed into several subsystems. Decomposing a system into subsystems reveals interconnections between the subsystems where the outputs of one subsystem are used as inputs to other subsystems. This is illustrated in Figure 1 for a system that is decomposed into three subsystems.

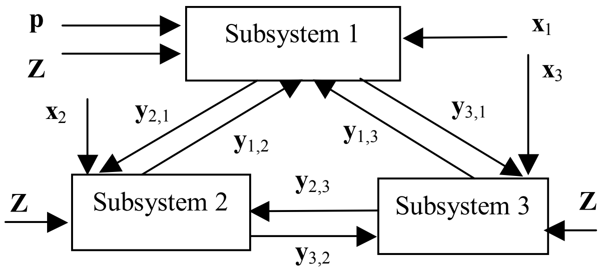

Fig. 1. Multi-Disciplinary Design Problem

In Figure 1, $\mathbf{Z}$ is a vector of shared design variables, $\mathbf{x}_i$ is a vector of local design variables for subsystem i and together they comprise the vector $\mathbf{x}$ of (1), $\mathbf{p}$ is a vector of design parameters and $\mathbf{y}_{ij}$ is a vector of outputs to subsystem i from subsystem j. System-level specifications may include outputs from several subsystems, i.e., $f(\mathbf{x}, \mathbf{p}, \mathbf{w})$ in (1) includes outputs from several subsystems.

Target cascading is used to specify nominal values for subsystem design targets and shared design variables, $\mathbf{Z}$, using surrogate models. The use of higher fidelity models in the subsystem design stage may result in the values of some of the design variables and parameters deviating from their nominal values. The deviation of design variables and parameters around their nominal values can be described using statistical properties, e.g., Probability Density Function (PDF). Since the subsystems are designed in parallel, the design team working on a particular subsystem design problem treats the design variables in other subsystems as uncertainties.

The objective of a subsystem optimization problem, assuming probabilistic uncertainties, is to minimize the kth percentile value of the subsystem objective function. This problem can be stated as:

$$\min_{x_i} \varphi_k$$

subject to $\qquad g(\mathbf{x}, \mathbf{p}) \leq \mathbf{0}$

$$(1-\delta)\,\mathbf{x}_0 \leq \mathbf{x}_i \leq (1+\delta)\,\mathbf{x}_0$$

where

ϕ_k	is such that k % of $f(\mathbf{x}, \mathbf{p}, \mathbf{w}) \le \phi_k$
x	vector of design variables with perturbations, δ, about their nominal values $\mathbf{x}_0$
$\mathbf{x}_i$	vector of design variables for subsystem i
p	design parameters having uncertainties Δ with known probability density functions

Monte Carlo simulation is used to evaluate a subsystem's objective function given the distribution of uncertainties. In Monte Carlo simulation, the random variables are sampled. The function of interest is then evaluated for the different sampled values. An estimate of the value of the function of interest is obtained by averaging the values corresponding to different samples. For further details on Monte Carlo simulation, the reader is referred to Kalos (1986).

A random search algorithm is used to minimize a subsystem's objective function with respect to design variables. Monte Carlo simulation is used to evaluate the subsystem objective function using the known PDFs of uncertainties. The random search algorithm used in the present work is Simulated Annealing. However, the proposed PRPD is not limited to the use of Simulated Annealing and other random search algorithms, e.g., Genetic Algorithm and Reactive Taboo Search, can be used.

In Simulated Annealing, a cooling law, resembling Boltzmann's law for energy states for atoms, is used to assign probabilities of accepting moves that result in an increase in the value of the objective function. The probability of accepting moves that increase the value of the objective function decreases uniformly over the course of the minimization process. The proposed Probabilistic Robust Parallel Design Approach will now be described and illustrated using a simple example consisting of a half-car suspension model.

In this paper, the system-level and subsystem objectives are assumed to be aligned, i.e., improving on subsystem objectives improves the system level objective. Having subsystem objectives aligned with system-level objectives can be done by weighting each subsystem objective with the derivative of the system-level objective with respect to the respective subsystem objective. Mahmoud *et al.* (2004b) propose an algorithm to efficiently calculate the sensitivity of a Noise-Vibration-Harshness (NVH) system-level objective with respect to subsystem objectives for linear systems. In the present work, the subsystem objective is assumed to be the same as the system-level objective, i.e., the sensitivity of the system-level objective with respect to subsystem objectives is unity.

The design teams working on the different subsystems are minimizing the (same) objective and hence are likely to select values for their local design variables that are minimizers of this objective. The Probability Density Function of design variables of other subsystems should reflect this, i.e., there should be a higher probability of values that improve on the objective function. The Probability Density Functions of design parameters can follow any form.

Each subsystem optimization problem minimizes the system-level objective at a certain percentile with respect to local design variables, a subset of the set of design variables. Thus the system-level performance is expected to be better than the best performance achieved in any of the subsystem design problems, i.e., ϕ_k for the overall system is expected to be less than, or equal to, the minimum ϕ_k achieved in any subsystem design.

4. CASE STUDY: VEHICLE SUSPENSION DESIGN

The design of active, passive and semi-active vehicle suspension systems using various vehicle models, corner-car, half-car and full-car, has been reported in the literature, e.g., Hrovat (1997), Sharp and Crolla (1987) and Ulsoy *et al.* (1994). Performance measures for a vehicle suspension include passenger comfort, suspension stroke ("rattlespace") and road handling. These are quantified by the acceleration of the sprung mass, the relative displacement of the sprung and unsprung masses, and the dynamic forces at the tires, respectively. In this paper, a passive suspension is decomposed into two subsystem design problems that can be solved in parallel using PRPD.

4.1 Road Excitation Model

A vehicle's suspension is subjected to various sources of excitations (e.g., road roughness, tire-wheel assembly imperfections and engine/transmission excitation). In the present work, road excitation is considered to be the only source of disturbance. The Power Spectral Density (PSD) of road roughness is obtained by applying a first-order filter to unit variance white noise. The first-order filter used is given by Zuo and Nayfeh (2003b):

$$G(s) = \frac{\left(2\pi A_r V\right)^{\frac{1}{2}}}{s + \left(2\pi V z_0\right)} \qquad (2)$$

where

A_r	is the road roughness coefficient. A value of 16×10^{-7} m^{2}cycle/m is used corresponding to a class B road.
V	is the longitudinal vehicle velocity, 30 mph
z_0	spatial cutoff frequency of 0.005 cycle/m to avoid infinite PSD at low frequencies.

4.2 Half-car Model

The four-degree-of-freedom (4 DOF) half car model shown in Figure 2 is used in this study. The equations of motion for the half-car model are provided in Mahmoud *et al.* (2004a). Nominal values of the different design variables and system parameters are given in Table 1, obtained from Mahmoud *et al.*, (2004a).

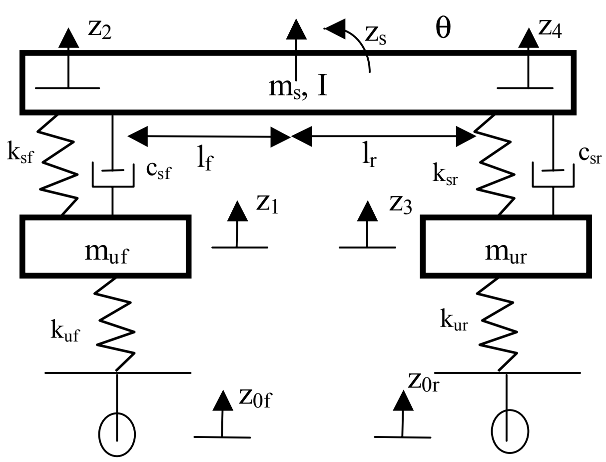

Fig. 2. Half-car suspension model

TABLE 1: NOMINAL VALUES AND UNCERTAINTIES FOR DESIGN VARIABLES AND DESIGN PARAMETERS

Description	Symbol	Value
Sprung mass	m_s	688 kg +/- 10 %
Sprung moment of inertia	I	1172 kg.m^2 +/- 10 %
Unsprung masses (front/rear)	m_{uf}/m_{ur}	40/40 kg +/- 10 %
Tire Stiffness (front/rear)	k_{uf}/k_{ur}	182/182 kN/m +/- 10 %
Suspension stiffness (front/rear)	k_{sf}/k_{sr}	20985/19122 N/m +/- 20 %
Suspension damping (front/rear)	c_{sf}/c_{sr}	1306/1470 N.s/m +/- 44 %
Distance between c.g and front/rear tires	l_f/l_r	1.125/1.511 m

4.3 Passenger Comfort

Passenger comfort is proportional to the amount of acceleration experienced by the vehicle passengers. International Standard ISO 2631-1 provides frequency weights that can be used to modify measured accelerations to account for human sensitivities to acceleration forces of various frequencies. The following second order filter is used to approximate ISO 2631-1 frequency weighting curves (Zuo and Nayfeh, 2003a).

$$W(s) = \frac{50s + 500}{s^2 + 50s + 1200} \qquad (3)$$

The performance index $f(\mathbf{x}, \mathbf{p}, \mathbf{w})$ consists of a weighted sum of the root-mean-square values of signals of interest. These include; the frequency-weighted acceleration of the sprung mass, the velocity of the sprung mass, the rotational velocity of the sprung mass, the suspension stroke and the tire dynamic forces, i.e.,

$$f(\mathbf{x}, \mathbf{p}, \mathbf{w}) = E\left\{ \begin{array}{l} r_1 \ddot{z}_s^2 + r_2 \dot{z}_s^2 + r_3 \dot{\theta}^2 + r_4\left[(z_2 - z_1)^2 + (z_4 - z_3)^2\right] \\ + r_5\left[k_{uf}^2 (z_1 - z_{0f})^2 + k_{ur}^2 (z_3 - z_{0r})^2\right] \end{array} \right\}$$

The values of the weights used, obtained from are

Weight	r_1	r_2	r_3	r_4	r_5
Value	1	8.3	120	120	8.3e-3

4.4 Application of PRPD Approach

The half-car suspension model is decomposed into two subsystems to be designed in parallel; front and rear suspension. Subsystem A consists of the front suspension stiffness and damping and subsystem B consists of the rear suspension stiffness and damping. Uncertain parameters in a subsystem design problem consist of the design parameters, whereas design variables of other subsystems are treated as constants. The subsystem design problems consist of minimizing the subsystems' objective functions with respect to local subsystem design variables. The optimization problem for subsystem A is given below, a similar optimization problem was obtained for subsystem B.

Subsystem A

$\min_{x_A} \phi_{k_A}$

subject to Equations of motion

$(1-\delta)\ \mathbf{x}_{A0} \le \mathbf{x}_A \le (1+\delta)\ \mathbf{x}_{A0}$

where

ϕ_{k_A} is such that k_A % of $f(\mathbf{x}_A, \mathbf{p}, \mathbf{w}) \le \phi_{k_A}$

$\mathbf{x}_A$ design variables for subsystem A with nominal values $\mathbf{x}_{A0}$

$\mathbf{p}$ uncertain parameters for subsystem A including design parameters and subsystem B design variables

In the present paper, the PDFs of the design variables of other subsystems are excluded from the uncertain parameters in a subsystem optimization problem. As mentioned earlier, the subsystem objective functions are the same but minimized with respect to different local design variables. This means that, in a subsystem optimization problem, the values of other subsystems' design variables have higher likelihoods of being minimizers of the subsystem objective function. Rather than using a PDF to reflect this, other subsystem design variables are treated as constants. This can be modified if suitable PDFs for other subsystems' design variables are available. The uncertain parameters are assumed to follow a normal distribution with means, μ, equal to the nominal values in Table 1 and variances, σ^2, such that $\mu \pm 3\sigma$ is equal to the limits in Table 1.

Simulated annealing was used to solve the subsystem optimization problems. Monte Carlo Simulation was used to calculate the value of the subsystem objective function for different values of design variables. An outer loop was used to update the temperature used to assign probabilities of accepting moves that result in an increase in the value of the objective function. The cooling law used was of the form:

$$t_i = \alpha\, t_{i-1}$$

At each temperature, a number of metropolis simulations were performed. In each metropolis simulation, the design variables were randomly sampled. Based on the value of the objective function corresponding to the sampled design variables and the temperature of the simulation, a decision was made whether to move to the new point or not. If the sampled design variables resulted in a lower value for the objective function, a move to the new point was accepted. If the sampled design variables resulted in an increase in the value of the objective function, a move to new point was accepted if the following condition was true:

$$r \leq \exp\left(-\frac{f(x_i) - f(x_{i-1})}{t} \right)$$

where

r random number between 0 and 1

t temperature

$f(x_i)$ value of objective function evaluated using Monte Carlo simulation

The following parameters were for Simulated Annealing:

Parameter	Value
Number of outer loop iterations	20
Number of metropolis simulations	200
α	0.95
Number of Monte Carlo simulations	100

Although only a 100 Monte Carlo simulation were used to evaluate the 95th percentile of a subsystem objective function when solving the subsystem optimization problems, the number of Monte Carlo simulations used to evaluate subsystem performance was increased to 2500.

Using the Probabilistic Robust Parallel Design approach, the following values for the design variables of subsystems A and B are obtained for the case when the uncertainty in the values of design parameters = 25 %.

5. RESULTS

The passive suspension design problem is solved by determining values for the design variables of subsystems A and B, respectively.

Subsystem A

For subsystem A the design variables are the front suspension stiffness and damping. Solving the optimization problem for subsystem A results in the following values for the front suspension stiffness and damping.

k_{sf} = 22,903 N/m, c_{sf} = 1,881 N.s/m

These new values of the design variables for subsystem A result in a 12 % decrease in the 95th percentile value of objective function for subsystem A, from 1,564 ± 2 to 1,373 ± 2, with a 95 % confidence level.

Subsystem B

For subsystem B the design variables are the rear suspension stiffness and damping. Solving the optimization problem for subsystem B results in the following values for the rear suspension stiffness and damping.

k_{sr} = 19,360 N/m, c_{sr} = 2,046 N.s/m

The new values of the design variables for subsystem B result in a 10 % reduction in the 95th percentile value of the objective function for subsystem B, from 1,564 ± 2 to 1,408 ± 2 at the 95 % confidence level.

The solution obtained by the PRPD approach is compared to the solutions of an All-At-Once (AAO) optimization and Parallel Design (PD), designing subsystems in parallel without accounting for uncertainties, in Table 2. The expected value of the performance index and its 95th percentile value are evaluated for the overall system, i.e., the solutions from both subsystem design problems are combined to obtain **x** for the overall system for the cases of PRPD and PD.

TABLE 2: COMPARISON OF PRPD TO AAO OPTIMIZATION AND PARALLEL DESIGN WHEN UNCERTAINTY = 25 %

	Nominal	**AAO**	**PRPD**	**PD**
k_{sf} (N/m)	20,985	16,788	22,903	16,788
c_{sf} (N.s/m)	1,306	1,881	1,881	1,881
k_{sr} (N/m)	19,122	18,394	19,360	19,228
c_{sr} (N.s/m)	1,470	2,117	2,117	2,117
E[P.I] (% change) Std Error	**1,331 2.85**	**1,183 (-11.1) 2.29**	**1,188 (-10.7) 2.22**	**1,188 (-10.7) 2.35**
95 % P.I (% change)	**1,564**	**1,374 (-12.2)**	**1,377 (12.0)**	**1,387 (-11.3)**

The half-car suspension model was solved again using PRPD assuming the magnitude of uncertainties is 50 %. The results are reported in Table 3.

TABLE 3: COMPARISON OF PRPD TO AAO OPTIMIZATION AND PARALLEL DESIGN WHEN UNCERTAINTY = 50 %

	Nom.	**AAO**	**PRPD**	**PD**
k_{sf} (N/m)	20,985	16,788	21,426	16,788
c_{sf} (N.s/m)	1,306	1,881	1,881	1,881
k_{sr} (N/m)	19,122	17,394	24,445	19,228
c_{sr} (N.s/m)	1,470	2,117	2,094	2,117
E[P.I] (% change) Std error	**1,353 5.76**	**1,195 (-11.68) 4.69**	**1,221 (-9.76) 4.53**	**1,204 (-11.07) 4.67**
95 % P.I (% change)	**1,859**	**1,612 (-13.3)**	**1,609 (-13.4)**	**1,611 (-13.36)**

The main motivation for the present work was to be able to decompose a system into subsystems, design the subsystems independently and guarantee achieving satisfactory system-level performance upon assembly of the subsystem. In such a situation, the use of AAO optimization may not be possible. The results reported in Tables 2 and 3 do not show a significant difference between PRPD and PD. This is caused by the large standard error of the Monte Carlo simulation used within the optimization loop to evaluate the 95th percentile of the value of a subsystem objective function. Increasing the number of Monte Carlo simulation, and of Simulated Annealing iterations, is expected to yield more significant difference between the results obtained using PRPD and PD.

6. CONCLUSIONS

The proposed PRPD approach allows a system design task to be decomposed into several subsystem design tasks that can be performed in parallel. A half-car example was used to illustrate the proposed approach. PRPD has the following three advantages over the RPD approach proposed by Mahmoud *et al.*, (2004a):

1. PRPD does not suffer from the curse of dimensionality to which RPD is suspect. Using the PRPD approach, subsystem optimization problems can be solved in polynomial time. This makes PRPD suitable for large scale problems.
2. PRPD approach is less conservative than the RPD approach. This can be seen by comparing the 95th percentile value of the system-level objective function to the worst-case value obtained using RPD. As the magnitude of uncertainty increases, the conservatism of the RPD approach becomes larger.
3. The use of random search and Monte Carlo simulation in PRPD makes it amenable to parallel computing, another feature that makes PRPD design suitable for large scale problems. Implementing PRPD design in a parallel computing environment can be done easily by using several computers to evaluate the Monte Carlo simulations for different values of design variables and/or parameters.

7. FUTURE WORK

The solution obtained using PRPD may deviate from the true solution due to the uncertainty in the results of Monte Carlo simulation. Monte Carlo simulation may return a value for a subsystem's objective function at the lower end of the confidence interval for a set of values of design variables and a value at the upper end of the confidence interval for a different set of values of design variables, possibly closer to the true solution. In this case, the solution returned by the random search algorithm will include the first set of values of design variables. Future work is needed to quantify the deviation of the solution obtained using the PRPD approach from the true solution.

8. REFERENCES

Chang, T.-S., and Ward, A. C., (1995), "Conceptual Robustness in Simultaneous Engineering: A Formulation in Continuous Spaces," *Research in Engineering Design*, **7**, pp. 67-85.

Chang, T.-S., Ward, A. C., and Lee, J., (1994), Conceptual Robustness in Simultaneous Engineering: An Extension of Taguchi's Parameter Design," *Research in Engineering Design*, **6**, pp. 211-222.

Chen, W., and Lewis, K., (1999), "Robust Design Approach for Achieving Flexibility in Multidisciplinary Design," *AIAA Journal*, **37**(8), pp. 982-989.

Hrovat, D., (1997), "Survey of Advanced Suspension Developments and Related Optimal Control Applications," *Automatica*, **33**(10), pp. 1781-1817.

Kalos, M. H., and Whitlock, P. A., (1986), *Monte Carlo Methods*, Wiley, New York.

Kim, H., (2001), *Target Cascading in Optimal System Design*, Ph.D. thesis, University of Michigan, Ann Arbor, MI.

Krishnamachari, R., and Papalambros, P., (1997), "Hierarchical Decomposition Synthesis in Optimal Systems Design," *Journal of Mechanical Design*, **119**(4), pp. 448-457.

Mahmoud, H., Kabamba, P., Ulsoy, A. G., and Brusher, G., (2004a), "Uncertainty in the Parallel Design of the Subsystems Constituting a Complex System," submitted to the *ASME Journal of Mechanical Design*.

Mahmoud, H., Kabamba, P., Ulsoy, A. G., and Brusher, G., (2004b), "Target Management in Complex System Design Using System Norms," accepted for publication in the *ASME Journal of Mechanical Design*.

Sharp, R. S., and Crolla, D. A., (1987), "Road Vehicle Suspension Systems Design – a Review," *Vehicle System Dynamics*, **16**, pp. 167-192.

Ulsoy, A. G., Hrovat, D., and Tseng, T., (1994), "Stability Robustness of LQ and LQG Active Suspensions," *Journal of Dynamic Systems, Measurement and Control*, **116**, pp. 123-131.

Zuo, L., and Nayfeh, S., (2003a), "Low-Order Continuous-Time Filters for Approximation of the ISO 2631-1 Human-Vibration Sensitivity Weightings," *Journal of Sound and Vibration*, **265**(2), pp. 459-465.

Zuo, L., and Nayfeh, S., (2003b), "Strucured H2 Optimization of Vehicle Suspensions Based on Multi-Wheel Models," *Vehicle System Dynamics*, **40**(5), pp. 351-37.