SLIDING MODE OBSERVER FOR TRIANGULAR INPUT HYBRID SYSTEM

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Abstract: In this paper a methodology to design an observer for a class of hybrid nonlinear systems with no continuous state reset is proposed. By using hybrid systems techniques for modelling and synthesis, we propose a solution to the challenging observer problem related to such system. Some simulations illustrate the proposed approach.

Keywords: Hybrid system, Nonlinear observer, Sliding mode, SISO systems.

1. INTRODUCTION

Physical system behavior follows the general principles of conservation of energy and continuity of power. They may exhibit nonlinearities that result from small parasitic effects or occur on a time scale much smaller than the time scale of interest. At a macroscopic level, the detailed continuous behavior may appear to be discontinuous. In fact, switching or impacting behaviors are met in many systems in engineering and applied science. Such systems describe special dynamical processes of mixed continuous and discrete natures and inherently combine logical and continuous process, usually coupled with finite automata and differential equations. Hence, those systems which are capable of exhibiting simultaneously several kinds of dynamic behavior in different parts of the system (e.g. continuous time dynamics, discrete-time dynamics, jump phenomena, logic commands, ...) are of great interest. In the literature such systems are called Hybrid Systems (HS).

Recently, there has been an increased interest in the study of HS. By this way considerable research effort has been devoted to fundamental topics such as modelling and simulation of HS. Among the main issues in hybrid systems (as it is the case of linear and nonlinear continuous and discrete time case) are the synthesis of control, observer, supervisory control schemes, and the formal verification for safety analysis, which aims at certifying that the hybrid system behaves as desired. Recently, some works devoted to algebraic properties of observability and controllability of HS have been developed.

In (Balluchi et *al.*, 1999) was highlighted the complexity of observability properties for hybrid systems through examples on MLD (Mixed Logic Dynamical) systems and PWA (Piecewise affine). Sontag in (Sontag, 1979) introduces a set of observability related definitions and examines the implications among the various concept of observability. In (Balluchi et *al.*, 2003), the authors focused on the property of the generic final state determinability of HS to construct an asymptotic state observer. They showed that this property can be verified even if each of the continuous subsystems of the HS is not observable. In (Vidal et *al.*, 2003), the authors defined the so called extended joint observability matrix to analyze the observability of jump linear systems.

Some sufficient geometrical conditions to analyze the observability of hybrid dynamical systems are given in (Boutat et *al.*, 2004). These conditions are refined for the particular class of the piecewise linear and nonlinear systems. In the same way other works have treated on the design of hybrid observers. In (De la Sen and Ningsu Luo, 2000) a design of linear observers for a class of linear hybrid systems is addressed. Two observers prototypes based on the prediction error are proposed. The first is based on the observation of an extended discrete-time system. The second one estimates the continuous-time sub-state for all time from initial conditions. A methodology for the design of a hybrid observer for generic hybrid plant with no continuous sate resets is proposed in (Balluchi et *al.*, 2001). The structure of the proposed hybrid observer is composed of a location observer and a continuous observer and applied for a non linear model of a driveline with discontinuous elasticity. Despite an abundant literature on the design of linear observers for hybrid systems, only few works concern the design of nonlinear hybrid observer for hybrid systems (see for example (Lin et *al.*, 2002)).

The main purpose of this paper lies in nonlinear observer design for HS without jump. We discuss the problem of designing a sliding mode observer for a class of nonlinear hybrid systems. In (Drakunov and Utkin, 1995) a new concept of sliding observers is introduced. The key point is that the equivalent control concept is extensively used. Moreover, in (Boukhobza et *al.*, 1996; Barbot et *al.*, 1996), we use a "classical" sliding mode observer in order to design an observer for the a largest class with the so-called output injection form (Krener and Isidori, 1983). Here, our purpose is to discuss the observer design by using a triangular input observer form introduced in (Boukhobza et *al.*, 1996; Barbot et *al.*, 1996) and (Drakunov and Utkin, 1995). The idea consists in using the step by step observer such as described here after : The $(n-1)$ first step consists in reconstructing the state vector while the step n is used to to define in which state P_i for $i = 1, ..., k$ the system is found. A complete scheme of the observer is given in figure 1.

Since the considered system is a particular class of HS, where the interactions of time-continuous models are governed by differential equations and inter-connected by switching functions, then, to characterize the system's observability, we will use the observability conditions developed by (Boutat et *al.*, 2004) for a such class of piecewise dynamical systems.

Hence the paper is organized as follow: after a recall on the observability study for hybrid systems, we present the non linear hybrid sliding mode observer design. Two illustrative examples will show the performances of the developed algorithm.

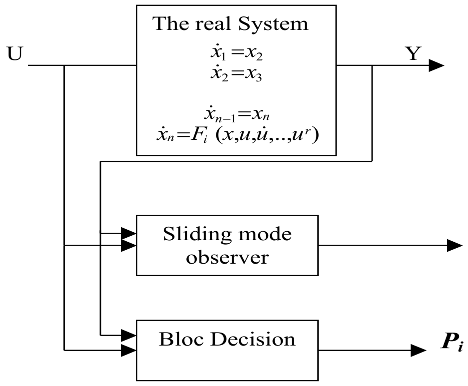

Fig. 1 : Hybrid observer structure

2. RECALLS ON OBSERVABILITY STUDY

In the following section, let us recall the main result of one of the authors in (Boutat et *al.*, 2004), on the observability of the class of hybrid system considered in this paper. The proof of the theorem will be found in the cited reference. Let us consider the dynamical systems formed with two dynamics inter-connected by a switch function :

$$\begin{cases} \dot{x} = f_1(x) \text{ and } y = h_1(x) \text{ if } \sigma(x) \leq 0 \\ \dot{x} = f_2(x) \text{ and } y = h_2(x) \text{ if } \sigma(x) > 0 \end{cases} \quad (1)$$

where $f_i(x)$ are smooth vector fields, $h_i(x)$ are smooth outputs and $\sigma(x)$ is a smooth switching function.

Assumption 1. : We assume throughout this paper that

a) All the evolution duration of each subsystem of (1) are measurable.
b) Each subsystem is observable. That is, for $i = 1 : 2$ the codistribution:

$$\left\{ dh_i, dL_{f_i}h_i, ..., dL_{f_i}^{(n-1)}h_i \right\}$$

is of *rank n*

The a) of assumption 1 means that systems with Zeno phenomenon are not considered. Also, the knew of duration of evolution τ_i of each system allows us to exclude the instability due to τ_i.

Under conditions of assumption 1, if we know which of the subsystem evolves, we can conclude on the observability of the global system (1).

Using Fliess's observability canonical form, each subsystem of (1) can be written as:

$$\begin{cases} \dot{z}_i^1 = z_{i+1}^1 \qquad \text{for } i = 1 : n-1 \\ \dot{z}_n^1 = g_1(z_1^1, z_2^1, ..., z_n^1) \end{cases} \tag{2}$$

if $\sigma_1 := \sigma(z_1^1, z_2^1, ..., z_n^1) \leq 0$, and

$$\begin{cases} \dot{z}_i^2 = z_{i+1}^2 \qquad \text{for } i = 1 : n-1 \\ \dot{z}_n^2 = g_2(z_1^2, z_2^2, ..., z_n^2) \end{cases} \tag{3}$$

if $\sigma_2 := \sigma(z_1^2, z_2^2, ..., z_n^2) > 0$.
Where $(z^j, j = 1 : 2)$ are the observability coordinates given by:

$$z_{i+1}^j = L_{f_j}^{(i)} h_j \qquad \text{for } 0 \leq i \leq n-1$$

where $L_{f_j}^{(i)} h_j$ is the i^{th} Lie derivative of σ_j in the direction of f_j.
One approach to analyze the observability of (1), presented in (Boutat et *al.*, 2004), is based on the comparison of g_1 and g_2 on the one hand and σ_1 and σ_2 on the other hand. For this, we need to evaluate such functions in terms of the same variables. These variables are given naturally by the output y and its successive time derivatives $y^{(i)} = \frac{d^i y}{dt^i}$ for $i = 1 : n-1$.
Let us consider the two submanifolds:

$$\mathcal{M} = \{v \in \mathbb{R}^n \ / \ g_1(v) = g_2(v)\}$$
$$\mathcal{S} = \{v \in \mathbb{R}^n \ / \ \sigma_1(v) = \sigma_2(v)\}$$

and finally, the submanifold of common singularities of subsystems of system (1):

$$\mathcal{L} = \{x \in \mathbb{R}^n \ / \ f_1(x) = f_2(x) = 0\}$$

The main result that we recall here is given in this theorem.

Theorem 1.

i) If $\mathcal{M}$ is a discrete set then system (1) is observable for any switch σ for which we have $\sigma(\mathcal{L}) \leq 0$ or else $\sigma(\mathcal{L}) > 0$.
ii) If dynamics (2) and (3) are transverse to $\mathcal{M}$ except on a discrete subset then the system is observable for any switch σ for which we have $\sigma(\mathcal{L}) \leq 0$ or else $\sigma(\mathcal{L}) > 0$.
iii) If $\mathcal{S} = \mathbb{R}^n$ then system (1) is observable.

The reader can see (Boutat et *al.*, 2004) for proof and more details. He can find also some algebraic sufficient conditions to analyze the observability of piecewise linear systems.

3. HYBRID OBSERVER

Let us consider the canonical observer form of the following nonlinear autonomous system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = f_i(x) \quad i \in \{1, .., p\} \\ \qquad if \quad \sigma_i(x) \text{ is verified.} \end{cases} \tag{4}$$

where $y = x_1$, and (4) is assumed to be bounded state in finite time.

Remark 1. The assumption of bounded state must concern the full system. Indeed, we can find subsystems perfectly stable, while the global system can be unstable.

Example 1. Consider the two stable subsystems

$$\Sigma_1 : \begin{cases} \dot{x}_1 = -x_2 \\ \dot{x}_2 = x_1 \end{cases} \quad \text{and} \quad \Sigma_2 : \begin{cases} \dot{x}_1 = -x_2 \\ \dot{x}_2 = 4x_1 \end{cases}$$

The hybrid system will be defined as

$$\Sigma = \begin{cases} \Sigma_1 \quad if \quad x_1 x_2 > 0 \\ \Sigma_2 \quad if \quad x_1 x_2 \leq 0 \end{cases}$$

It is easy to verify that each subsystem Σ_1 and Σ_2 is stable and bounded state.(see figure 2) while the system Σ is unstable (see figure 3). This unstability is due to the switching function.

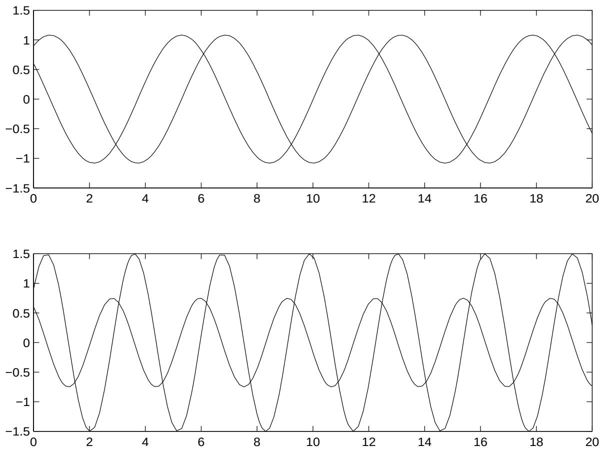

Fig. 2 : x_1, x_2 of Σ_1 and x_1, x_2 of Σ_2

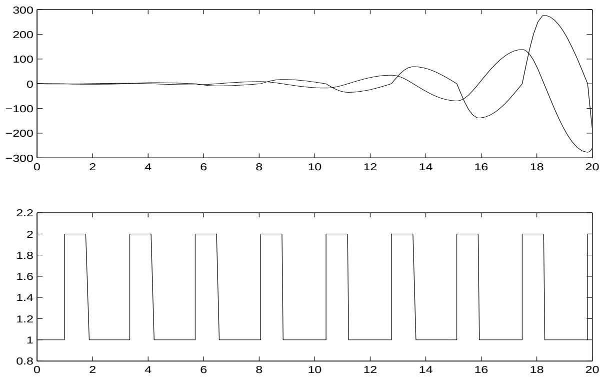

Fig. 3 : x_1 and x_2 and the switching indicator S

From the work (Drakunov and Utkin, 1995) and (Boukhobza et *al.*, 1996; Barbot et *al.*, 1996), we propose the following type of sliding mode observer

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + \lambda_1 sign(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_2 = \hat{x}_3 + E_1\lambda_2 sign_1(\tilde{x}_2 - \hat{x}_2) \\ \vdots \\ \dot{\hat{x}}_{n-1} = \hat{x}_n + E_{n-2}\lambda_{n-1} sign_{n-2}(\tilde{x}_{n-1} - \hat{x}_{n-1}) \\ \dot{\hat{x}}_n = f_i(x_1, \tilde{x}_2 ..., \tilde{x}_n) \\ \qquad +E_{n-1}\lambda_n sign_{n-1}(\tilde{x}_n - \hat{x}_n) \\ \qquad i \in \{1, .., p\} \quad if \quad \sigma_i(x) \text{ is verified.} \end{cases} \tag{5}$$

where $\tilde{x}_i = \hat{x}_i + E_i\lambda_{i-1} sign(x_{i-1} - \hat{x}_{i-1})$ for $i = 2, ..., n-1$, and the $sign(x)$ function denotes the usual sign function. We note that we use a low pass filter of the x variable (Drakunov and Utkin, 1995) and an anti-peaking structure (Boukhobza et *al.*, 1996; Barbot et *al.*, 1996; Khalil, 1996). This anti-peaking structure issues from the idea that we do not inject the observation error information before reaching the sliding manifold linked with this information. Moreover, we reach the manifold one by one. Doing this we obtain a subdynamic of dimension one and consequently, we do not have peaking phenomena (Sussmann and Kokotovic, 1991). More precisely $E_i = 0$ is equal to zero if there exists $j \in \{1, i-1\}$ such that $\tilde{x}_j - \hat{x}_j \neq 0$ (by definition $\tilde{x}_1 = x_1$), else $E_i = 1$.

Theorem 2. Considering the system (4) supposed to be bounded state in finite time, and the observer (5). For any initial conditions $x(0)$, $\hat{x}(0)$, there exists a choice of λ_i such that the observer state $\hat{x}$ converges in finite time to x, and $\sigma(\tilde{x})$ converge to $\sigma(x)$.

Proof: See (Boukhobza et *al.*, 1996) for more details and proof. △

4. SIMULATIONS AND COMMENTS

Example 2. Let us consider the triangular input observer system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = F_i(x) \end{cases} \tag{6}$$

for $i = 1, 2, 3$ with $y = x_1$.
The dynamic F_i is defined as follows

$$F_i = \begin{cases} F_1 & if \quad x_2 < 0 \\ F_2 & if \quad x_2 \geq 0 \end{cases}$$

with $F_1 = -\cos(30x_2) + 0.4$ and $F_2 = -40\cos(300x_3 + \pi/2) - 0.5$. The associated observer (5) takes the form

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + \lambda_1 sign(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_2 &= \hat{x}_3 + E_1\lambda_2 sign(\tilde{x}_2 - \hat{x}_2) \\ \dot{\hat{x}}_3 &= F_i(x) + E_2\lambda_3 sign(\tilde{x}_3 - \hat{x}_3) \end{aligned} \tag{7}$$

with $\tilde{x}_2 = \hat{x}_2 + E_1\lambda_1 sign(x_1 - \hat{x}_1)$ and $\tilde{x}_3 = \hat{x}_3 + \lambda_2 sign(\tilde{x}_2 - \hat{x}_2)$.

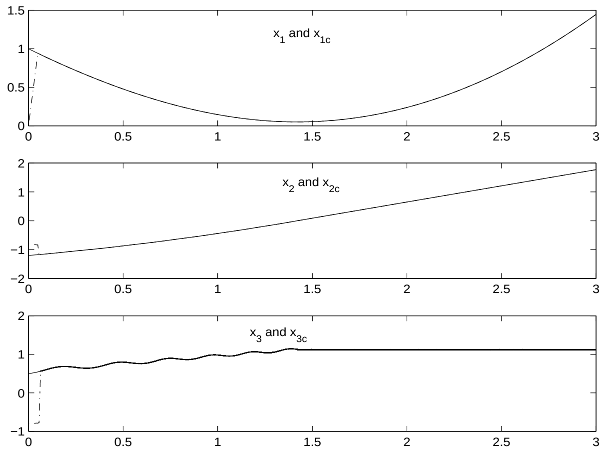

Fig. 4 : The states $x_1,\ x_2,\ x_3$, with x_i and $\hat{x}_i$

The performance of the proposed hybrid observer is shown in figure 4 by the dashed line with the true states shown by the solid line.

In the case when a law pass filter is used for $\tilde{x}_2$ and $\tilde{x}_3$ during the computation of switching condition $\sigma(\tilde{x})$ (see figure 5), the results show a delay occurring for the switching decision. In fact this delay is between the switching indicators S calculated on the basis of $\sigma(x)$ and S_o calculated on the basis of $\sigma(\tilde{x})$. Moreover, the same delay also held between x_2 and $\tilde{x}_2$.

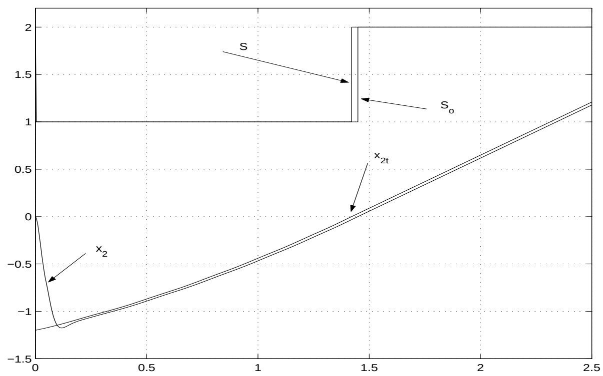

Fig. 5 : Switching indicator S and S_o

Figure 6 represents the same simulation without law pass filter. The delay is completely removed but we have a chattering phenomena which generate some widely commutations (see figure 9 of the next example).

Example 3. Let us consider the following system

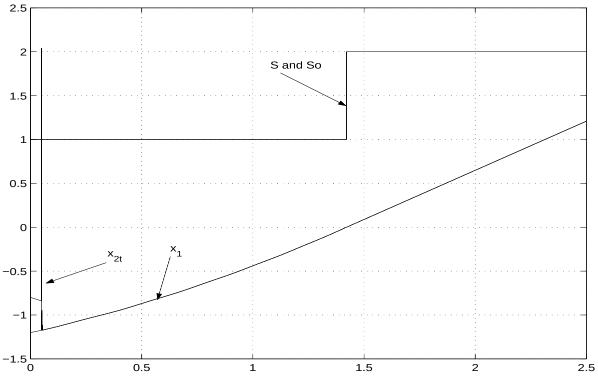

Fig. 6 : Switching indicator S and S_o and switching function $\sigma(x)$ without filter

$$\dot{x} = \begin{cases} A_1x + B_1 & if \quad x_1 \geq 1 \\ A_2x + B_2 & if \quad |x_1| < 1 \\ A_3x + B_3 & if \quad x_1 \leq -1 \end{cases}$$
$$y = Cx = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} x$$

where $A_\alpha = \begin{pmatrix} \alpha & 9 & 0 \\ 1 & -1 & 1 \\ 0 & -\dfrac{100}{7} & 0 \end{pmatrix}$; $B_\beta = \begin{pmatrix} \beta \\ 0 \\ 0 \end{pmatrix}$, with $A_1 = A_{\alpha=-\frac{18}{7}}$, $A_2 = A_{\alpha=\frac{9}{7}}$, and $A_3 = A_{\alpha=-\frac{18}{7}}$, and $B_1 = B_{\beta=\frac{27}{7}}$, $B_2 = B_{\beta=0}$, and $B_3 = B_{\beta=-\frac{27}{7}}$,.
It is easy to verify that each subsystem is observable.
Now to use the canonical form, let us consider the following diffeomorphism:
The system has the general following form:

$$\dot{x}_1 = a_1x_1 + a_2x_2 + a_3$$
$$\dot{x}_2 = x_1 - x_2 + x_3$$
$$\dot{x}_3 = c_1x_2$$

we consider the diffeomophism:

$$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & c_1 & 0 \\ c_1 & -c_1 & c_1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$
$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 & \dfrac{1}{c_1} & \dfrac{1}{c_1} \\ 0 & \dfrac{1}{c_1} & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix}$$

which gives:

$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = z_3$$
$$\dot{z}_3 = \alpha_1z_1 + \alpha_2z_2 + \alpha_3z_3 + \beta$$

with $\alpha_1 = -a_1c_1$, $\alpha_2 = a_1 + a_2 + c_1$, $\alpha_3 = a_1 - 1$ and $\beta = a_3c_1$, and the switching condition: $\sigma(z) = \frac{1}{c_1}(z_3 + z_2) - z_1$.
For all values of α and β, the observer will have the following form

$$\frac{d\hat{z}_1}{dt} = z_2 + \lambda_1 sign(z_1 - \hat{z}_1)$$
$$\frac{d\hat{z}_2}{dt} = z_3 + E_1\lambda_2 sign(\tilde{z}_2 - \hat{z}_2)$$
$$\frac{d\hat{z}_3}{dt} = \alpha_1z_1 + \alpha_2z_2 + \alpha_3z_3 + \beta + E_2\lambda_3 sign(\tilde{z}_3 - \hat{z}_3)$$

with $\tilde{z}_2 = \hat{x}_2 + E_1\lambda_1 sign(z_1 - \hat{z}_1)$ and $\tilde{z}_3 = \hat{z}_3 + E_2\lambda_2 sign(\tilde{z}_2 - \hat{z}_2)$

$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = z_3$$
$$\dot{z}_3 = \begin{cases} P_1(z) & if \quad \sigma(z) \geq 1 \\ P_2(z) & if \quad |\sigma(z)| < 1 \\ P_3(z) & if \quad \sigma(z) \leq -1 \end{cases}$$

with: $P_1(z) = -\frac{1800}{49}z_1 - \frac{55}{7}z_2 - \frac{25}{7}z_3 + \frac{2700}{49}$; $P_2(z) = \frac{100}{49}z_1 - \frac{36}{7}z_2 - \frac{6}{7}z_3$; and $P_3(z) = -\frac{1800}{49}z_1 - \frac{55}{7}z_2 - \frac{25}{7}z_3 + \frac{2700}{49}$.

The chaotic behavior of the considered system is represented by its phase portrait in figure 7 while figure 8 highlights the efficiency of the proposed observer and shows the finite time step by step convergence.

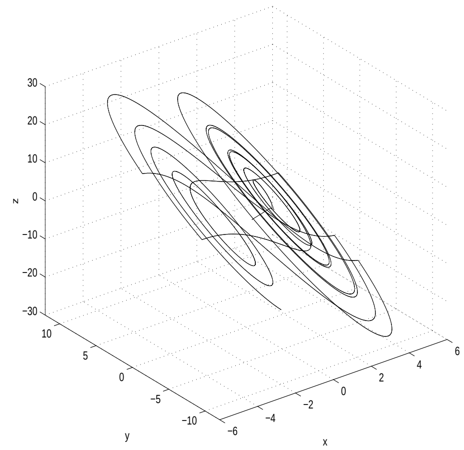

Fig. 7 : Phase portrait of double scroll hybrid system

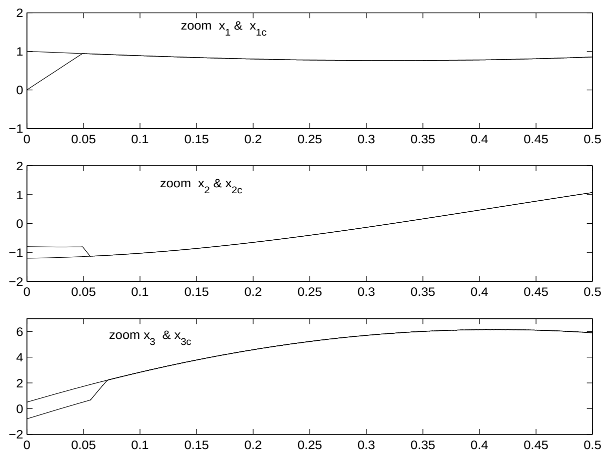

Fig. 8 : Zoom on x_i and $\hat{x}_i$ with $x_{ic} = \hat{x}_i$

Figure 9 shows respectively S, S_o and a zoom of a switching manifold $\sigma(\tilde{x})$ when it is close to the switching value −1. This justifies the undesirable switching appearing on S_o (at t close to 5 s).

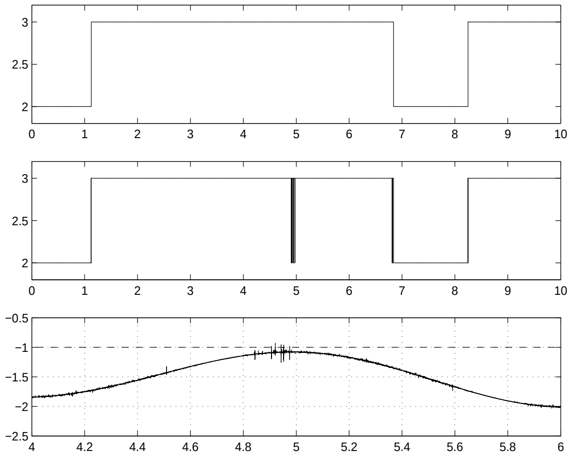

Fig. 9 : Switching indicators S and So and zoom on $\sigma(\tilde{x})$

5. CONCLUSION

In this paper, a sliding mode observer for a class of hybrid systems is proposed. The considered class is bounded state in finite time without jumps and does't concern Zeno phenomena. A step by step sliding mode observer was used for the main both reasons: the final time convergence and the ability to take into account naturally the variable structure. Nevertheless, some difficulties as chattering phenomena occur. It induces some irrelevant decision of switching between the subsystems when the trajectory is in the neighborhood of the switching manifold. Obviously, this problem can be overcome by using a law pass filter during the computation of the equivalent vector; unfortunately, this solution introduces a delay. For future works, the authors intend to consider non autonomous hybrid systems with and without jumps.

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