

DISTURBANCE REJECTION WITH LTI INTERNAL MODELS FOR PASSIVE NONLINEAR SYSTEMS

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Abstract: First, the internal model principle is applied for plants that are passive nonlinear systems, to solve a disturbance rejection problem (without reference signal), where the exogeneous signal is a finite superposition of sine waves of arbitrary known frequencies. The proposed controller assures that both the state of the plant and the error signal converge to zero.

Based on the above result, we solve the tracking and disturbance rejection problem for a class of fully actuated passive mechanical systems. Here, we combine our first result with the ideas behind the *Slotine-Li* controller. We assume that the reference signal r and its first two derivatives are available to the controller. The internal model-based compensator that we propose causes the tracking error to converge to zero. *Copyright*© 2005 IFAC

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1. INTRODUCTION

The internal model principle for LTI systems suggests that the dynamic structure of the exosystem must be included in the controller (see also (Francis and Wonham, 1975)). For example, to eliminate the steady-state error for step reference or disturbance signals, we need integrators in the loop. If an internal model with transfer function $s/(s^2 + \omega^2)$ (with suitable multiplicity) is in the feedback loop and the closed-loop system is stable, then we obtain tracking and/or disturbance rejection for sinusoidal reference and disturbance signals of frequency ω . If the reference and disturbance signals are periodic, then the internal model principle leads to repetitive

control (see for example (S. Hara, 1988), (Weiss and Hafele, 1999)).

The idea of internal model has been generalized for output regulation of nonlinear systems by Byrnes *et al.* (C.I. Byrnes, 1997). The controller design relies on the solution of the regulator equations, which make this controller design impractical for many systems. This drawback has been relaxed by Huang in (Huang and Lin, 1994) where he only requires the approximate solution of the regulator equations in designing the controller, at the price of having a small steady state error. Other results related to the output regulation problem for nonlinear systems can also be found in (Byrnes and Isidori, 2003), (Huang and Rugh, 1992) and (Priscoli, 2004).

In the first part of this paper, we propose a simple controller design method leading to an LTI controller (based on the internal model principle) for a

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disturbance rejection problem for nonlinear passive plants. In the second part of the paper, we combine this LTI controller with a *Slotine-Li* type controller (Slotine and Li, 1988) used for tracking a smooth reference signal r . Here, we assume that the plant is a fully actuated mechanical system, and the signals r , \dot{r} and \ddot{r} are available.

Passive systems have a \mathcal{C}^1 storage function H (defined on the state space) which has the intuitive meaning of stored energy. The input signal u and the output signal y take values in the same inner product space. We denote the state of the system at time t by $x(t)$. The defining property of a passive system is that

$$\dot{H} \leq \langle y, u \rangle, \quad \text{where } \dot{H} = \left\langle \frac{\partial H(x)}{\partial x}, \dot{x} \right\rangle. \quad (1)$$

The function H is often used as a Lyapunov function in analyzing the system stability. The interconnection of several passive systems leads to a passive closed-loop system if the interconnection is neutral with respect to the power supply, see (van der Schaft, 2000). Many physical systems (electrical circuits, mechanical systems, etc.) are passive if the input and output variables are chosen carefully such that their product represents the flow of power into the system.

For nonlinear plants, passivity can be used for controller design, see for example (Byrnes and Isidori, 1991), (R. Ortega and Sira-Ramirez, 1998) and (van der Schaft, 2000).

2. PRELIMINARIES

Notation. Throughout this paper, the inner product on any Hilbert space is denoted by $\langle \cdot, \cdot \rangle$ and $\mathbb{R}_+ = [0, \infty)$. For a finite-dimensional vector x , we use the norm $\|x\| = (\sum_n |x_n|^2)^{\frac{1}{2}}$ and for matrices, we use the operator norm induced by $\|\cdot\|$ (the largest singular value). For any $\epsilon \geq 0$, we denote $\mathbf{B}_\epsilon = \{x \in \mathbb{R}^n \mid \|x\| \leq \epsilon\}$. For a square matrix A , $\sigma(A)$ denotes the set of its eigenvalues.

The set $L^2[0, \infty)$ denotes the space of all the measurable functions $f : \mathbb{R}_+ \rightarrow \mathbb{R}$ which satisfy $\int_0^\infty |f(t)|^2 dt < \infty$. We denote by f_T the truncation of f to the interval $[0, T]$. The space $L^2_{loc}[0, \infty)$ consists of all the measurable functions $f : \mathbb{R}_+ \rightarrow \mathbb{R}$ such that $f_T \in L^2[0, \infty)$, for all $T > 0$. For any finite-dimensional vector space \mathcal{V} endowed with a norm $\|\cdot\|_{\mathcal{V}}$, the space $L^2([0, \infty), \mathcal{V})$ consists of all the measurable functions $f : \mathbb{R}_+ \rightarrow \mathcal{V}$ such that $\int_0^\infty \|f(t)\|_{\mathcal{V}}^2 dt < \infty$. The space $\mathcal{C}^1(\mathbb{R}^l, \mathbb{R}^p)$ consists of continuously differentiable functions $f : \mathbb{R}^l \rightarrow \mathbb{R}^p$, while $\mathcal{C}^2[0, \infty)$ consists of all the twice continuously differentiable functions $r : \mathbb{R}_+ \rightarrow \mathbb{R}$.

We consider a nonlinear plant \mathbf{P} described by

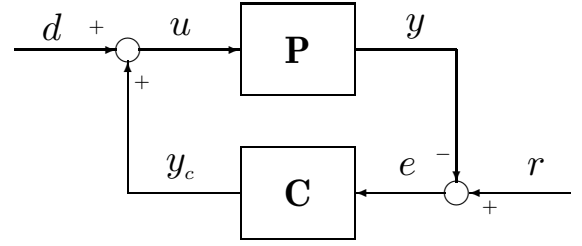


Figure 1. The tracking and disturbance rejection problem for a certain class of signals d and r is to find a controller \mathbf{C} for the plant \mathbf{P} such that this closed-loop system \mathbf{L} is stable and $e(t) \rightarrow 0$ as $t \rightarrow \infty$.

$$\dot{x} = f(x) + g(x)u, \quad (2)$$

$$y = h(x), \quad (3)$$

where the state x , the input u and the output y are functions of $t \geq 0$, such that $x(t) \in \mathbb{R}^n$, $u(t), y(t) \in \mathbb{R}^m$, $m \leq n$, $f \in \mathcal{C}^1(\mathbb{R}^n, \mathbb{R}^n)$ with $f(0) = 0$, $g \in \mathcal{C}^1(\mathbb{R}^n, \mathbb{R}^{n \times m})$, $g(x)$ has rank m for all $x \in \mathbb{R}^n$ and $h \in \mathcal{C}^1(\mathbb{R}^n, \mathbb{R}^m)$ with $h(0) = 0$. We assume that there exists a storage function $H \in \mathcal{C}^1(\mathbb{R}^n, \mathbb{R}_+)$ such that

$$\frac{\partial H(x)}{\partial x} [f(x) + g(x)u] \leq \langle h(x), u \rangle. \quad (4)$$

Equivalently, the storage function H satisfies the *Hill-Moylan conditions*:

$$\frac{\partial H(x)}{\partial x} f(x) \leq 0, \quad \frac{\partial H(x)}{\partial x} g(x) = h^T(x), \quad (5)$$

see (van der Schaft, 2000). H is called *proper* if $H(x) \rightarrow \infty$ whenever $\|x\| \rightarrow \infty$.

\mathbf{P} is said to be *zero-state observable* if $u(t) = 0, y(t) = 0$ for all $t \geq 0$ implies that $x(t) = 0$ for all $t \geq 0$, and \mathbf{P} is *zero-state detectable* if $u(t) = 0, y(t) = 0$ for all $t \geq 0$ implies that $\lim_{t \rightarrow \infty} x(t) = 0$.

Consider the feedback system \mathbf{L} as shown in Figure 1, which consists of the above plant \mathbf{P} with a controller \mathbf{C} and with $r = 0$. If we regard y as the output function of the closed-loop system and use a proportional gain $K = K^* > 0$ as our controller, i.e., $y_c = -Ky$, then the closed-loop system becomes *strictly output passive*, which means that

$$\dot{H} \leq \langle y, d \rangle - K\|y\|^2. \quad (6)$$

Remark 2.1. (van der Schaft, 2000) If \mathbf{P} is described by (2), (3), $H \in \mathcal{C}^1(\mathbb{R}^n, \mathbb{R}_+)$ and \mathbf{L} satisfies (6), then \mathbf{L} has L^2 -gain $\leq 1/K$ (from d to y), as defined in (van der Schaft, 2000). Indeed, it can be shown that

$$\|y_T\| \leq \frac{1}{K} \|d_T\| + \sqrt{\frac{2}{K} H(x(0))}, \quad (7)$$

for all $T > 0$.

The following proposition is a consequence of Lemma 3.2.8 in (van der Schaft, 2000).

Proposition 2.2. Suppose that the plant \mathbf{P} is described by (2), (3), with $H(x) > 0$ for all $x \neq 0$,

$H(0) = 0$ and \mathbf{P} is zero-state detectable. Let the controller \mathbf{C} be a proportional gain $K = K^* > 0$. Consider the feedback system \mathbf{L} as in Figure 1, with $r = 0$ and $d = 0$. Then the origin of \mathbf{L} is asymptotically stable. If H is proper, then the origin of \mathbf{L} is globally asymptotically stable.

PROOF. The closed-loop system \mathbf{L} is described by $\dot{x} = f_L(x) + g(x)d$, $y = h(x)$, where $f_L(x) = f(x) - g(x)Kh(x)$, which is of class \mathcal{C}^1 and $f_L(0) = 0$. It is easy to see that \mathbf{L} is zero-state detectable. It follows from (6) that for $d = 0$, $\dot{H} \leq -K\|y\|^2 \leq 0$ for all $t \geq 0$. This implies (using H as a Lyapunov function) that \mathbf{L} is stable. It follows that there exists $\delta > 0$ such that $x(0) \in \mathbf{B}_\delta \Rightarrow x(t) \in \mathbf{B}_1$ for all $t \geq 0$. By the La-Salle invariance principle (van der Schaft, 2000) and the zero-state detectability of \mathbf{L} , it is easy to derive that the origin of \mathbf{L} is asymptotically stable.

When H is proper, then every state trajectory of \mathbf{L} with $d = 0$ remains bounded, as it is easy to see. Thus, for any state trajectory x , we can apply the preceding argument with \mathbf{B}_1 replaced by a ball \mathbf{B}_ϵ that contains this state trajectory. Then, we conclude that $\lim_{t \rightarrow \infty} x(t) = 0$. \square

3. THE INTERNAL MODEL PRINCIPLE IN PASSIVE SYSTEMS

Let us consider the feedback system in Figure 1, where $r = 0$ and the disturbance d is given by

$$\begin{aligned} \dot{w} &= Sw \\ d(t) &= C_w w(t) \end{aligned} \quad (8)$$

where $C_w \in \mathbb{R}^{m \times p}$, $w(t) \in \mathbb{R}^p$ is the exosystem state, $S \in \mathbb{R}^{p \times p}$ has its eigenvalues on the imaginary axis and e^{St} is uniformly bounded for $t \geq 0$. An equivalent way of expressing our assumptions on S is the following: $\sigma(S) \subset i\mathbb{R}$ and all its Jordan blocks are of dimension 1 (i.e., there are no generalised eigenvectors for S).

Let the plant \mathbf{P} be defined as in (2), (3). We choose the controller \mathbf{C} as follows:

$$\dot{x}_c = Ax_c + Be, \quad y_c = B^*x_c + De, \quad (9)$$

where $x_c \in \mathbb{R}^l$, $l \geq p$, $e \in \mathbb{R}^m$, $y_c \in \mathbb{R}^m$, $A \in \mathbb{R}^{l \times l}$, $A^* + A = 0$, $B \in \mathbb{R}^{l \times m}$, (B^*, A) is observable and $D = kI^{m \times m}$, $k > 0$. Let \mathbf{L} be the closed-loop system as in Figure 1, with $u = d + y_c$ and $e = -y$. Then the closed-loop system \mathbf{L} is described by

$$\begin{aligned} \dot{x} &= f(x) + g(x)B^*x_c - kg(x)h(x) + g(x)d, \\ \dot{x}_c &= Ax_c - Bh(x), \quad y = h(x). \end{aligned} \quad (10)$$

The controller \mathbf{C} solves the *disturbance rejection problem* for the plant \mathbf{P} with d as in (8) if, in the closed-loop system \mathbf{L} shown in Figure 1 with $r = 0$,

all state trajectories of the closed-loop system are bounded and $x \rightarrow 0$ as $t \rightarrow \infty$ (and hence $y \rightarrow 0$ as $t \rightarrow \infty$). \mathbf{C} solves the *disturbance rejection problem locally*, if it solves the disturbance rejection problem for all initial conditions $x(0)$ in some neighborhood of the origin and for $x_c(0) \in \mathcal{X}_c$ (an open set which may be dependent on $w(0) \in \mathbb{R}^p$). \mathbf{C} solves the *disturbance rejection problem globally*, if it solves the disturbance rejection problem for any initial conditions $x(0) \in \mathbb{R}^n$ and $x_c(0) \in \mathbb{R}^m$.

Lemma 3.1. Suppose that the plant \mathbf{P} defined by (2), (3) is zero-state detectable. Let the controller \mathbf{C} be given by (9) and consider the control system \mathbf{L} as in Figure 1, with $r = 0$. Then, the following two conditions are equivalent.

- (1) \mathbf{L} is zero-state detectable (with output y).
- (2) For any $x_{co} \in \mathbb{R}^l$, $x_{co} \neq 0$ and for any $x(0) = x_0 \in \mathbb{R}^n$, the plant \mathbf{P} satisfies $u(t) = B^*e^{At}x_{co} \Rightarrow \exists t \geq 0$ such that $y(t) \neq 0$.

PROOF. The proof (1) \Rightarrow (2) is omitted due to space constraint. For our main result, it is sufficient to show that (2) \Rightarrow (1). From Figure 1, if $y = 0$ and $d = 0$, then we have

$$u(t) = y_c(t) = B^*e^{At}x_c(0) \quad \text{where } x_c(0) \in \mathbb{R}^l.$$

This together with the condition (2) implies that $x_c(0) = 0$, hence $x_c(t) = 0$ for all $t \geq 0$ and $u = 0$. By the zero-state detectability of \mathbf{P} , $u = 0$ and $y = 0$ implies that $x(t) \rightarrow 0$ as $t \rightarrow \infty$. Hence, $\begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix} \rightarrow 0$ as $t \rightarrow \infty$. \square

Theorem 3.1. Let the plant \mathbf{P} defined by (2), (3) be zero-state detectable with a storage function H such that $H(x) > 0$ for $x \neq 0$, $H(0) = 0$ and satisfying (4). Let the controller \mathbf{C} be given by (9) and consider the control system \mathbf{L} as in Figure 1, with $r = 0$. We assume that \mathbf{P} has property (2) from Lemma 3.1. Suppose that the disturbance d is generated by the exosystem (8). Then \mathbf{C} solves the disturbance rejection problem if and only if there exists a matrix $\Sigma \in \mathbb{R}^{l \times p}$ which satisfies

$$\Sigma S = A\Sigma \quad \text{and} \quad B^*\Sigma + C_w = 0. \quad (12)$$

If such a Σ exists, then it is unique.

PROOF. The necessity proof is omitted due to space constraint. The proof can be obtained by observing that if \mathbf{C} solves the disturbance rejection problem, then $[w \ x \ x_c]^T$ is bounded and converges to an invariant set Ω , where $x = 0$ in Ω . In Ω , we have $x = 0$ and $x_c = \Sigma w$, and Σ satisfies (12).

(Sufficiency) First, let us denote $\rho = x_c - \Sigma w$, then by evaluating (10) – (12) and using x, ρ as the state variables of \mathbf{L} , we have

$$\dot{x} = f(x) + g(x)B^*\delta - kg(x)h(x), \quad (13)$$

$$\dot{\rho} = A\rho - Bh(x), \quad y = h(x). \quad (14)$$

Consider the storage function $H_{cl}(x, \rho) = H(x) + \frac{1}{2}\|\rho\|^2$. Then, using the Hill-Moylan conditions in (5) and (13) – (14), $\dot{H}_{cl} = -k\|y\|^2$. By the assumptions of the theorem and using Lemma 3.1, the system described by (13) – (14) is zero-state detectable. Then, by using the same argument as in Proposition 2.2, there exists $\delta > 0$ such that $\begin{bmatrix} x(0) \\ \rho(0) \end{bmatrix} \in \mathbf{B}_\delta \Rightarrow \begin{bmatrix} x(t) \\ \rho(t) \end{bmatrix} \rightarrow 0$ as $t \rightarrow \infty$. This implies $x_c - \Sigma w \rightarrow 0$ and $y \rightarrow 0$. \mathbf{C} solves the disturbance rejection problem locally. Moreover, if H is proper, it implies that H_{cl} is also proper. Then, by using the same argument as in Proposition 2.2, it can be shown that \mathbf{C} solves the disturbance rejection problem globally.

It is easy to derive that Σ as in (12) is unique. The proof is omitted due to space constraint. \square

Remark 3.2. The solution of nonlinear regulator equations, as given in (Byrnes and Isidori, 1991), for the nonlinear plant \mathbf{P} above is trivial, i.e., the mappings $\pi(w) = 0$ and $c(w) = -C_w w$ satisfy

$$\begin{aligned} \frac{\partial \pi}{\partial w} S w = 0 &= f(\pi(w)) + g(\pi(w))(c(w) + d) \\ 0 &= h(\pi(w)). \end{aligned}$$

Let $\chi(s) = s^q + a_{q-1}s^{q-1} \dots + a_1s + a_0$ be the minimal polynomial of $S \in \mathbb{R}^{p \times p}$, so that

$$S^q + a_{q-1}S^{q-1} + \dots + a_2S^2 + a_1S + a_0 = 0, \quad (15)$$

where $a_{q-1}, \dots, a_0 \geq 0$, $q \leq p$ and χ has only simple zeros, all on $i\mathbb{R}$.

Suppose that $S_{min} \in \mathbb{R}^{q \times q}$ is such that $S_{min} + S_{min}^* = 0$ and its characteristic polynomial is χ . If $0 \in \sigma(S)$, then the simplest choice would be

$$S_{min} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 0 & \Omega_1 & 0 & \dots & 0 \\ 0 & 0 & \Omega_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \Omega_\nu \end{bmatrix}, \quad (16)$$

where for each $k = 1, \dots, \nu$, $\Omega_k = \begin{bmatrix} 0 & -\omega_k \\ \omega_k & 0 \end{bmatrix}$ for some $\omega_k \in \mathbb{R} \setminus \{0\}$ and $\omega_k \neq \omega_j$ for $k \neq j$. The set $\sigma(S_{min}) = \sigma(S)$ contains 0 and $\pm i\omega_k$ ($k = 1, \dots, \nu$) (0 and ω_k are the known frequencies of the disturbance signal). If $0 \notin \sigma(S)$, then we omit the first line and the first column in (16), so that $\sigma(S_{min})$ contains only $\pm i\omega_k$.

For $i = 1, \dots, m$, let $\Gamma_i \in \mathbb{R}^{q \times 1}$ be such that (Γ_i^*, S_{min}) is observable (the m vectors Γ_i may be taken equal).

Consider the controller \mathbf{C} described in (9) where $x_c \in \mathbb{R}^{qm}$ (q is as in (15)), the matrices $A \in \mathbb{R}^{qm \times qm}$ and $B \in \mathbb{R}^{qm \times m}$ are given by

$$A = \begin{bmatrix} S_{min} & 0 & \dots & 0 \\ 0 & S_{min} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & S_{min} \end{bmatrix}, B = \begin{bmatrix} \Gamma_1 & 0 & \dots & 0 \\ 0 & \Gamma_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \Gamma_m \end{bmatrix}, \quad (17)$$

and $D = kI^{m \times m}$ where $k > 0$.

Theorem 3.3. Suppose that the plant \mathbf{P} defined by (2), (3) satisfies (4) (passivity) with a storage function H such that $H(x) > 0$ for $x \neq 0$, $H(0) = 0$. Assume that \mathbf{P} is zero-state detectable. Suppose that the disturbance d is generated by the exosystem (8) and denote by χ the minimal polynomial of S . Let the controller \mathbf{C} be given by (9) where the matrices A, B and D are as in (17), S_{min} has characteristic polynomial χ and satisfies $S_{min} + S_{min}^* = 0$. Consider the control system \mathbf{L} as in Figure 1, with $r = 0$. We assume that \mathbf{P} has property (2) from Lemma 3.1. Then \mathbf{C} solves the disturbance rejection problem locally for \mathbf{P} . Moreover, if H is proper, then \mathbf{C} solves the disturbance rejection problem globally.

PROOF. The theorem is proved by showing that there exists a mapping Σ satisfying (12) and then using Theorem 3.1. It can be shown that

$$\Sigma = -(\phi_c)^{-1}\phi_w,$$

where

$$\phi_c = \begin{bmatrix} B^* \\ B^*A \\ \vdots \\ B^*A^{q-1} \end{bmatrix}, \quad \phi_w = \begin{bmatrix} C_w \\ C_w S \\ \vdots \\ C_w S^{q-1} \end{bmatrix},$$

satisfies both identities in (12). The proof is omitted due to space constraint. \square

Remark 3.4. Theorem 3.3 shows that we can design a controller \mathbf{C} knowing only the minimal polynomial of the exosystem, and \mathbf{C} achieves asymptotic disturbance rejection for \mathbf{P} . The simple choice of S_{min} as in (16) can be used in Theorem 3.3 where ω_k is assigned based on the known frequencies of the disturbance signal. The proposed high-order compensator assures that the identity (12) have a (unique) solution.

4. TRACKING AND DISTURBANCE REJECTION IN FULLY-ACTUATED MECHANICAL SYSTEMS

We consider a plant \mathbf{P} described by the second-order differential equation

$$\mathcal{M}(q)\ddot{q} + \mathcal{D}(q, \dot{q})\dot{q} + g(q) = u, \quad (18)$$

which often corresponds to a fully actuated mechanical system, see (A. Astolfi, 1997), (Koivo, 1989). Here, $q \in \mathbb{R}^n$ is the vector of generalized coordinates, $\mathcal{M}(q)$ is a positive definite inertia matrix

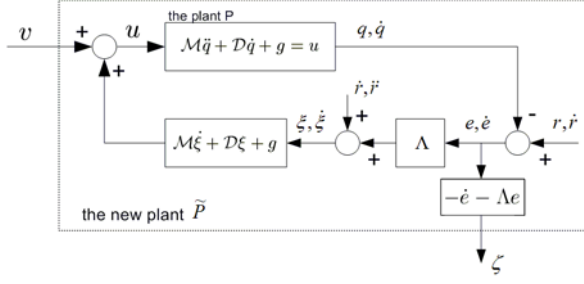


Figure 2. The new plant obtained after the feedback (20), which contains the signal generator for the reference r . The tracking error is e . This is a time-varying passive system with input v , state $[e \ \zeta]^T$, and output ζ .

which satisfies $m_1 I \leq M(q) \leq m_2 I$ for some positive constants m_1 and m_2 , $g(q)$ is a continuous function (which usually represents forces due to the potential energy) and $u \in \mathbb{R}^n$ is the input (usually, forces or torques). The function $\mathcal{M}(\cdot)$ is assumed to be continuously differentiable and $\mathcal{D}(\cdot, \cdot)$ is assumed to be continuous. As usual, we denote $\dot{\mathcal{M}}(q, \dot{q}) = \sum_{j=1}^n \frac{\partial \mathcal{M}}{\partial q_j} \dot{q}_j$. The state of this system is the vector $\begin{bmatrix} q \\ \dot{q} \end{bmatrix}$. We assume that $J(q, \dot{q}) = \dot{\mathcal{M}}(q, \dot{q}) - 2\mathcal{D}(q, \dot{q})$ satisfies $J^*(q, \dot{q}) + J(q, \dot{q}) \leq 0$, so that

$$\left\langle \left(\frac{1}{2} \dot{\mathcal{M}} - \mathcal{D} \right) \zeta, \zeta \right\rangle \leq 0 \quad \forall \zeta \in \mathbb{R}^n. \quad (19)$$

Note that the matrix $J(q, \dot{q})$ may contain terms which are due to friction from $\mathcal{D}(q, \dot{q})$.

Let us describe a first feedback loop which is based on the *Slotine-Li* controller and which eliminates r from the picture, so that the problem is reduced to the disturbance rejection problem discussed in Section 3. We assume that $r \in \mathcal{C}^2([0, \infty), \mathbb{R}^n)$ and the signals \dot{r}, \ddot{r} are available to the controller. Consider the static feedback law

$$u = \mathcal{M}(q)\dot{\xi} + \mathcal{D}(q, \dot{q})\xi + g(q) + v, \quad (20)$$

where

$$\xi := \dot{r} + \Lambda(r - q), \quad \Lambda = \Lambda^* > 0$$

and v is a new input signal, see Figure 2. Substitution of (20) into (18) gives

$$\mathcal{M}(q)\dot{\zeta} + \mathcal{D}(q, \dot{q})\zeta = v, \quad (21)$$

where $\zeta = \dot{q} - \xi$. A simple computation shows that denoting $e = r - q$,

$$-\dot{e} - \Lambda e = \zeta. \quad (22)$$

For this new system $\tilde{\mathbf{P}}$ (shown in Figure 2), we may choose e and ζ as state variables and then the system is described by the differential equations (21) and (22). The old state variables q and \dot{q} may be expressed in terms of the new ones: $q = r - e$, $\dot{q} = \dot{r} + \zeta + \Lambda e$ (remember that r and \dot{r} are regarded

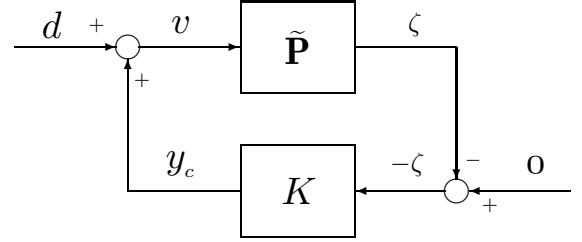


Figure 3. The modified plant $\tilde{\mathbf{P}}$ with a proportional gain controller K and a disturbance d .

as known functions) and then it is possible to rewrite (21) as

$$\mathcal{M}_r(e)\dot{\zeta} + \mathcal{D}_r(e, \zeta)\zeta = v, \quad (23)$$

where, by definition,

$$\mathcal{M}_r(e) = \mathcal{M}(r - e), \quad \mathcal{D}_r(e, \zeta) = \mathcal{D}(r - e, \dot{r} + \zeta + \Lambda e).$$

We denote

$$\dot{\mathcal{M}}_r = \frac{\partial \mathcal{M}_r}{\partial t} + \sum_{j=1}^n \frac{\partial \mathcal{M}_r}{\partial e_j} \dot{e}_j,$$

where $\frac{\partial \mathcal{M}_r(e)}{\partial t} = \sum_{j=1}^n \frac{\partial \mathcal{M}}{\partial q_j} (r - e) \dot{r}_j$. Note that $\dot{\mathcal{M}}_r(e) = \dot{\mathcal{M}}(r - e)$, so that (as in (19)),

$$\left\langle \left(\frac{1}{2} \dot{\mathcal{M}}_r - \mathcal{D}_r \right) \zeta, \zeta \right\rangle \leq 0, \quad \forall \zeta \in \mathbb{R}^n. \quad (24)$$

Using $\tilde{H}(e, \zeta) = \frac{1}{2} \langle \mathcal{M}_r(e) \zeta, \zeta \rangle$ as a storage function, the new plant $\tilde{\mathbf{P}}$ is a time-varying passive system with input v , state $\begin{bmatrix} e \\ \zeta \end{bmatrix}$ and output ζ . Indeed, using (24),

$$\dot{\tilde{H}} = -\langle \zeta, \mathcal{D}_r \zeta \rangle + \langle \zeta, v \rangle + \frac{1}{2} \langle \dot{\mathcal{M}}_r \zeta, \zeta \rangle \leq \langle \zeta, v \rangle.$$

Assume that a disturbance d acts on the new system $\tilde{\mathbf{P}}$ and we connect a proportional controller to it, as shown in Figure 3, with $y_c = -K\zeta$, $K = K^* > 0$. Then the closed-loop system is strictly output passive with input d and output ζ . Thus, for every $d \in L^2[0, \infty)$, the signal ζ will be in $L^2[0, \infty)$, as follows from Remark 2.1. Since $\dot{e} = -\Lambda e - \zeta$, $\Lambda = \Lambda^* > 0$, and $\zeta \in L^2[0, \infty)$, it follows that $e \in L^2[0, \infty)$ and $\dot{e} \in L^2[0, \infty)$. This implies that $e(t) \rightarrow 0$ as $t \rightarrow \infty$.

Suppose now that the disturbance d is as in (8). In practice, the disturbance d can be produced, for example, if an unknown load is given to the manipulator while tracking a periodic signal r .

Proposition 4.1. Consider the system \mathbf{P} as in (18) with outputs q and \dot{q} , the reference $r \in \mathcal{C}^2([0, \infty), \mathbb{R}^n)$ and the disturbance d generated by the exosystem (8).

Let the controller \mathbf{C} be given by the state equation

$$\dot{x}_c = Ax_c - B\zeta, \quad (25)$$

where $x_c \in \mathbb{R}^l$, $l \geq p$, $\zeta \in \mathbb{R}^n$ as in (22), $A \in \mathbb{R}^{l \times l}$, $A^* + A = 0$, $B \in \mathbb{R}^{l \times n}$. The controller generates the signal

$$y_c = \mathcal{M}(q)\dot{\xi} + \mathcal{D}(q, \dot{q})\xi + g(q) + B^*x_c - k\zeta,$$

where $y_c \in \mathbb{R}^n$, $\xi := \dot{r} + \Lambda e$ and $k > 0$.

If (B^*, A) is detectable and there exists $\Sigma \in \mathbb{R}^{l \times p}$ which satisfies

$$\Sigma S = A\Sigma \quad \text{and} \quad B^*\Sigma + C_w = 0, \quad (26)$$

then trajectory of the closed-loop system \mathbf{L} as in Figure 3 with state variables (e, ζ, x_c) is bounded and $\lim_{t \rightarrow \infty} e(t) = 0$.

PROOF. Let us denote $\rho = x_c - \Sigma w$, then by evaluating (23), (25) – (26) and (8), we have

$$\mathcal{M}_r \dot{\zeta} = -\mathcal{D}_r \zeta + B^* \rho - k\zeta, \quad (27)$$

$$\dot{\rho} = A\rho - B\zeta, \quad (28)$$

$$\dot{e} = -\rho e - \zeta, \quad (29)$$

where $[e \ \zeta \ \rho]^T$ is the new state of the closed-loop system.

By using the storage function $H_{cl} = \frac{1}{2} \langle \mathcal{M}_r(e)\zeta, \zeta \rangle + \frac{1}{2} \|\rho\|^2$, it can be easily evaluated that $\dot{H}_{cl} \leq -k\|\zeta\|^2$. The system described by (27) – (29) is zero-state detectable (with output ζ), as can be easily verified. By La-Salle invariance principle, the state $[e \ \zeta \ \rho]^T$ converges to zero. Indeed, by Remark 2.1, $\|\zeta_T\| \leq \sqrt{\frac{2}{k} H_{cl}(e(0), \zeta(0), \rho(0))}$ for all $T \geq 0$ which implies $\zeta \in L^2[0, \infty)$. Since $\dot{e} = -\Lambda e - \zeta$, $\Lambda = \Lambda^* > 0$, and $\zeta \in L^2[0, \infty)$, it follows that $e \in L^2[0, \infty)$ and $\dot{e} \in L^2[0, \infty)$. This implies that $e(t) \rightarrow 0$ as $t \rightarrow \infty$. \square

Similar to the development of high-order compensator as in Theorem 3.3, we can have the high-order compensator version of Proposition 4.1.

Proposition 4.2. Consider the system \mathbf{P} as in (18) with outputs q and \dot{q} and the reference $r \in \mathcal{C}^2([0, \infty), \mathbb{R}^n)$. Suppose that the disturbance d is generated by the exosystem (8) and denote by χ the minimal polynomial of S .

Let the controller \mathbf{C} be given by the state equation

$$\dot{x}_c = Ax_c - B\zeta, \quad (30)$$

where $x_c \in \mathbb{R}^{qn}$ (q is the degree of χ), $\zeta \in \mathbb{R}^n$ is as in (22). The matrices $A \in \mathbb{R}^{qn \times qn}$ and $B \in \mathbb{R}^{qn \times n}$ are as in (17) where S_{min} has characteristic polynomial χ and satisfies $S_{min} + S_{min}^* = 0$. Let the controller output be given by

$$y_c = \mathcal{M}(q)\dot{\xi} + \mathcal{D}(q, \dot{q})\xi + g(q) + B^*x_c - k\zeta,$$

where $y_c \in \mathbb{R}^n$, $\xi := \dot{r} + \Lambda e$ and $k > 0$.

Then trajectory of the closed-loop system \mathbf{L} as in Figure 3 with state variables (e, ζ, x_c) is bounded and $\lim_{t \rightarrow \infty} e(t) = 0$.

PROOF. The theorem is proved by using Proposition 4.1 where the solution of (26) can be obtained

by using a similar approach to the proof in Theorem 3.3, hence it is omitted. \square

5. CONCLUSION

In this paper, internal model-based compensator has been presented for dealing with the disturbance of a finite superposition of sine waves of arbitrary known frequencies for passive nonlinear plants. The internal model-based compensator is used to solve the tracking and disturbance rejection problem for a class of fully actuated passive mechanical systems.

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