SMOOTH SLIDING MODE CONTROL FOR CONSTRAINED MANIPULATOR WITH JOINT FLEXIBILITY

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Abstract: This paper presents a smooth sliding mode control plus backstepping method for the motion/force control of a constrained robotic manipulator with flexible joints. The control scheme is presented based on that overall system parameters are with uncertainties whereas only the constrained force, positions and velocities of links and rotors are measurable. The smooth sliding controller can achieve zero motion and force tracking errors. The simulation results for a planar robot illustrate the expected satisfactory performance.

Keywords: Nonlinear Control, Sliding Mode Control, Robotics, Manipulators.

1. INTRODUCTION

In many industrial tasks such as deburring, writing, grinding, painting, etc, it is necessary to drive the robot's end-effector to move along the environmental constraint and track both the desired force and the desired motion. The initial study on this issue was conducted in (Hemami and Wyman, 1979), and then a number of approaches were proposed. These theoretical frameworks and stability analysis for constrained robots have stemmed from the reduced-state position/force control described by (McClamroch and Wang, 1988). In (Su, *et al.,* 1995; Kwan, 1996; Yu and Lloyd, 1997), adaptive control has been proposed to deal with parametric uncertainty in the form of a robust approach. However, in most schemes the concerned force tracking error is either arbitrarily small at best by using high gain of force feedback or steered to zero under the condition of persistent excitation (PE). In order to achieve zero motion/force errors, some methods shown in (Lian and Lin, 1998; Yuan, 1997; Yao, *et al.*, 1994) use different error space decomposition techniques, i.e., the motion/force decomposition or the nonlinear curvilinear coordination transformation, etc.

Most papers on the manipulator control neglect the actuator dynamics. Actually, robot manipulators are usually with flexible joints due to gear elasticity, shaft windup, etc. It was pointed out in (Spong and Vidyasagar, 1989; Spong, 1989) that the joint flexibility must be considered in designing the control law if high performance is to be achieved. For a constrained flexible joint robot (FJR), the rigid manipulator subsystem is driven by the rotor subsystem. The dynamics form performs a strictly feedback structure. Based on this feature, the integrator backstepping control technique would be straight forward applied in the problem. However, the full-order model with parametric uncertainty results that the controller is difficult designed by only using reasonable measurements, e.g. constrained force, positions and velocities of the manipulator and rotor.

2. KINEMATICS, DYNAMICS, AND RELATED PROPERTIES

In the section, we first give the kinematic relation of constrained motion and the dynamics of constrained FJRs. Then, some properties are addressed. Here we assume that all end-effectors are rigidly attached to the environment. Hence the motion of a robot is governed by the constrained dynamics, i.e., a dynamic system subject to algebraic constraint equations.

2.1 Kinematics and Dynamical Models

Consider the system of an *n*-link rigid robot with environmental constraints. Let $q \in R^n$ be the vector of generalized coordinates and let $\sigma(q)=0$, where σ: $R^n \mapsto R^m$, denote the environmental constraints. In order to perform the subsequent derivation, the following assumptions concerning the constraint equations $\sigma(q)=0$ are made:

Assumption 1: The constraint are assumed in the absence of friction and other disturbance.

Assumption 2: The environmental constraints can be described by a smooth and exactly known equation $\sigma(q)=0$. There exists an open set $\Theta \subset R^{n-m}$ and a function $\Omega : \Theta \mapsto R^m$; $q_1 = \Omega(q_2)$, so that $\sigma(q_1,\, q_2) = \sigma(\Omega(q_2),\, q_2)$. Moreover, the function $\frac{\partial\Omega}{\partial q_2}$ and $\frac{\partial^2\Omega}{\partial^2 q_2}$ are assumed bounded in the working space.

Since the dimension of the constraint is *m*, the robot losses *m* degrees of freedom and is left with (*n*-*m*) degrees of freedom. One can always partition the generalized coordinates to $q_1 \in R^m$ and $q_2 \in R^{n-m}$ such that the Jacobian matrix of the constraint equations $A(q) = \partial\sigma(q)/\partial q$ is denoted in the form:

$$A(q) = \left[\frac{\partial\sigma(q)}{\partial q_1} \quad \frac{\partial\sigma(q)}{\partial q_2}\right] \equiv \left[A_1(q_2) \quad A_2(q_2)\right],$$

where q_1 has been substituted by the kinematics $q_1 = \Omega(q_2)$. From Assumption 2, that the environmental constraints $\sigma(q)=0$ are always satisfied, the manipulator dynamics is constrained by an invariant submanifold defined by $\{(q,\dot{q}) \,:\, \sigma(q)=0,\, A(q)\dot{q}=0\}$. Then velocities $\dot{q}_1 = -A_1^{-1}A_2\dot{q}_2 = \frac{\partial\Omega(q_2)}{\partial q_2}\dot{q}_2$ is well-defined according to that $A_1 \in R^{m\times m}$ is nonsingular, i.e., $A(q)$ is full row rank, and the boundedness of $\left\|\frac{\partial\Omega}{\partial q_2}\right\|$ in working space. Consequently, we have the following kinematics:

$$q = \left(\Omega(q_2),\, q_2\right);$$

$$\dot{q} = \begin{bmatrix} \dfrac{\partial\Omega(q_2)}{\partial q_2} \\ I_{n-m} \end{bmatrix} \dot{q}_2 \equiv J\dot{q}_2 \qquad (1)$$

Accordingly, one has $A(q)\dot{q} = A(q)J\dot{q}_2 = 0$. Since $\dot{q}_2$ are independent variables, it follows that:

Fact 1: $AJ = J^T A^T = 0$.

Consider the flexible joint constrained robot which has been modeling as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = K(q_m - q) + A^T\lambda \qquad (2)$$

$$J_m\ddot{q}_m + K(q_m - q) = \tau + \tau_d \qquad (3)$$

where $q \in R^n$, $q_m \in R^n$ are generalized coordinates of the robot and the rotor subsystem, respectively; $M(q)$ is an $n\times n$ inertia matrix, which is symmetric and positive definite for all $q \in R^n$; $C(q,\, \dot{q})\dot{q}$ is the Coriolis/centripetal force vector; $G(q)$ and $K(q_m - q)$ are $n\times 1$ vectors containing gravity force and elastic force transmitted through joints, respectively; K is an $n\times n$ diagonal matrix representing the joint stiffness; J_m is actuator inertial matrix; $\tau \in R^n$ is the input force vector, τ_d is the external disturbance of the server system; and $\lambda \in R^m$ is a vector that represents the generalized force multipliers associated with the constraints, namely the constrained force. Then the term $f = A^T(q_2)\lambda$; $f \in R^n$ denotes the contact force due to of the environmental constraints. It is reasonable that the constrained force is required to be always positive under proper control.

2.2 Reduced Dynamics and Some Properties

Since motion equations (2) is subjected to the kinematic constraints (1), the reduced dynamics for the rigid manipulator subsystem is derived to facilitate the control design. Let $\left[E_1^T \,\vdots\, E_2^T\right]$ be a partition of an identity matrix I_n with $E_1 = \left[I_m \quad 0\right] \in R^{m\times n}$ and $E_2 = \left[0 \quad I_{n-m}\right] \in R^{(n-m)\times n}$. Define

$$T = \begin{bmatrix} I_m & \dfrac{\partial\Omega}{\partial q_2}(q_2) \\ 0 & I_{n-m} \end{bmatrix}.$$

Then some facts about the transformation T and Jacobian matrix J are stated below:

Fact 2: $T^T E_2^T = E_2^T$.

Fact 3: $T^T A^T = \left[A_1 \quad 0_{n\times(n-m)}\right]^T$.

Fact 4: Matrix J can be factorized as $J = TE_2^T$. It is also noted that $J^T E_2^T = I_{n-m}$.

Then (2) can be rewritten in the form:

$$MJ\ddot{q}_2 + (M\dot{J} + CJ)\dot{q}_2 + G = Kq_m - B + A^T\lambda$$

where $\quad B \equiv Kq$.

Applying the kinematics (1), the equations of motion (2) can be rewritten as

$$J^T MJ\ddot{q}_2 + J^T(M\dot{J} + CJ)\dot{q}_2 + J^T G = J^T Kq_m - J^T B \ (4)$$

which, by multiplying T^T on the both sides, further results in

$$E_2(\bar{M}E_2^T\ddot{q}_2 + \bar{C}E_2^T\dot{q}_2 + \bar{G}) = E_2(T^T Kq_m - T^T B), \quad (5)$$

where

$$\bar{M} = T^T MT;\ \bar{C} = T^T(M\dot{T} + CT);\ \bar{G} = T^T G.$$

According to Fact 1, 3 and 4, the reduced dynamics can be written in the following form after multiplying E_2^T on the both sides of (5):

$$\underline{M}\ddot{q}_2 + \underline{C}\dot{q}_2 + \underline{G} = J^T K q_m - J^T B \tag{6}$$

where

$\underline{M} = E_2\bar{M}E_2^T = J^T MJ;\ \underline{C} = E_2\bar{C}E_2^T = J^T CJ;\ \underline{G} = E_2 G = J^T G.$

Now, to be beneficial for the stability analysis and control design, some useful properties are addressed below:

Property 1: $\bar{M}$ is a symmetric and positive definite matrix. Also, $\bar{M}^{-1}(q_2)$ is uniformly bounded.

Property 2: $\bar{M}(q_2)$ and $\bar{M}^{-1}(q_2)$ are uniformly bounded. Further, matrix $\bar{M}(q_2)$ is bounded by $M_m I_{n-m} \le \bar{M} \le M_M I_{n-m}$, with $M_m,\ M_M > 0$.

Property 3: $\bar{C}(q_2,\ \dot{q}_2)$ and $\dot{\bar{M}}(q_2,\ \dot{q}_2)$ are uniformly bounded functions if q_2 and $\dot{q}_2$ are uniformly bounded. If q_2, $\dot{q}_2$ and $\ddot{q}_2$ are uniformly bounded, $\dot{\bar{C}}(q_2,\ \dot{q}_2,\ \ddot{q}_2)$ is a uniformly bounded function.

Property 4: For an appropriate choice of C, $\dot{\bar{M}} - 2\bar{C}$ can be a skew-symmetric matrix. This means that $z^T(\dot{\bar{M}} - 2\bar{C})z = 0,\ \forall z \in R^n$.

Property 5 (Linear Parameterization): It follows that

$$\begin{aligned}\bar{M}\dot{q}_r + \bar{C}q_r + \bar{G} &= T^T\left[M\left(T\dot{q}_r + \dot{T}q_r\right) + C\left(Tq_r\right) + G\right] \\ &= T^T Y\left(q_2,\dot{q}_2,\nu,\dot{\nu}\right)\phi\end{aligned}$$

where q_r will be defined later, $\nu = Tq_r$,the regression matrix $Y \in R^{n\times r}$ is a known function and $\phi \in R^r$ is unknown but constant.

The hyperbolic tangent function has the following property

Property 6: The hyperbolic tangent function defined as $\tanh(\mu) = \left(e^{\mu} - e^{-\mu}\right)/\left(e^{\mu} + e^{-\mu}\right)$ satisfies the following properties:

(a) $\mu\tanh(a\mu) \ge 0$, for $a > 0$, this further implies for any odd positive integer b with $\mu\tanh\left(a\mu^b\right) \ge 0$

(b) $|\tanh(\mu)| = \tanh(|\mu|) \ge 0$

(c).if $\tanh(c|\mu|) \ge d$, for $c > 0$, $0 \le d < 1$,then $|\mu| \ge \frac{1}{2c}\ln\left(\frac{1+d}{1-d}\right)$ must be satisfied.

Let $z_1 = Kq_m$ such that the complete system (2) (3) is expressed in the new coordinates as

$$\underline{M}\ddot{q}_2 + \underline{C}\dot{q}_2 + \underline{G} = J^T z_1 - J^T B$$
$$J_m K^{-1}\ddot{z}_1 + z_1 - B = \tau + \tau_d$$

3. SMOOTH SLIDING MODE CONTROL FOR A CONSTRAINED FJR

For a constrained FJR, the rigid manipulator subsystem is driven by the actuator subsystem. Hence the dynamical equations (1) and (2) perform the strictly feedback structure. In this section, a two-stage approach,or namely the integrator backstepping approach is applied to design the controller.

3.1 Design a Virtual Control Input z_1 for the First Stage

Consider the stiffness matrix K is uncertainty and M, C, G is with the unknown parameters.

Let us define the following error signals:

$$e_p = q_{2d} - q_2,\ \text{and}\ \tilde{\lambda} = \lambda_d - \lambda$$

Define a force filter as

$$\dot{e}_\lambda + \alpha e_\lambda = \Lambda_\lambda\tilde{\lambda}$$

where $e_\lambda \in R^m$ and $\Lambda_\lambda \in R^{m\times n}$ is positive definite. Then the error measure is

$$\begin{aligned}s &= E_2^T(\dot{e}_p + \Lambda e_p) + E_1^T e_\lambda = \begin{bmatrix} e_\lambda \\ \dot{e}_p + \Lambda e_p\end{bmatrix} \\ &= q_r - E_2^T\dot{q}_2\end{aligned}$$

where $q_r = E_2^T(\dot{q}_{2d} + \Lambda e_p) + E_1^T e_\lambda = \begin{bmatrix} e_\lambda \\ \dot{q}_{2d} + \Lambda e_p\end{bmatrix}$.

Therefore, if the error measure can be converge to zero, the error signal e_λ , e_p , and $\dot{e}_p$ will converge to zero and thus our objective is achieved. To this end, the error dynamics of the error measure is rewritten as

$$\bar{M}\dot{s} = \bar{M}\dot{q}_r + \bar{C}E_2^T q_2 + \bar{G} - T^T z_1 + T^T B - T^T A^T\lambda$$

If let z_1 as a virtual control input, and consider the feedback control $z_1 = z_{1d}$ with z_{1d} defined as

$$z_{1d} = (T^T)^{-1}\left[\bar{M}\dot{q}_r + \bar{C}q_r + \bar{G} + T^T B - T^T A^T\lambda + \psi s\right]$$

Because the system has the parametric uncertainty, the equation can not be implemented. Hence the LP property is used for solve above problems. Therefore, if assume the uncertain parameters can be partition into $\phi = \phi_0 + \Delta\phi$ which are known priori, then the virtual control input z_{1d} can be rewritten as

$$\begin{aligned}z_{1d} &= (T^T)^{-1}\left[T^T Y\phi_0 + T^T Y\varphi_1 + T^T B - T^T A^T\lambda + \psi s\right] \\ &= Y\phi_0 + Y\varphi_1 + B - A^T\lambda + (T^T)^{-1}\psi s\end{aligned} \tag{7}$$

where the control parameter ψ is positive defined and $\varphi_1 = \left[\varphi_{11},\varphi_{12},\cdots,\varphi_{1r}\right]^T$ is the robust function designed as follows:

$$\varphi_{1i} = (\overline{\Delta\phi_i} + \eta_{1i})\tanh\left[(u + vt)\left(\sum_{j,k=1}^{n,n} s_j (T^T)_{jk} Y_{ki}\right)\right]$$

$i = 1,2,\cdots,r$ with $\eta_{1i}, u, v > 0$ and $\overline{\Delta\phi_i} \ge |\Delta\phi_i|$, then the error dynamics can be rewritten as follow:

$$\bar{M}\dot{s} + \bar{C}s + \psi s = T^T Y\Delta\phi - T^T Y\varphi_1$$

Further, using the Lyapunov function

$$V_1 = \frac{1}{2}s^T\bar{M}s$$

is positive.

$$\begin{aligned}\dot{V}_1 &= s^T\bar{M}\dot{s} + \frac{1}{2}s^T\dot{\bar{M}}s \\ &= -s^T\psi s + \sum_{i=1}^{r}\left(\sum_{j,k=1}^{n,n} s_j (T^T)_{jk} Y_{ki}\right)\Delta\phi_i \\ &\quad - \sum_{i=1}^{r}\left(\sum_{j,k=1}^{n,n} s_j (T^T)_{jk} Y_{ki}\right)(\overline{\Delta\phi_i} + \eta_{1i})\tanh\left[(u + vt)\left(\sum_{j,k=1}^{n,n} s_j (T^T)_{jk} Y_{ki}\right)\right]\end{aligned}$$

$$\dot{V}_1 \le -s^T\psi s + \sum_{i=1}^{r} \Big| \sum_{j,k=1}^{n,n} s_j (T^T)_{jk} Y_{ki} \Big| |\Delta\phi_i|$$
$$-\sum_{i=1}^{r} \Big| \sum_{j,k=1}^{n,n} s_j (T^T)_{jk} Y_{ki} \Big| (\overline{\Delta\phi_i} + \eta_{1i}) \tanh\left[(u+vt) \Big| \sum_{j,k=1}^{n,n} s_j (T^T)_{jk} Y_{ki} \Big| \right]$$

Since V_1 is positive definite and $\dot{V}_1 < 0$ if (8) sustains, it can be concluded that all error signals s, e_λ and $\dot{s}, \dot{e}_\lambda, \tilde{\lambda}$ are all uniformly bounded.

$$\tanh\left[(u+vt) \Big| \sum_{j,k=1}^{n,n} s_j (T^T)_{jk} Y_{ki} \Big| \right] \ge \frac{|\Delta\phi_i|}{\overline{\Delta\phi_i} + \eta_{1i}} \qquad (8)$$

$$\dot{V}_1 = -s^T\psi s \qquad (9)$$

Applying Property 6.(c), the measure of the ultimated ball can be described by the radius about the origin as

$$\Big| \sum_{jk}^{n} s_j (T^T)_{jk} Y_{ki} \Big| \ge b_i = \left(\frac{1}{2(u+vt)} \ln\left(\frac{\overline{\Delta\phi_i} + \eta_{1i} + |\Delta\phi_i|}{\overline{\Delta\phi_i} + \eta_{1i} - |\Delta\phi_i|} \right)\right)$$

Or

$$\|s\| \ge b / \|T^T Y\| \equiv b_s$$

Equation (9) can be rewritten in the following form

$$\dot{V}_1 \le -\lambda_{\min}(\psi)\|s\|^2 \le -\frac{2\lambda_{\min}(\psi)}{M_M} V_1$$

where the Property 3 had been used and integrating both sides of the above inequality with respect to time. We obtain $V_1 \le V_0 e^{-(2\lambda_{\min}(\psi)/M_M)t}$ and this means using the property again, the reaching transient response is shaped by

$$\|s(t)\| \le \sqrt{\frac{M_M}{M_m}} \|s(0)\| e^{\left(-\frac{\lambda_{\min}(\psi)}{M_M}\right)t} \qquad (10)$$

As a result, the motion tracking error exponentially decay if $\|s\| \ge b_s$. Obviously, the ball b_s will converge to zero linearly in time. Therefore , the error signals s will asymptotically converge to zero.

We now show that $\dot{s}$ also converge to zero by using contradiction argument. Assume that $s(t)$ is small enough and $\dot{s}(t) \ne 0$ at some $t > 0$. Since $\bar{M}^{-1}(T^T Y \Delta\phi - T^T Y \varphi_1) \approx \dot{s}$ is continuous, without loss of generality we can say $\bar{M}^{-1}(T^T Y \Delta\phi - T^T Y \varphi_1) \ge \delta$ for some $\delta > 0$ in the time interval $(t, t+\Delta T)$. By Integrating error equation, it follows that $s(t+\Delta T) = \int_t^{t+\Delta T} \bar{M}^{-1}(T^T Y \Delta\phi - T^T Y \varphi_1) dt$. Thus $s(t+\Delta T) \ge \delta \cdot \Delta T$, which contradicts with the fact that s converges to zero. Hence $\dot{s} \to 0$ as $t \to \infty$. This mean $\dot{e}_\lambda \to 0$ as $t \to \infty$. Accordingly, the force error $\tilde{\lambda}$ will asymptotically converge to zero as $t \to \infty$.

Now, since z_1 is not the input, consists an error term related to $\tilde{z}_1 = z_{1d} - z_1$ and is shown below the practical error dynamics is

$$\bar{M}\dot{s} + \bar{C}s + \psi s = T^T Y \Delta\phi - T^T Y \varphi_1 + T^T \tilde{z}_1 \qquad (11)$$

If $\tilde{z}_1$ can be driven to zero, the stability can be obtained.

3.2 Regulate z_1 Converge to z_{1d} for the Second Stage.

Step 2: We now have the error dynamics of $\tilde{z}_1$ as

$$\dot{\tilde{z}}_1 = \dot{z}_{1d} - \dot{z}_1 \qquad (12)$$

where $\dot{z}_{1d}$ is with the uncertainty of the manipulator, so $\dot{z}_{1d}$ is not realizable. To let the control input be realized by position, velocity and force measurements only, the following relations are introduced:

$$\dot{s} = \bar{M}^{-1}(T^T Y \Delta\phi - T^T Y \varphi_1 + T^T \tilde{z}_1 - \bar{C}s - \psi s)$$
$$\ddot{q}_2 = \underline{M}^{-1}(J^T z_1 - J^T B - \underline{C}\dot{q}_2 - \underline{G})$$

Then the terms $\dot{Y}$, where $\ddot{q}_2$ and $\dot{s}$ replaced by $q_2, \dot{q}_2$ signals, yields

$$\dot{Y}(q_2,\dot{q}_2,\ddot{q}_2,q_r,\dot{q}_r,\ddot{q}_r) = \bar{Y}_1(q_2,\dot{q}_2,q_r,\dot{q}_r,\phi_0) + \bar{Y}_2(q_2,\dot{q}_2,q_r,\dot{q}_r,\Delta\phi,\dot{\lambda})$$

Additionally, the time derivative of the virtual control input can be partitioned into the known part N_0 and unknown part ΔN, and is expressed as

$$\dot{z}_{1d} = N_0 + \Delta N$$

where

$$\Delta N = \bar{Y}_2(q_2,\dot{q}_2,q_r,\dot{q}_r,\Delta\phi,\dot{\lambda})(\phi_0 + \varphi_1) + Y\left[\frac{d\varphi_1}{dt}\right]_\Delta - A^T\dot{\lambda} + \Delta d$$
$$d = (T^T)^{-1}\psi \bar{M}^{-1}(T^T Y \Delta\phi - T^T Y \varphi_1 + T^T \tilde{z}_1 - \bar{C}s - \psi s)$$
$$d_0 = (d)_0$$
$$\Delta d = d - d_0$$

Note that $\Delta(\cdot) = (\cdot) - (\cdot)_0$ denote the uncertain term can not be obtained. The term N_0 is realizable on-line, and ΔN is an uncertain term with unknown parameters which is not satisfying the LP property. Hence the typical adaptive backstepping can not be applied in this problem. According to the stability analysis in Step 1, the state variables and the time derivative of the constrained force are all bounded once the $\tilde{z}_1$ can be steered to zero. This means that when a suitable control is applied, the uncertain term ΔN has a upper bound as

$$|\Delta N_i(q_2,\dot{q}_2,q_r,\dot{q}_r,\Delta\phi,\dot{\lambda},\tilde{z}_1)| \le \overline{\Delta N_i} \quad \text{for i=1,2,}\cdots\text{,n}$$

Next, let us consider the error dynamics (12) again, yields

$$\dot{\tilde{z}}_1 = \dot{z}_{1d} - \dot{z}_1 = N_0 + \Delta N - \dot{z}_1 \qquad (13)$$

Now, we assume $\dot{z}_1$ as the virtual control input in the form:

$$\dot{z}_1 = z_{2d} = N_0 + \beta_1\tilde{z}_1 + Ts + (\overline{\Delta N} + \eta_2)\tanh[(u+vt)\tilde{z}_1] \qquad (14)$$

with

$\overline{\Delta N} = diag\{\overline{\Delta N}_1,\cdots,\overline{\Delta N}_n\}; \eta_2 = diag\{\eta_{21},\cdots,\eta_{2n}\}$ and $\beta_1, \eta_{2i} > 0$.

Then (13) can be further rewritten as

$$\dot{\tilde{z}}_1 + \beta_1\tilde{z}_1 = -Ts + \Delta N - (\overline{\Delta N} + \eta_2)\tanh[(u+vt)\tilde{z}_1] \qquad (15)$$

Consider $V_2 = V_1 + \frac{1}{2}\tilde{z}_1^T\tilde{z}_1$ as a Lyapunov function candidate for the error system (15) with the virtual input (14). Let $e = \begin{bmatrix} s^T & \tilde{z}_1^T \end{bmatrix}^T$. We have the time derivative of V_2 as

$$\dot{V}_2=-e^T\begin{bmatrix}\psi & T^T\\ -T & \beta_1 I_n\end{bmatrix}e+\sum_{i=1}^{r}\Big(\sum_{j,k=1}^{n}s_j(T^T)_{jk}Y_{ki}\Big)\Delta\phi_i+\sum_{i=1}^{n}\tilde{z}_{1i}\Delta N_i$$
$$-\sum_{i=1}^{r}\Big(\sum_{j,k=1}^{n,n}s_j(T^T)_{jk}Y_{ki}\Big)(\overline{\Delta\phi}_i+\eta_{1i})\tanh\Big[(u+vt)\Big(\sum_{j,k=1}^{n,n}s_j(T^T)_{jk}Y_{ki}\Big)\Big]-\sum_{i=1}^{n}\tilde{z}_{1i}(\overline{\Delta N}_i+\eta_{2i})\tanh[(u+vt)\tilde{z}_{1i}]$$

In the same way, if both the inequality (8) and following inequality

$$\tanh\left[(u+vt)\,|\,\tilde{z}_{1i}\,|\right]\geq\frac{|\,\Delta N_i\,|}{\overline{\Delta N}_i+\eta_{2i}} \qquad (16)$$

are satisfied, it follows that

$$\dot{V}_2=-e^T\begin{bmatrix}\psi & T^T\\ -T & \beta_1 I_n\end{bmatrix}e \qquad (17)$$

Since $\psi>0$ and $\beta_1 I_n+T\psi T^T>0$, $\dot{V}_2\leq 0$ is assured. Using the facts that V_2 is upper bounded and $\dot{V}_2\leq 0$, the Lyapunov function V_2 is decreasing. this implies all error signals s, $\tilde{z}_1$ are uniformly bounded. From the analysis of step 1, this means all error signals are uniformly bounded. Moreover, the error signal e will asymptotically converge to an ultimated bound defined by (8) and (16) as follows:

$$\|e\|\geq\max\left(b_s,b_{\tilde{z}_1}\right),\ \text{with}\ b_{\tilde{z}_1}=\Big(\frac{1}{2(u+vt)}\ln\left(\frac{\overline{\Delta N}_i+\eta_{2i}+|\,\Delta N_i\,|}{\overline{\Delta N}_i+\eta_{2i}-|\,\Delta N_i\,|}\right)\Big)$$

Obviously, the ball $\max\left(b_s,b_{\tilde{z}_1}\right)$ will converge to zero linearly in time. Therefore, the error signals e will asymptotically converge to zero as long as $t\to\infty$. The remaining of the proof is to show the control law is smooth as $t\to\infty$. From the facts that V_2 satisfies the inequality as

$$c_1\|e\|^2\leq V_2\leq c_2\|e\|^2$$
$$\dot{V}_2\leq -c_3\|e\|^2$$

with $c_1=\lambda_{\min}\left(M_m,1\right), c_1=\lambda_{\max}\left(M_M,1\right)$ and $c_3=\lambda_{\min}\left(\begin{bmatrix}\psi & T^T\\ -T & \beta_1 I_n\end{bmatrix}\right)$

,we can easily show that the error signal e is shapped by $\|e(t)\|\leq\sqrt{\frac{c_2}{c_1}}\|e(0)\|e^{-\frac{c_3}{2c_2}t}$ Hence, using the same technique of the proof in step 1, the robust function and its time derivative are bounded and smooth.

Since $\dot{z}_1$ is not the true input, we look again at the error $\tilde{z}_2=z_{2d}-\dot{z}_1$, where z_{2d} defined in (14). The error dynamics of $\tilde{z}_1$ is rewritten in

$$\dot{\tilde{z}}_1+\beta_1\tilde{z}_1=-Ts+\Delta N-(\overline{\Delta N}+\eta_2)\tanh[(u+vt)\tilde{z}_1]+\tilde{z}_2$$

In Step 3, the actual designing a control input will be determined to let $\tilde{z}_2$ converge to zero.

Step 3: Before applying the same design scheme as Step 2, let us reformulate $\dot{z}_{2d}$ into a nominal part and a uncertain part to obtain actual control input only form position, velocity, and force. Assume J_m is nominally known as J_{m0}, and the unknown part ΔJ_m. Thus the dynamics of $\tilde{z}_2$ is

$$\begin{aligned}J_mK^{-1}\dot{\tilde{z}}_2&=J_mK^{-1}\dot{z}_{2d}-J_mK^{-1}\ddot{z}_1\\&=J_{m0}K^{-1}(w_0+\Delta w+E\Delta N)+z_1-B-\tau\end{aligned} \qquad (18)$$

where

$$w_0(q_2,\dot{q}_2,q_r,\dot{q}_r,z_1,\dot{z}_1,\phi_0)=[\dot{N}_0+T\dot{s}]_0+\dot{T}s+(\overline{\Delta N}+\eta_2)P\{v\tilde{z}_1+(u+vt)(N_0-\dot{z}_1)\}+\beta_1(N_0-\dot{z}_1)$$

$$\Delta w(q_2,\dot{q}_2,q_r,\dot{q}_r,z_1,\dot{z}_1,\Delta\phi,\dot{\lambda},\tau_d)=[\dot{N}_0+T\dot{s}]_\Delta-KJ_{m0}^{-1}\tau_d+J_{m0}^{-1}(J_m-J_{m0})\dot{z}_{2d}$$
$$P=diag\{\sec h^2[(u+vt)\tilde{z}_{11}],\cdots,\sec h^2[(u+vt)\tilde{z}_{1n}]\}$$
$$E=(u+vt)(\overline{\Delta N}+\eta_2)P+\beta_1 I_n$$

Note that w_0 is a known part in the error system and the term $(u+vt)\tilde{z}_1^{\,2}$ is bounded has been shown in step2. Once the error $\tilde{z}_2$ can be drived to zero, which is achieved in this step. Moreover, the uncertain term Δw is assumed with a upper bound $|\,\Delta w_i(\cdot)|\leq\overline{\Delta w_i}$.

Let the control input as $\tau=z_1-B+\beta_2\tilde{z}_2+\tilde{z}_1+J_{m0}K^{-1}\upsilon$, where υ will be defined later. Thus we obtain the overall error system:

$$\bar{M}\dot{s}+\bar{C}s+\psi s=T^TY\Delta\phi-T^TY\varphi_1+T^T\tilde{z}_1 \qquad (19)$$
$$\dot{\tilde{z}}_1+\beta_1\tilde{z}_1=-Ts+\Delta N-(\overline{\Delta N}+\eta_2)\tanh[(u+vt)\tilde{z}_1]+\tilde{z}_2 \qquad (20)$$
$$J_mK^{-1}\dot{\tilde{z}}_2+\beta_2\tilde{z}_2=J_{m0}K^{-1}(w_0+\Delta w+E\Delta N-\upsilon)-\tilde{z}_1 \qquad (21)$$

In (21), the new control υ will be used to cancel Δw, ΔN. Therefore let us consider the Lyapunov function $V_3=V_2+\frac{1}{2}\tilde{z}_2^{\,T}\tilde{z}_2$. From the analysis of V_2, applying the inequality (8) and (16), the time derivative of V_3 yields

$$\dot{V}_3\leq-\bar{e}^T\begin{bmatrix}\psi & T^T & 0\\ -T & \beta_1 I_n & I_n\\ 0 & -I_n & \beta_2 I_n\end{bmatrix}\bar{e}+\tilde{z}_2^{\,T}J_{m0}K^{-1}(w_0+\Delta w+E\Delta N-\upsilon)$$

where $\bar{e}=\left[s^T\ \ \tilde{z}_1^{\,T}\ \ \tilde{z}_2^{\,T}\right]^T$. To obtain $\dot{V}_3\leq 0$, we choose $\upsilon=w_0+(\overline{\Delta w}+\eta_3)\tanh[(u+vt)\tilde{z}_2]+E(\cdot)\delta$, where

$\overline{\Delta w}=diag\{\overline{\Delta w_1},\cdots,\overline{\Delta w_n}\};\eta_3=diag\{\eta_{31},\cdots,\eta_{3n}\}$ and

$$\delta=[\delta_1,\cdots,\delta_n];\delta_i=(\overline{\Delta N}_i+\eta_{4i})\tanh\{(u+vt)[\sum_{j=1}^{n}\tilde{z}_{2j}E_{ji}]\},\ \eta_{4i}>0\ \text{for}\ i=1,\cdots,n$$

In this setting, the derivative of V_3 can be rewritten as follows:

$$\dot{V}_3\leq-\bar{e}^T\begin{bmatrix}\psi & T^T & 0\\ -T & \beta_1 I_n & I_n\\ 0 & -I_n & \beta_2 I_n\end{bmatrix}\bar{e}$$
$$+\sum_{i=1}^{n}J_{m0}K^{-1}(|\,\tilde{z}_{2i}\,||\,\overline{\Delta w}_i\,|-\left(\tilde{z}_{2i}\left(\tanh\{(u+vt)\,|\,\tilde{z}_{2i}\,|\}(\overline{\Delta w}_i+\eta_{3i}\right)+P_1-P_2\right)$$

where

$$P_1=|\sum_{j=1}^{n}\tilde{z}_{2j}E_{ji}\,||\,\Delta N_i\,|;P_2=|\sum_{j=1}^{n}\tilde{z}_{2j}E_{ji}\,|\tanh\{(u+vt)|\sum_{j=1}^{n}\tilde{z}_{2j}E_{ji}\,|\}(\overline{\Delta N}_i+\eta_{4i})$$

Once the following inequality is satisfied

$$|\,z_{2i}\,|\geq b_{\tilde{z}_2}\equiv\max\Big(\Big(\frac{1}{2(u+vt)}\ln\left(\frac{\overline{\Delta w}_i+\eta_{3i}+|\,\Delta w_i\,|}{\overline{\Delta w}_i+\eta_{3i}-|\,\Delta w_i\,|}\right)\Big)$$
$$,\Big(\frac{1}{2(u+vt)}\ln\left(\frac{\overline{\Delta N}_i+\eta_{2i}+|\,\Delta N_i\,|}{\overline{\Delta N}_i+\eta_{2i}-|\,\Delta N_i\,|}\right)\Big)\Big)/\,|\sum_{i=1}^{n}E_{ji}\,|$$

then we have

$$\dot{V}_3\leq-\bar{e}^T\begin{bmatrix}\psi & T^T & 0\\ -T & \beta_1 I_n & I_n\\ 0 & -I_n & \beta_2 I_n\end{bmatrix}\bar{e}$$

this means that all error signals are uniformly bounded, and the signal $\bar{e}$ will exponentially converge to the ultimated ball as

$\|\bar{e}\| \geq \max\left(b_s, b_{\bar{z}_1}, b_{\bar{z}_2}\right)$. Since the ball shrinks to the origin $\bar{e}$ will converge to zero along the ball as $t \to \infty$. Applying this fact, we have

$$d_1 \|e\|^2 \leq V_3 \leq d_2 \|e\|^2$$

$$\dot{V}_3 \leq -d_3 \|e\|^2$$

with $d_1 = \min\left(M_m, 1\right), d_2 = \max\left(M_M, 1\right)$

and $d_3 = \lambda_{\min}\left(\begin{bmatrix} \psi & T^T & 0 \\ -T & \beta_1 I_n & I_n \\ 0 & -I_n & \beta_2 I_n \end{bmatrix}\right)$

Hence $\|\bar{e}(t)\| \leq \sqrt{d_2 / d_1}\, \|\bar{e}(0)\| e^{-\frac{d_3}{2d_2}t}$, this meaning that the control law is smooth and bounded as $t \to \infty$. Therefore the results of step1 is achieved such that the e_p, $\dot{e}_p$ will converge to zero as $t \to \infty$. Moreover, the contact force asymptotically tracks to $\lambda_d(t)$.

4. SIMULATION RESULTS

Here a two-link constrained robotic manipulator with revolute flexible joint, as shown in Fig.1. Take $m_1 = m_2 = a_1 = a_2 = 1$ and gravity $g = 9.8$ m/s^2, and the actuator inertial, stiffness matrices of the rotor subsystem are set as:

$$J_m = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, K = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}.$$

The constraint which is indeed by restricting the robot end-effector to keep contact with the environment can be described in terms of the Cartesian coordinate as x=1. Through the forward kinematics, we can obtain the implication

$$x = 1 \;\Rightarrow\; \cos(q_2) + \cos(q_1 + q_2) = 1$$

The control design is performed in assumption that m_1, m_2 are unknown. The initial states are chosen to be $q_2(0) = 0.5,\ \dot{q}_2(0) = 0$, $\hat{m}_1$=1.2,$\hat{m}_2$=1.2. Select $\psi = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}$, $\beta_1 = \begin{bmatrix} 30 & 0 \\ 0 & 35 \end{bmatrix}$, $\beta_2 = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$, $\Lambda = 20,\ \eta = 0.001$.

The desired joint trajectory and force trajectory are chosen as $\lambda_d = 5 + \sin(\pi t),\ q_{2d} = \pi(\frac{1}{4} + \frac{1}{24}\sin(\pi t))$ .The desired trajectory q_{2d} and the actual joint position q_2 of the simulation are shown in Fig. 2. The desired velocity $\dot{q}_{2d}$ and the joint velocity $\dot{q}_2$ are shown in Fig. 3, while the desired force λ_d and the constraint force λ are shown in Fig. 4. The tokes of robot are shown in Fig. 5.

The expected performance can be verified from these numerical results.

5. CONCLUSIONS

An approach based on a smooth sliding-mode controller has been successfully developed for motion control and force control of constrained robots having parametric uncertainty. Some properties of the reduced dynamics of constrained robots have been presented and exploited to demonstrate the motion and tracking errors to zero.

The numerical simulation of a two-link constrained manipulator driven through flexible shafts has been set up to demonstrate the performance of the proposed controllers.

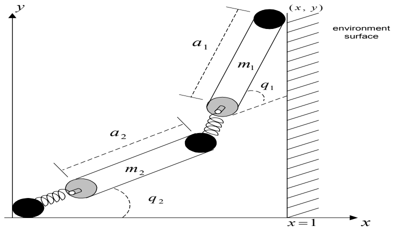

Fig 1 A two-link planar constrained FJRs

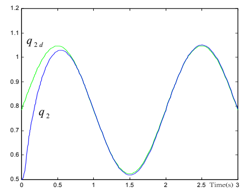

Fig. 2 The joint position

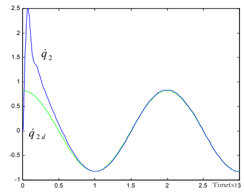

Fig. 3 The joint velocity

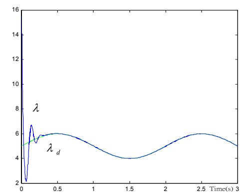

Fig. 4 The constraint forces

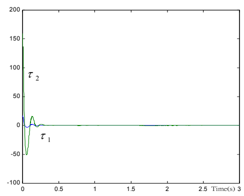

Fig. 5 The joint tokes

REFERENCES

Hemami, H. and B.F. Wyman, "Modeling and control of constrained dynamic system with application to biped locomotion in the frontal plane," *IEEE Trans. Automat. Cont.*, AC-24, pp. 526-535, 1979.

Kwan, C. M., "Robust Adaptive Force/Motion Control of Constrained Robots," *IEE Proceedings Control Theory Appl.*, vol. 143, no. 1, pp. 103-109, January 1996.

Lian, K.-Y., J. -H. Jean and L. -C. Fu, "Adaptive Force Control of Single-link Mechanism with Joint Flexibility," *IEEE Trans. on Robotics and Automation*, vol. 7, no. 4, pp. 540-545, 1991.

Lian, K.-Y.and C. -R. Lin, "Sliding-Mode Motion/Force Control of Constrained Robots," *IEEE Trans. on Automat. Contr.*, vol. 43, no. 8, pp. 1101-1103, 1998.

McClamroch, N. H. and D. Wang, "Feedback stabilization and tracking of constrained robots," *IEEE Trans. Automat. Control.*, vol. 33, pp. 419-426, May 1988.

Spong, M. and M. Vidyasagar, *Robot Dynamics and Control.* Wiley, New York, 1989.

Spong, M., "Adaptive control of flexible joint manipulators," *System and Control Letters.*, 13, pp.15-21, 1989.

Su, C. Y.,Y. Stepanenko and T. P. Leung, "Combined Adaptive and Variable Structure Control for Constrained Robots," *Automatica,* vol. 31, no. 3, pp. 483-488, 1995.

Yao, B., S. P. Chan, and D. Wang, "Variable Structure Adaptive Motion and Force Control of Robot Manipulators," *Automatica,* vol. 30, no. 9, pp. 1473-1477, 1994.

Yao, B. and M. Tomizuka, "Adaptive Control of Robot Manipulators in Constrained Motion - Controller Design," *ASME J. Dynam. Syst. Meas. & Contr.*, pp.320-327, 1995.

You, L.-S. and B.-S. Chen, "Tracking control design for both holonomic and nonholonomic constrained mechanical systems: a unified viewpoint" *Int. J. Control*, vol. 58, no. 3, pp. 587-612, 1993.

Yu, H. and S. Lloyd, "Combined Direct and Indirect Adaptive Control of Constrained Robots," *Int. J. Control*, vol. 68, no. 5, pp. 955-970, 1997.

Yuan, J., "Composite Adaptive Control of Constrained Robots," *IEEE Trans. Automat. Cont.*, vol. 12, no. 4, pp. 640-645, August 1996.

Yuan, J., "Adaptive Control of A Constrained Robot - Ensuring Zero Tracking and Zero Force Errors," *IEEE Trans. Automat. Cont.*, vol. 42, no. 12, pp. 1709-1714, December 1997.