A CHAIN-SCATTERING APPROACH TO LMI MULTIOBJECTIVE CONTROL

Rémi DRAI * **Martine OLIVI** **
Jean-Paul MARMORAT *

* *C.M.A., Ecole des Mines de Paris*
remi.drai@cma.inria.fr
** *INRIA Sophia-Antipolis*

BP 93, 06902 Sophia-Antipolis Cedex, France

Abstract: This paper revisits, from a chain-scattering perspective, the LMI solution based on Youla-Kucera parametrisation of the general multi-objective control problem. The conceptual and computational advantages of the chain-scattering formalism are demonstrated by allowing a more direct derivation of some known results as well as by hinting to some new research directions.

Keywords: LMI, multi-objective control, chain-scattering, Youla-Kucera parametrisation.

1. INTRODUCTION

The linear matrix inequality (LMI) formulation of multi-objective control, as exposed for example in (Scherer *et al.*, 1997), has become a very popular design method and is by now widely used in the control literature and its applications. More recently one of its most significant limitations that consist to formulate all the closed-loop control objectives in terms of a single Lyapunov function has been overcome, in (Scherer, 2000), by using the well-known Youla-Kucera parametrisation to transform the approach of (Scherer *et al.*, 1997) into a more versatile and effective control technique.

This improvement comes with a cost in terms of the order of the resulting controller, this issue being related to the ability to efficiently represent and optimize the Youla-Kucera parameter. One of the main goals of this paper is to show how some of the main results of (Scherer, 2000) receive an interesting interpretation when expressed in a chain-scattering framework and how this same framework can also be used to point out research directions that may significantly expand its current scope and applicability.

The chain-scattering formalism originated from circuit theory where it is still widely used. Its use in control is more recent and more limited; it has mostly been applied for H_∞ control, see eg. the articles (Ball-Helton-Verma, 1991), (Verma-Zames, 1991). The reader can also consult the textbook (Kimura, 1997) for a nice and more extensive treatment of the chain-scattering approach in control.

The article is organized as follows : well-known facts on the chain-scattering formalism and Youla-Kucera parametrisation are first recalled and discussed in order to motivate the approach proposed. The LMI multi-objective approach of (Scherer, 2000) is summarized in Section III and then revisited in Section IV. Possible extensions of the proposed approach are then mentioned before concluding.

2. PRELIMINARIES AND NOTATIONS

In the following, both notations $G(z) = \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right)$ and $G(z) = (A, B, C, D)$ will be used to denote a state-space realisation of the transfer matrix $G(z)$. RH_∞ will denote the set of rational, real, stable transfer matrices. It is also assumed that all plants are defined in a discrete-time setting; the extension to the continuous-time domain being straightforward.

2.1 Chain Scattering Formalism

Starting from a classical two-port plant $P(z)$ relating some input variables (w, u) to output variables (z, y), one can introduce the chain-scattering operator $chain(.)$ so that $\hat{P} = chain(P)$ is defined through

$$\begin{pmatrix} z \\ w \end{pmatrix} = chain(P) \begin{pmatrix} u \\ y \end{pmatrix} \qquad (1)$$

as represented in Figure 1 below.

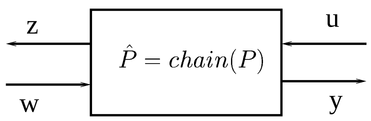

Fig. 1. *Chain-Scattering Representation*

It is readily shown that a chain-scattering representation of a plant $P = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}$ exists if and only if the transfer P_{21} is invertible in which case

$$\hat{P} = chain(P) = \begin{pmatrix} P_{12} - P_{11}P_{21}^{-1}P_{22} & P_{11}P_{21}^{-1} \\ -P_{21}^{-1}P_{22} & P_{21}^{-1} \end{pmatrix}$$

In the case where P_{21} is not invertible, some output augmentation techniques can be used in order to define $chain(P)$, see eg. (Kimura, 1997).

Likewise, starting from a chain-wave representation $\hat{P} = G$, one can also define the scattering operator

$$\check{G} = chain^{-1}(G) = \begin{pmatrix} G_{12}G_{22}^{-1} & G_{11} - G_{12}G_{22}^{-1}G_{21} \\ G_{22}^{-1} & G_{22}^{-1}G_{21} \end{pmatrix} \qquad (2)$$

which associates to $\hat{P} = G$ its usual input/output (I/O) representation $P = \check{G}$. Moreover, it is also useful to introduce the dual chain-scattering operator

$$dchain(P) = chain(P)^{-1} = \begin{pmatrix} P_{12}^{-1} & P_{12}^{-1}P_{11} \\ P_{22}P_{12}^{-1} & P_{21} - P_{22}P_{12}^{-1}P_{11} \end{pmatrix}$$

which relates the variables, in Figure 1, according to

$$\begin{pmatrix} u \\ y \end{pmatrix} = dchain(P) \begin{pmatrix} z \\ w \end{pmatrix}$$

A formula similar to (2) relating the operators $dchain(.)$ and $dchain^{-1}(.)$ being readily available.

2.2 Plants Interconnexion

One of the main advantages of the chain-scattering formalism is that it simplifies considerably the feedback interconnexion of two-port plants. From the definition of $chain(.)$, one can indeed immediately verify that

$$chain(P_1 \star P_2) = chain(P_1).chain(P_2)$$

where : $P_1 \star P_2$ denotes the classical Redheffer or star product of the plants P_1 and P_2 illustrated in Figure 2. Moreover, one can also easily check that

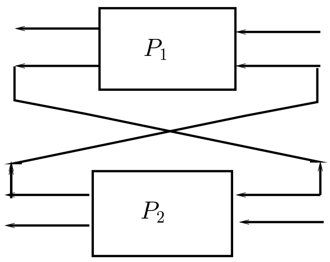

Fig. 2. *Redheffer or star product*

$$F_l(P, K) = Hom(\hat{P}, K)$$

with

$$F_l(P, K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}$$
$$Hom(\hat{P}, K) = (\hat{P}_{11}K + \hat{P}_{12})(\hat{P}_{21}K + \hat{P}_{22})^{-1}$$

Another interest of using the chain scattering formalism in control is that it provides a natural framework for the classical Youla-Kucera parametrization, cf. (Kučera, 1979) and (Youla *et al.*, 1976).

2.3 Parametrization of All Stabilizing Controllers

We recall the well-known facts: a controller internally stabilizes the plant $P = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}$ i.f.f. it stabilizes the resulting representation of P_{22} supposed to be minimal. Moreover if P_{22} admits a (right) coprime factorization

$$P_{22}(z) = N(z)M(z)^{-1}, N, M \in RH_\infty$$

then starting from any arbitrary stabilizing controller $K_0(z) = X(z)Y(z)^{-1}$, the Youla-Kucera parametrization states that all stabilizing controllers are of the form

$$K(z) = (X + MQ).(Y + NQ)^{-1} \qquad (3)$$

where the matrix $U(z) := \begin{pmatrix} M & X \\ N & Y \end{pmatrix}$ is normalized into a doubly-coprime factorization and that

$Q(z) \in RH_\infty$.

Using a chain-scattering vocabulary, Youla-Kucera parametrization says that stabilizing controllers can be written

$$K(z) = Hom(U, Q) \tag{4}$$

where $U(z)$ is a particular unit of RH_∞.

The closed-loop structure of the standard problem of modern control theory can thus be represented as in Figure 3 below.

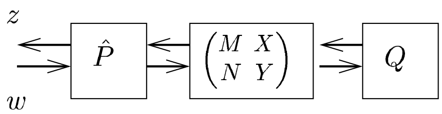

Fig. 3. *Closed-loop Structure in the Chain-scattering Framework*

State-space Formulas (observer-based case)

In the special case where K_0 is an observer-based stabilizing controller for $P_{22}(z) = \left(\begin{array}{c|c} A & B_u \\ \hline C_y & D_{yu} \end{array}\right)$ defined by the two gain matrices (K_c, K_f) simple state-space formulas for $U(z)$ are those given by Nett et al. in (Nett-Jacobson-Balas, 1984), namely

$$U = \left(\begin{array}{c|cc} A - B_u K_c & B_u & -K_f \\ \hline -K_c & I & 0 \\ C_y & 0 & I \end{array}\right)$$

where it is assumed for the sake of simplicity and without loss of generality that $D_{yu} = 0$.

The usual (but somewhat less natural) formulation of Youla parametrization $K(z) = F_l(J, Q)$ used in robust control is obtained by applying the scattering operator $chain^{-1}(.)$ to the unit matrix $U(z)$, viz.

$$J = \check{U} = \left(\begin{array}{c|cc} A - B_u K_c - K_f C_y & K_f & B_u \\ \hline -K_c & 0 & I \\ -C_y & I & 0 \end{array}\right)$$

3. LMI MULTI-OBJECTIVE CONTROL

Using the usual assumptions and notations of multi-objective control, cf. for example (Scherer *et al.*, 1997) or (Scherer, 2000), one considers a generalized plant of the form :

$$\begin{pmatrix} x_{k+1} \\ z_i \\ y \end{pmatrix} = \begin{pmatrix} A & B_{w_i} & B_u \\ C_{z_i} & D_{z_i w_i} & D_{z_i u} \\ C_y & D_{y w_i} & 0 \end{pmatrix} \begin{pmatrix} x_k \\ w_i \\ u \end{pmatrix}$$

where a controller $u = K(z)y$ is searched such that a different control objective is satisfied on each closed-loop transfer $T_{z_i w_i}$.

In the LMI approach, these control objectives are expressed by some closed-loop matrix inequalities constraints involving a Lyapunov matrix associated to the $T_{z_i w_i}$ constraint and the controller parameters. The main difficulty being then to transform these nonlinear constraints into LMI constraints in some newly defined variables.

The main contribution of the paper (Scherer, 2000) is to show how to reduce the search of a multi-objective controller $K(z)$ into a tractable LMI problem by restricting the parameter $Q(z) \in RH_\infty$ to the form

$$Q(z) = \sum_{i=1}^{N} \bar{Q}_j q_j(z)$$

with fixed transfers $q_j(z)$ (e.g. $q_j(z) = 1/z^j$ which corresponds to a Finite Impulse Response or FIR expansion).

The decision variables of the obtained LMI problem are:

- the coefficients $\bar{Q}_j$ of the above expansion.
- some new variables related to the Lyapunov matrix $\mathcal{X}_i > 0$ associated to the $T_{z_i w_i}$ constraint.

This LMI formulation of the multi-objective problem is made possible by taking advantage of the well-known linearity property of the closed loop transfers $T_{z_i w_i}$ w.r.t. the $Q(z)$ parameter as well as thanks to an *ad-hoc* linearizing change of variables.

These two key points of (Scherer, 2000), the first one being central in the approach of (Boyd and Barratt, 1991), are now examined from a chain-scattering perspective.

4. CHAIN-SCATTERING FORMULATION

4.1 Linearity w.r.t. the Q Parameter

This classical linearity property

$$T_{z_i w_i} = T_{11}^i + T_{12}^i Q T_{21}^i$$

follows from a straightforward (but tedious) computation of

$$T = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = P \star J$$

which shows that $T_{22} = 0$.

This result is immediate in the chain-scattering formalism: from Youla-Kucera parametrization (3) and Figure 3, one has

$$T_{z_i w_i} = Hom(\hat{P}, K) = Hom(S, Q) \text{ with}$$
$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \hat{P}.U$$

and $S_{21} = -P_{21}^{-1} P_{22} M + P_{21}^{-1} N = 0$,
as : $P_{22} = NM^{-1}$.

While the above are classical results, it should also be mentionned that any factorization: $\hat{P} = \begin{pmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{pmatrix} . U^{-1}$, with S_{11}, S_{12}, S_{22}^{-1} stable and U a unit of RH_∞, yields to a (possibly alternative) parametrization of all stabilizing controllers in the form (4), cf. (Ball-Helton-Verma, 1991).

4.2 *Linearizing Change of Variables*

A major contribution of (Scherer, 2000) was to observe that if for each constraint on the transfer $T_{z_i w_i}$ is associated a Lyapunov matrix partitioned according to the closed-loop equations:

$$\mathcal{X}_i = \begin{pmatrix} X_i & Z_i \\ Z_i^T & Y_i \end{pmatrix}$$

Then the introduction of the new variables

$$\begin{pmatrix} Q_i & S_i \\ S_i^T & R_i \end{pmatrix} := \begin{pmatrix} X_i^{-1} & -X_i^{-1} Z_i \\ -Z_i^T X_i^{-1} & Y_i - Z_i^T X_i^{-1} Z_i \end{pmatrix}$$

provides an LMI solution to the multi-objective control problem. This results from the key triangular factorization

$$\Pi_{i1}^T \mathcal{X}_i = \Pi_{i2} \qquad (5)$$

with

$$\Pi_{i1} = \begin{pmatrix} Q_i & S_i \\ 0 & I \end{pmatrix} \qquad \Pi_{i2} = \begin{pmatrix} I & -S_i \\ 0 & R_i \end{pmatrix}$$

Indeed, assuming that: $T_{z_i w_i} = (\mathcal{A}, \mathcal{B}_{w_i}, \mathcal{C}_{z_i}, \mathcal{D}_{z_i w_i})$ and $Q(z) = (A_Q, B_Q, C_Q, D_Q)$ it can be the shown that any matrix constraint on $T_{z_i w_i}$ involving the terms :

$$\mathcal{X}_i \mathcal{A}, \mathcal{X}_i \mathcal{B}_{w_i}, \mathcal{C}_{z_i}, \mathcal{D}_{z_i w_i}$$

and their transpose can be translated into an LMI in Q_i, S_i, R_i, C_Q, D_Q after a proper congruence transformation involving Π_{i1} has been performed.

In the FIR case, one has simply $[C_Q, D_Q] = [\bar{Q}_N, ..., \bar{Q}_0]$. More importantly, it should be noted that the linearization relies on the fact that the pair (A_Q, B_Q) is supposed to be fixed.

It turns out that the change of variables presented above, and similar ones, can be readily obtained and interpreted within the chain-scattering framework.

4.3 *Chain-Scattering Factorizations*

A basic property of the chain-scattering matrices, cf. (Kimura, 1997), is that they admit factorization of the form

$$chain(P) = \begin{pmatrix} P_{12} & P_{11} \\ 0 & I \end{pmatrix} \begin{pmatrix} I & 0 \\ P_{22} & P_{21} \end{pmatrix}^{-1} \qquad (6)$$

and

$$dchain(P) = \begin{pmatrix} P_{12} & 0 \\ -P_{22} & I \end{pmatrix}^{-1} \begin{pmatrix} I & -P_{11} \\ 0 & P_{21} \end{pmatrix} \qquad (7)$$

Now if

$$\hat{P} = dchain(P) = \begin{pmatrix} X & Z \\ Z^T & Y \end{pmatrix}$$

one can check that $P = \begin{pmatrix} -X^{-1}Z & X^{-1} \\ Y - Z^T X^{-1} Z & Z^T X^{-1} \end{pmatrix}$
and the factorization (7) is simply

$$\begin{pmatrix} X^{-1} & 0 \\ -Z^T X^{-1} & I \end{pmatrix} \begin{pmatrix} X & Z \\ Z^T & Y \end{pmatrix} = \begin{pmatrix} I & X^{-1} Z \\ 0 & Y - Z^T X^{-1} Z \end{pmatrix}$$

which is the key (Gaussian elimination-like) factorization in (Scherer, 2000) and (5).

In other words, the linearizing change of variables $(X_i, Y_i, Z_i) \to (Q_i, R_i, S_i)$ of (Scherer, 2000) is nothing else but the (dual) scattering operation $\mathcal{X}_i \to \check{\mathcal{X}}_i = dchain^{-1}(\mathcal{X}_i)$ in reference to the map

$$\begin{pmatrix} X_i & Z_i \\ Z_i^T & Y_i \end{pmatrix} \to \begin{pmatrix} -S_i & Q_i \\ R_i & S_i \end{pmatrix} \qquad (8)$$

A similar factorization and a related linearizing change of variables can of course be obtained using the formula (2) and factorization (6).

4.4 *State-space Computations*

A state-space realization of $\hat{P}(z) = chain(P)$ can readily be obtained from a state-space realizations of $P(z)$ and vice-versa. One can thus assume that $T_{z_i w_i} = Hom(S_i, Q)$ with

$$S_i = \hat{P}_i.U = \begin{pmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{pmatrix}$$

has a state-space realization in the form

$$S_i = \left(\begin{array}{cc|cc} A_1 & \hat{A} & \hat{B} & B_{1i} \\ 0 & A_2 & 0 & B_{2i} \\ \hline C_{1i} & C_{2i} & D_i & E_i \\ 0 & \hat{C} & 0 & F_i \end{array}\right) \qquad (9)$$

Mimicking the approach in (Scherer, 2000), one can then easily show that closing the loop with a *static* feedback $u = Ny$ gives the closed-loop representation

$$T_{z_i w_i} = \left(\begin{array}{cc|c} A_1 & \hat{A} + \hat{B}N\hat{C} + B_{1i}\hat{C} & (\hat{B}N + B_{1i})F_i \\ 0 & A_2 + B_{2i}\hat{C} & B_{2i}F_i \\ \hline C_{1i} & C_{2i} + (D_iN + E_i)\hat{C} & (D_iN + E_i)F_i \end{array}\right)$$

The *dynamic* case where : $Q(z) = (A_Q, B_Q, C_Q, D_Q)$ fits into the above framework by setting :

$$u = N\tilde{y} = [C_Q\ D_Q]\begin{pmatrix} x_Q \\ y \end{pmatrix}$$

and by incorporating the fixed (A_Q, B_Q) matrices into the terms $\hat{A}$ and $\hat{B}$ in (9).

Then just like in (Scherer, 2000), direct computations show that the quantities :

$$\Pi_{i1}^T \mathcal{X}_i \Pi_{i1}\ ,\ \Pi_{i1}^T \mathcal{X}_i \mathcal{A} \Pi_{i1}\ ,\ \Pi_{i1}^T \mathcal{X}_i \mathcal{B}_{wi}\ ,\ \mathcal{C}_{zi}\Pi_{i1}$$

and their transpose are linear expressions of the variables Q_i, R_i, S_i, C_Q, D_Q.

4.5 Parametrization of the Q Parameter

Limiting the search of the parameter $Q(z) \in RH_\infty$ to the form $\sum \bar{Q}_j q_j(z)$ is essential in order to transform the multiobjective control problem into a tractable LMI problem. It turns out however that this is also the main limitation of the approach as high order expansions might be necessary, due notably to the fact that the pole structure is fixed through the pair (A_Q, B_Q).

The chain scattering framework can be useful in this important issue. Indeed a FIR expansion is nothing but a cascade expansion of the form

$$Q(z) = Hom(\Theta_1(z) ... \Theta_{N-1}(z), \Theta_N)$$

with

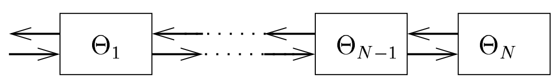

Fig. 4. *Cascade Representation of* $Q(z)$

$$\Theta_j(z) = \begin{pmatrix} I & \bar{Q}_j \\ 0 & I \end{pmatrix}\begin{pmatrix} I & 0 \\ 0 & z^{-1} \end{pmatrix} \text{ and } \Theta_N = Q_N.$$

The above remark stresses the *recursive* nature, cf. (Antoulas, 1986) of the problem at hand which needs to be further exploited. It also suggests that other cascade expansions might be more judicious, like the one resulting from the classical Schur algorithm, viz.

$$\Theta_j(z) = \begin{pmatrix} K_j^\star & K_j^c \\ K_j^c & -K_j \end{pmatrix}\begin{pmatrix} I & 0 \\ 0 & z^{-1} \end{pmatrix}$$

with $K_j^c = (I - K_j^2)^{1/2}$ and $\Theta_N = K_N$.

The matrices K_j are usually called the Schur or reflection coefficients associated to $Q(z)$ and are obtained by the tangential Schur algorithm, see e.g. (Ball *et al.*, 1990). While the main advantage of using the FIR coefficients $\bar{Q}_j$ is that they allow a direct LMI treatment of the multi-objective control problem, the use of reflection coefficients K_j yields a more compact description of the free parameter $Q(z)$.

The Schur algorithm is indeed known to provide a compact (IIR and actually continued fraction-like) representation of the stable rational plants. They moreover can be used to completely describe the differential stucture of the sub-space RH_∞ made of the all-pass functions for which they can provide charts, cf. (Alpay-Baratchart-Gombani, 1994).

Using the local coordinates of these charts it becomes thus possible to perform some optimization search within the manifold RH_∞. Encouraging results have already been obtained in that direction, for L_2 approximation problems see (Marmorat-Olivi-Hanzon-Peeters, 2002) and (Hanzon-Olivi-Peeters, 2003).

In the multiobjective problem studied here, this optimization concern the poles of the *lossless part* of $Q(z)$ via the matrices $(A_Q,\ B_Q)$ and can be performed within a sequence of LMI problems. This work is currently under investigation and will be reported later, cf. (Marmorat-Olivi-Drai, 2004).

4.6 Extensions

It should also be noted that the proposed interpretation for the change of variables of (Scherer, 2000) is not restricted to the Youla-Kucera approach. Indeed the one proposed in (Scherer, 2001) for the so-called structured case can also be obtained likewise.

More precisely the map: $\mathcal{X} \rightarrow R = R^t = (R_{ij})_{i,j=1...3}$ defined by the key factorization property

$$\begin{pmatrix} R_{11} & 0 & 0 \\ R_{12}^T & I & 0 \\ R_{13}^T & 0 & I \end{pmatrix}\mathcal{X} = \begin{pmatrix} I & -R_{12} & -R_{13} \\ 0 & R_{22} & R_{23} \\ 0 & R_{23}^T & R_{33} \end{pmatrix}$$

can be obtained by a "reversing arrows process" that generalizes the dual-scattering operation and is illustrated in Figure 5 below.

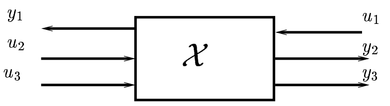

Fig. 5. *Structured Change of Variables*

The I/O variables being related by

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ -R_{12}^T & R_{22} & R_{23} \\ -R_{13}^T & R_{23}^T & R_{33} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

Similar *structured* linearizing change of variables can thus be derived in a more systematic manner using similar remarks. This is especially true for control problems with strong interaction and communication structures like the ones studied in (Voulgaris, 2001).

5. CONCLUSION

This paper demonstrated the relevance of the chain-scattering formalism to LMI multiobjective control in the spirit of various references that did the same for H_∞ control.

The advantages of this formalism are believed to be both conceptual and computational as the chain-scattering approach typically simplifies the algebra and provides some insight on some known results. As an example, it brings some light on the derivation of the various *ad-hoc* change of variables which remains one of the most obscure and frustrating aspects of the LMI approach in control and is not fully understood to this day, cf. for example (de Oliveira Helton, 2003) for the change of variables of (Scherer *et al.*, 1997).

Another contribution of this paper is to point out the possibility to fertilize the purely elementary and algebraic LMI approach with the rich and vast topic of Schur analysis which was very instrumental in the pioneering days of H_∞ control. For these various reasons it is believed that the chain-scattering interpretation provided in this paper can be helpful and deserve to be further investigated.

REFERENCES

Alpay-Baratchart-Gombani (1994). On the differential structure of matrix-valued rationnal inner functions. In: *Operator Theory: Advances and Applications*.

Antoulas, A.C. (1986). On recursiveness and related topics in linear systems. *IEEE Trans. Automatic Control* **31**, pp. 1121–1135.

Ball-Helton-Verma (1991). A factorization principle for stabilization of linear control systems. *International Journal of Robust and Nonlinear Control* **1**, 229–294.

Ball, J. A., I. C. Gohberg and L. Rodman (1990). *Interpolation of rational matrix functions*. Birkhäuser.

Boyd, S. P. and C. H. Barratt (1991). *Linear Controller Design: Limits of Performance*. Prentice Hall.

de Oliveira Helton (2003). Computer algebra tailored to matrix inequalities in control. In: *Proceedings of* 42^{nd} *IEEE CDC*.

Hanzon-Olivi-Peeters (2003). Balanced realizations of discrete-time stable all-pass systems and the tangential schur algorithm. In: *INRIA Technical Report 5111*.

Kimura, H. (1997). *Chain -Scattering Approach to* H_∞ *Control*. Birkhauser, Boston, MA.

Kučera, V. (1979). *Discrete Linear Control: The Polynomial Equation Approach*. Wiley.

Marmorat-Olivi-Drai (2004). Applications of nudelman synthesis to lmi optimization. In: *under preparation*.

Marmorat-Olivi-Hanzon-Peeters (2002). Matrix rational h_2 approximation : A state-space approach using schur parameters. In: *Proceedings of* 41^{th} *IEEE CDC*.

Nett-Jacobson-Balas (1984). A connection between state-space and doubly coprime representations. *IEEE Trans. Automatic Control*.

Scherer, C. (2000). An efficient solution to multi-objective control problems with lmi objectives. *Systems Control Letters* pp. 75–84.

Scherer, C. (2001). Multi-objective control without youla parametrization. In: *Perspectives in Robust Control, Moheimani Ed., LNCIS Springer-Verlag, pp. 311-325*.

Scherer, C., P. Gahinet and M. Chilali (1997). Multi-objective output feedback control via lmi optimization. *IEEE Trans. Automatic Control, vol. 42, pp. 896-911*.

Verma-Zames (1991). An energy and two-port framework for h_∞ control. In: *Control of Uncertain Dynamic Systems, Bhattacharayya and Keel (Eds.), pp. 39-51*.

Voulgaris (2001). A convex characterization of classes of problems with specific interaction and communication structures. In: *Proceedings of the American Control Conference*.

Youla, D. C., J. J. Bongiorno and H. A. Jabr (1976). Modern Wiener-Hopf design of optimal controllers, Part II: The multivariable case. *IEEE Trans. Automat. Control* **21**, 319–338.