A STABLE REAL-TIME OPTIMAL MULTIPLE-MODEL BASED CONTROL OF A NONLINEAR PROCESS

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Abstract: This article presents a real-time H_2 optimal control platform which guarantees the stability of a system and also is capable of controlling some complex nonlinear processes using a multiple model based concept. The idea is based on developing local linear models for the whole operating range of the process being controlled in Matlab/Simulink/Real-Time-Workshop environment using Visual C++ and Watcom compilers and a DAQ interface. Two real nonlinear plants are approximated by sets of local models where each model is valid for a small operating region and connected together to form a global continuous model using proper Gaussian validity and interpolation functions. Stability of the global system based on the local stability of the sub-systems is also addressed and results are presented.

Keywords: Optimal Control, Real Time, Multiple Model, Data Acquisition.

1. INTRODUCTION

Almost every linear or nonlinear control strategy centers around a process model, with the real processes being usually nonlinear functions. The accuracy of the model could directly influence the performance of the final controller. Linear models are often easy to develop and lead to relative simplicity in the design of controllers. Also the stability with linear models can be derived easier than with nonlinear ones. There has always been a compromise between the simplicity of the model and its accuracy. This is to say that a very accurate model but highly nonlinear could also lead to a dramatic increase in the complexity of the controller and therefore not very appealing.

Multiple modeling based control has been a research tool in various applications. One conventional method that guarantees the stability of a global system and includes some local sub-systems or controllers is addressed by Fernandez-Anaya and Escandon-Alcazar (1997) and is called Simultaneous Stabilization. Johansen and Foss (1995, 1997) present an empirical modeling of a heat transfer process using local models and interpolation. Gregorcic and Lightbody (2000) compared pole-placement self-tuning control with the multiple model approach for the control of a highly nonlinear process. A nonlinear Continuously Stirred Tank Reactor (CSTR) process is used to highlight some of the difficulties associated with self-tuning control. Doya *et al* (2002) introduced a modular reinforcement learning architecture for non-linear, non-stationary control tasks which is called Multiple Model-based Reinforcement Learning (MMRL). There are two other methods used for complex system with several operation behaviors. The first method is introduced by Aarhus (1994) called Partial Least Squares regression (PLS) model that usually is used for nonlinear empirical modeling. The second one is addressed by Angelis (2001) named Polytopic Linear Models (PLM) which is used for modeling, control and identification. One of the latest applications of the multiple modeling approach in radar and communication is introduced by Bar-Shalom and Blair (2000) which is called Interacting Multiple Model (IMM) estimator and provide superior tracking performance compared to maneuver detection schemes.

Each of the above methods has some advantages and special complexity. Besides, none of the methods has been applied to Real-Time applications. In this article, it is tried not only to develop the optimal multiple model based method in a simple way, but also to present its real-time implementation. This paper is organized as follows. In Section 2 of this paper, the real time processes are introduced and in Section 3 the multiple model based approach is presented. Section 4 reviews the optimal control method for sample data systems. Stability issue is addressed in Section 5. Experimental results are provided is Section 6, and concluding remarks are considered in Section 7.

2. REAL-TIME PLATFORM FOR THE REAL PROCESSES

The systems used in this article are an experimental Heating plant and a Twin Rotor MIMO System (TRMS).
The Heating plant is the main case study and composes of an “air tube” which contains a “heating element” as input, “temperature sensor” as output and an “air damper” as a disturbance. The aim is to control the output temperature of this system which has a special nonlinear behavior.

Consider the experimental Heating plant depicted in Figure 1. The practical process consists of a tube, an air damper, a heating element and a temperature measuring device. Air enters the tube and is warmed up by the heating element. The temperature of the air is measured by the a sensor and is sent to the controllers to make a proper signal. The variables are:

- Input voltage *u(t)* which is applied to the heater and changes by the fire angle of a BT-137 Triac.
- Fan driver *v(t)* that is considered as a disturbance and changes by the potentiometer which controls the fan driver containing two BD-140 and 2N-3055 Transistors.
- Output temperature *y(t)* which is measured by an LM-35 Transistor and amplified by an OP-07 Op-Amp. The measured output sensitivity is 1V/20ºC.
- For implementation of identification and control algorithms, the thermal process is connected to a computer via a PCL-818HG DAQ-card of Advantech (1994). The platform to implement the control and identification procedures is developed within Matlab/Simulink/Real-Time-Workshop. This process was built in Shiraz University.

The TRMS is a laboratory scale set-up designed for control experiments by Feedback Company.

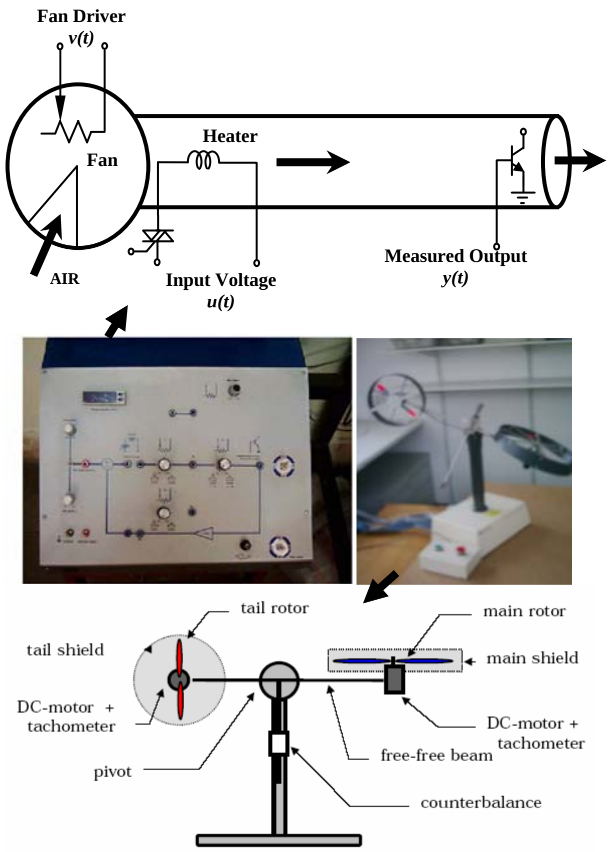

Fig. 1: The thermal-process and the Twin Rotor MIMO System (TRMS).

In certain aspects this plant’s behavior resembles that of a helicopter (see Feedback, 2004). From the modeling and control point of view it exemplifies a high order non-linear system with significant cross couplings (Figure 1). Iit consists of a beam pivoted on its base in such a way that it can rotate freely both in horizontal and vertical planes. At both ends of the beam there are rotors (the main and tail rotors) driven by DC motors. A counterbalance arm with a weight at its end is fixed to the beam at the pivot.

The states of the beam are described by four process variables: horizontal and vertical angles measured by position sensors fitted at the pivot, and two corresponding angular velocities. The control outputs are the voltages applied to the DC motors. A change in the voltage value results in a change of the rotation speed of the propeller which results in a change of the corresponding position of the beam. The plant can be fixed as two have a one DOF or two DOF. This plant is connected to a computer via a PCL-812PG DAQ-card of Advantech. In the TRMS for the one DOF study, the input is the voltage applied to the main motor (U_v) and output is the vertical position of the TRMS beam (pitch position, $y=\alpha_v$). This behavior is quite nonlinear and oscillatory (see Feedback, 2004, for detail).

3. NONLINEAR MODELING USING MULTIPLE MODELS

Consider a general stable nonlinear system. The system operating range under all possible operating conditions are divided into several regimes (Johansen,1994) where in each regime the system can be represented by a linear model. The system's full operating range is completely covered by these regimes. The operating regimes could depend on states, inputs and outputs of the process.

One linear model is associated with each of the regimes and describes the system behavior in that regime. At some operating regions there may be overlaps between the regimes where several linear models are valid. In these cases a single model cannot be used to represent the system. In order to solve this problem a validity (weighting) function is associated with each of the local linear models and then the nonlinear process is approximated with a weighted combination of the local linear models using validity functions.

Assume that m linear ARMAX models have been identified to explain the plant behavior at different operating regimes and that all information about the plant is contained in the local models. Each of the local models has the following form:

$$y_p(t)=\sum_{i=1}^{n}(a_i^p\, y(t-i)+b_i^p\, u(t-i)+c_i^p\, v(t-i)) \quad (3.1)$$

The above equation shows the pth local model, where y_p is the output, u is the input, v is the disturbance, and a_i^p , b_i^p and c_i^p are the model coefficients. In general the global system is represented as:

$$y(t)=\sum_{p=1}^{m}\sum_{i=1}^{n}[\mu_p\,(a_i^p\, y(t-i)+b_i^p\, u(t-i+1)+c_i^p\, v(t-i)] \quad (3.2)$$

μ_ps are relative validity functions with the following characteristics:

$$\mu_p=f(z(t))\rightarrow[0,1],\forall p$$
$$\sum_{p=1}^{m}\mu_p=1 \qquad z(t):(y(t),u(t),v(t)) \quad (3.3)$$

where $z(t)$ is the operating point. The relative validity of each of the local models at a particular operating point is indicated by μ_p. If at a given operating point the local model j is accurate then $\mu_j = 1$ and $\mu_p=0,\forall p\neq j$. Also under some operating conditions there may be several local models which are valid. The local ARMAX model structures are combined into an NARMAX model structure. The input, disturbance, and output deviations are thus decomposed into separate regimes for both plants as shown in Table 1.

Table 1: The local regimes of the plants.

Regime	Heating Plant	TRMS
1	$u(t)\in[0,1]$, $v(t)\in[3,4]$	$y(t)\in[-0.4,-.2]$
2	$u(t)\in[0,1]$, $v(t)\in[4,5]$	$y(t)\in[-0.2,0]$
3	$u(t)\in[1,2]$, $v(t)\in[3,4]$	$y(t)\in[0,0.2]$
4	$u(t)\in[1,2]$, $v(t)\in[4,5]$	$y(t)\in[0.2,0.4]$
5	-	$y(t)\in[0.4,0.6]$
6	-	$y(t)\in[0.6,0.8]$

To combine the local ARMAX model structures into an NARMAX model in a smooth manner as mentioned before, one needs to define a validity function which shows the relative validation of each local model. The validity functions (ρ) are considered two dimensional Gaussian functions and the interpolation functions (μ) are also given below (assuming $z(t)=(u(t),v(t))$, for instance):

$$\rho_i(u,v)=e^{-\frac{1}{2}\left[\left(\frac{u-u_i}{\sigma_u}\right)^2+\left(\frac{v-v_i}{\sigma_v}\right)^2\right]} \quad (3.4)$$

$$\mu_i(u,v)=\frac{\rho_i(u,v)}{\sum_{j=1}^{j=4}\rho_j(u,v)} \quad (3.5)$$

The output of the model can be found by combining the outputs of local models with interpolating functions which depend on the value of input and disturbance for the Heating plant and output for the TRMS. This can be done by an off-line algorithm. One important task is to choose the best variance for the validity functions such that minimum NRMSE can be achieved. The performance of multiple model approach can be seen in Figure 3 for both real systems. In Figure 3(b) depicts the true data versus their approximations to clearly see the approximation quality, since the TRMS has a very oscillatory behavior. A single ARMAX (i.e. for the whole regions) model approximation quality is also shown for a better comparison.

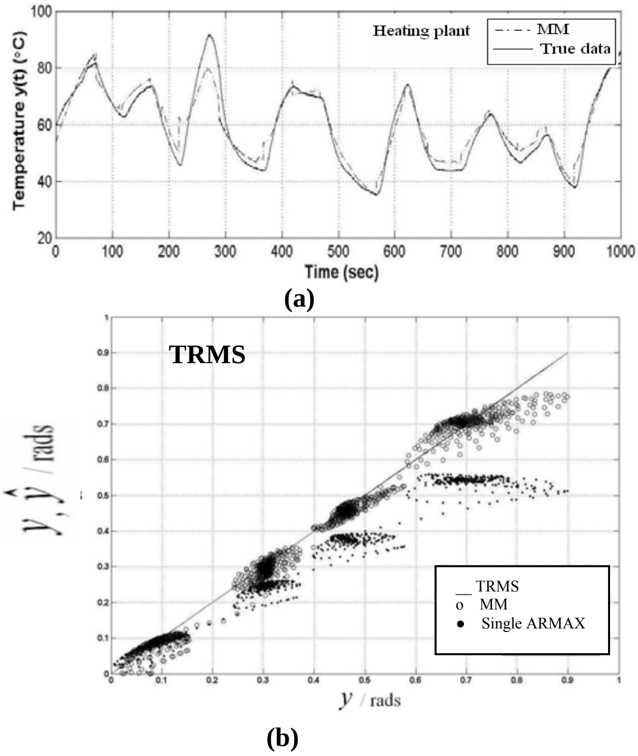

Fig. 3: Multiple model (MM) approximation capability. (a) Heating plant (True data and MM), (b) TRMS plant (True data, MM, and a single ARMAX for the whole region). Here the discrepancy from the line shows errors. $y=\alpha_v$ (in radians) and input is U_v (in volts).

4. H_2 OPTIMAL SAMPLED DATA CONTROL

In this section the procedure for designing the optimal control system is presented. First an optimal H_2 controller is designed (see Chen and Francis, 1995) for each of the local models and then these controllers are combined together to obtain the global control law. Each of the optimal controllers produces a control law that minimizes a cost function and provides a local stable closed loop system. Assume all of the local models are completely stabilizable and detectable and all of the states are available. Consider the block diagram shown in Figure 4.

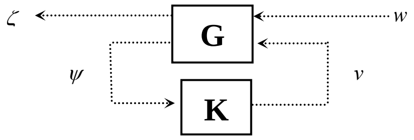

Fig. 4: Block-diagram of a typical closed loop system for H_2-optimal control (Chen and Francis, 1995).

where *w, v,* ψ and ζ are respectively the input, disturbance, output and the cost function or desired error to be minimized to meet the best performance. Consider ξ as the state vector of the system in discrete time and the control sequence *v(k)* is to be chosen to minimize the following cost function:

$$J=\sum_{k=0}^{\infty}\left[\xi_k{}^T Q\xi_k+v_k{}^T R v_k\right] \qquad (4.1)$$

To find the optimal control solution, the following equations should be solved:

$$-\xi_{k+1}+A\xi_k+Bv_k=0 \qquad (4.2)$$

$$v_k^T R+\lambda_{k+1}^T B=0 \qquad (4.3)$$

$$\xi_{k+1}Q-\lambda_k^T+\lambda_{k+1}^T A=0 \qquad (4.4)$$

where A and B are parameters of the state space models and λ is the co-state vector. Combining these equations we have:

$$\begin{bmatrix} I & BR^{-1}B^T \\ 0 & A^T \end{bmatrix}\begin{bmatrix} \xi_{k+1} \\ \lambda_{k+1} \end{bmatrix}=\begin{bmatrix} A & 0 \\ -Q & I \end{bmatrix}\begin{bmatrix} \xi_k \\ \lambda_k \end{bmatrix} \qquad (4.5)$$

or: $$\begin{bmatrix} \xi_{k+1} \\ \lambda_{k+1} \end{bmatrix}=S_2\begin{bmatrix} \xi_k \\ \lambda_k \end{bmatrix} \qquad (4.6)$$

where: $$S_2=\begin{bmatrix} I & BR^{-1}B^T \\ 0 & A^T \end{bmatrix}^{-1}\begin{bmatrix} A & 0 \\ -Q & I \end{bmatrix} \qquad (4.7)$$

Half of the eigenvalues of S_2 are inside the unit circle and, that is, they are stable and should be chosen. Thus, a matrix is defined whose columns are the generalized eigenvectors of S_2 corresponding to stable eigenvalues as:

$$T=\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \qquad (4.8)$$

where T_2 is invertible, and let:

$$X=Ric(S_2)=T_2T_1^{-1} \qquad (4.9)$$

The optimal state feedback control can be obtained as:

$$v=F\xi \qquad (4.10)$$

$$F=-(R+B^TXB)^{-1}B^TXA \qquad (4.11)$$

Since disturbances occur in the process and all of the states are not available, an *Output Feedback* for each local model should be used. First, the following notation is defined which is called a *Symplectic Pair*:

$$S_2=(S_l,S_r)=S_r^{-1}S_l \qquad (4.12)$$

$$S_l=\begin{bmatrix} A & 0 \\ -Q & I \end{bmatrix},\ S_r=\begin{bmatrix} I & P \\ 0 & A^T \end{bmatrix} \qquad (4.13)$$

as is used in Equation (4.7). So a dynamic feedback should be designed, which is called a *Finite-Dimensional Linear-Time-Invariant* (FDLTI) controller *K*, and also is causal and admissible if it achieves internal stability. Now consider the general output feedback case with:

$$\hat{g}=\left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & I & D_1 \\ C_2 & D_2 & 0 \end{array}\right] \qquad (4.14)$$

and with the following assumption:

i. $M=D_1^TD_1$ and $N=D_2D_2^T$ are nonsingular.

ii. The matrices $\begin{bmatrix} A-\lambda I & B_2 \\ C_1 & D_1 \end{bmatrix}$ and $\begin{bmatrix} A-\lambda I & B_1 \\ C_2 & D_2 \end{bmatrix}$ have full column and row rank, respectively, $\forall\lambda$.

These three assumptions guarantee that the symplectic pair, such as S_2, belongs to domain of $Ric(S_2)$. Also, the second part means that the sensor noise weighing is nonsingular. Finally, the last assumption is related to the condition in the *Model-Matching Problem* for the defined transfer functions on their domain. So, we can define:

$$S_2=\left(\begin{bmatrix} A-B_2M^{-1}D_1^TC_1 & 0 \\ -C_1^T(I-D_1M^{-1}D_1^T)C_1 & I \end{bmatrix},\begin{bmatrix} I & B_2M^{-1}B_2^T \\ 0 & (A-B_2M^{-1}D_1^TC_1)^T \end{bmatrix}\right)$$
(4.15)

$$X=Ric(S_2) \qquad (4.16)$$

$$F=-(M+B_2^TXB_2)^{-1}(B_2^TXA+D_1^TC_1) \qquad (4.17)$$

$$F_0=-(M+B_2^TXB_2)^{-1}(B_2^TXB_1+D_1^T) \qquad (4.18)$$

$$\hat{g}_c(\lambda)=\left[\begin{array}{c|c} A+B_2F & B_1+B_2F_0 \\ \hline C_1+D_1F & I+D_1F_0 \end{array}\right] \qquad (4.19)$$

and also:

$$T_2=\left(\begin{bmatrix} (A-B_1D_2^TN^{-1}C_2)^T & 0 \\ -B_1(I-D_2^TN^{-1}D_2)B_1^T & I \end{bmatrix},\begin{bmatrix} I & C_2^TN^{-1}C_2 \\ 0 & A-B_1D_2^TN^{-1}C_2 \end{bmatrix}\right)$$
(4.20)

$$Y=Ric(T_2) \qquad (4.21)$$

$$L=-(AYC_2^T+B_1D_2^T)(N+C_2YC_2^T)^{-1} \qquad (4.22)$$

$$L_0=(FYC_2^T+F_0D_2^T)(N+C_2YC_2^T)^{-1} \qquad (4.23)$$

$$R=(M+B_2^TXB_2)^{1/2} \qquad (4.24)$$

$$\hat{g}_f(\lambda)=\left[\begin{array}{c|c} A+LC_2 & B_1+LD_2 \\ \hline R(L_0C_2-F) & R(L_0D_2-F_0) \end{array}\right] \qquad (4.25)$$

Then the unique optimal controller is:

$$\hat{k}_{opt}(\lambda)=\left[\begin{array}{c|c} A+B_2F+LC_2-B_2L_0C_2 & L-B_2L_0 \\ \hline L_0C_2-F & L_0 \end{array}\right] \qquad (4.26)$$

Moreover:

$$\min_K\left\|\hat{t}_{\zeta w}\right\|_2^2=\left\|\hat{g}_c\right\|_2^2+\left\|\hat{g}_f\right\|_2^2 \qquad (4.27)$$

where the state vector of the gain k_{opt}, is $\begin{bmatrix} \eta \\ w \end{bmatrix}$ and η denote the estimated states and w is the measured disturbance from the plant. This gain estimates a new vector $\begin{bmatrix} \eta \\ v \end{bmatrix}$ for each local model where v is the input that should be applied to the plant.

5. STABILITY ANALYSIS

The stability of the global system is straightforward when the local models are stable and number of linearized models is sufficient. This can be shown by the following theorem:

Theorem: Consider the set M of m stable subsystems:

$x_{k+1} = A_p x_k$ for $p = 1,2,\ldots,m$ and $A_p \in M$ (5.1)

or using *Lyapunov Stability* for discrete systems (Slotine, and Li, 1991):

$$A_p^T S A_p - S < 0 \ , \ \forall p, \tag{5.2}$$

for positive definite matrix S , and μ_p 's are the interpolation functions such that:

$$\sum_p \mu_p = 1 \quad , \quad 0 \le \mu_p \le 1 \tag{5.3}$$

Then the global system

$$x_{k+1} = \sum_p \mu_p A_p x_k \tag{5.4}$$

is stable and:

$$\left(\sum_p \mu_p A_p \right)^T S \left(\sum_p \mu_p A_p \right) - S < 0, \tag{5.5}$$

for positive definite matrix S.

Proof: Considering equation (5.3), since $\sum_p \mu_p = 1$, by expanding $\left(\sum_p \mu_p \right)^2$, the left hand side of the inequality (5.5) can be presented in the following form:

$$\left(\sum_p \mu_p A_p \right)^T S \left(\sum_p \mu_p A_p \right) - \left(\sum_p \mu_p \right)^2 S = \cdots$$

$$\sum_p \mu_p^2 (A_p^T S A_p - S) + \sum\sum_{p \ne q} \mu_p \mu_q (A_p^T S A_q - 2S) < 0$$

$$A_p, A_q \in M \ , \forall p, \forall q \tag{5.6}$$

The derivation is straightforward and thus the theorem can be proved. Two other approaches to stability are presented in (Wan and Kothare, 2004; McConley *et al*,2000).

6. REAL-TIME IMPLEMENTATION RESULTS

Consider, **first, the Heating plant** described in Figure 1. The state-space representation for each local model of the plant was worked out and as a typical result two of the models are as follows:

Local Model of Regime #1:

$$A = \begin{bmatrix} 1.9838 & -0.9839 \\ 1 & 0 \end{bmatrix}, \ B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -0.383 \cdot 10^{-3} & 0.4508 \cdot 10^{-3} \end{bmatrix}, D_1 = \begin{bmatrix} -4.582 \cdot 10^{-4} \end{bmatrix}$$

$$C_2 = \begin{bmatrix} -0.2549 \cdot 10^{-5} & 0.1264 \cdot 10^{-5} \end{bmatrix} D_2 = \begin{bmatrix} -1.285 \cdot 10^{-6} \end{bmatrix}.$$

Local Model of Regime #2:

$$A = \begin{bmatrix} 1.9765 & -0.9766 \\ 1 & 0 \end{bmatrix}, \ B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, ,$$

$$C_1 = \begin{bmatrix} -0.6207 \cdot 10^{-3} & 0.70396 \cdot 10^{-3} \end{bmatrix} D_1 = \begin{bmatrix} -7.208 \cdot 10^{-4} \end{bmatrix}$$

$$C_2 = \begin{bmatrix} -0.5046 \cdot 10^{-5} & 0.2493 \cdot 10^{-5} \end{bmatrix}, \ D_2 = \begin{bmatrix} -2.553 \cdot 10^{-6} \end{bmatrix}.$$

Now, by following the straightforward procedure from Equations (4.15)-(4.18) and (4.20)-(4.23), one can calculate S_2, X, F and F_0 for each local model presented above and also T_2, Y, L and L_0. Finally, the four optimal control gains $\hat{k}_{opt}(\lambda)$ are computed. The first two gains as part of the results are:

$$\hat{k}_{opt1} = \begin{bmatrix} -2.6732 & 1.3262 & 10.4898 \\ 0.8463 & 0.07618 & 6.0253 \\ 2.6732 & -1.3262 & -10.4820 \end{bmatrix} \hat{k}_{opt2} = \begin{bmatrix} -1.7957 & 0.8877 & 3.5604 \\ 0.8472 & 0.07547 & 3.0270 \\ 1.7957 & -8.8774 & -3.5565 \end{bmatrix}$$

The optimal gains are implemented in SIMULINK/MATLAB/Real-Time environment using DAQ-Card to apply online optimal control for the thermal process. These controllers are combined via the validity and interpolation functions to obtain the global controller for the thermal plant.To illustrate the performance of the optimal multiple model controller three random setpoint changes at 15, 120 and 160 sec. together with disturbance changes at 35, 140 and 270 sec. with 20 sec. duration were applied. The data sequences are shown in Fig.5. The results of an experiment with multiple PID controllers (Aminzadeh *et al*, 2003) and its comparison to the optimal multiple-model based control are shown in Fig.6.

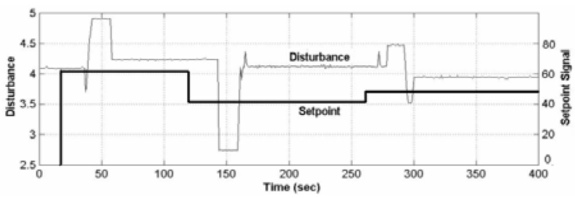

Fig. 5: Applied setpoint and disturbance for multiple optimal controller of the Heating plant.

To check the stability of multiple model based controller, it is sufficient to the check the stability of the local controllers and observers. Following the procedure will guarantee the stability but for demonstrations eigenvalues of two of the local controllers associated with the local models previously given are shown in Table 2.

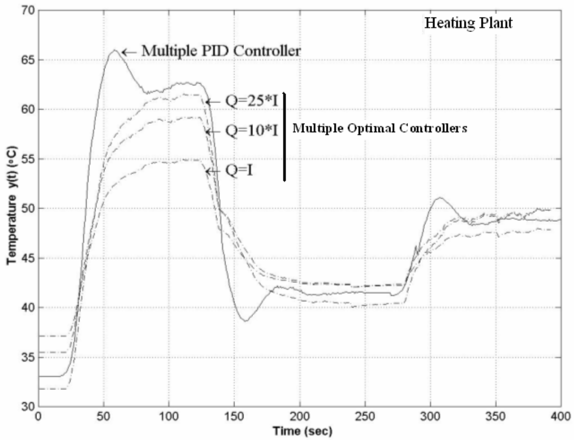

Fig. 6: Mult. optimal cont. responses of the Heat. P.

For the TRMS plant, a typically similar study for setpoint following optimal control was performed and the results are shown in Figure 7. In this figure both the multiple optimal controller results and a single model based optimal controller were presented. Though the single optimal controller seems to act reasonable at these setpoint regions it is clear to see at the end of the setpoint its oscillation is increasing while the multiple model seems to act properly.

7. CONCLUSIONS

In this article a study of real-time control of two nonlinear processes were presented. The method was based on multiple linear models. A global multiple model optimal control was designed. Results of implementation of real-time control for the nonlinear heating plant and a Twin Rotor MIMO System (TRMS) were provided to show the effectiveness of the technique. It is seen that the process can be controlled satisfactorily using the resulting optimal controllers. The heating plant is to some extent a slow dynamic a reasonably nonlinear plant. The TRMS is a highly nonlinear and fast process. Nevertheless the technique was quite effective in real-time control of both plants.

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Table 2: Stability in the local regimes of Heat.plant

Stability for Regime #1	
$\hat{\lambda}_1(S_2) = 5.3977 \cdot 10^{-4}$	$\hat{\lambda}_2(S_2) = 0.8711$
$\hat{\lambda}_1(T_2) = 0.07935$	$\hat{\lambda}_2(T_2) = 7.4905 \cdot 10^{-15}$

Stability for Regime #2	
$\hat{\lambda}_1(S_2) = 7.9444 \cdot 10^{-5}$	$\hat{\lambda}_2(S_2) = 0.89651$
$\hat{\lambda}_1(T_2) = 0.07860$	$\hat{\lambda}_2(T_2) = 1.3853 \cdot 10^{-17}$

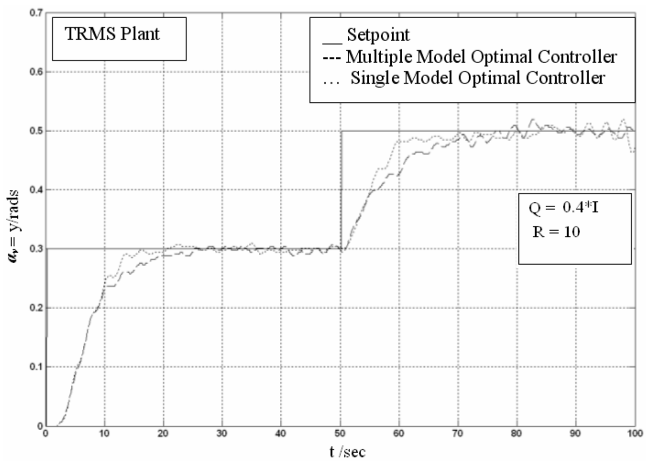

Fig.7: Multiple optimal control responses of the TRMS plant. Here, the TRMS the input is the voltage applied to the main motor (U_v) and output is the vertical position of the TRMS beam (pitch position, $y=\alpha_v$).