# BALANCE-BASED ADAPTIVE CONTROL OF THE ELECTRIC FLOW HEATER

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Abstract: This paper concentrates on the Balance-Based Adaptive Control (B-BAC) methodology and on its application to the control of the electric flow heater. The B-BAC methodology is dedicated for the control of a wide range of biochemical and heat exchange processes and the final form of the control law results from the general and simplified first-order model of a process, in which all unknown nonlinearities and modelling inaccuracy are replaced by the only one time-varying parameter. The value of this parameter is estimated on-line by RLS with very high accuracy and its application in the control law ensures the adaptability. The application of the B-BAController to the electric flow heater shows the generality of this methodology. The simulation results prove its very good control performance and robustness in spite of the fact that the complete nonlinear description of the system is assumed to be unknown. *Copyright* © 2005 IFAC

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# 1. INTRODUCTION

The electric flow heater is a unit that is used in all cases when the flowing water is to be warmed by the electric power supply. Because we always expect this device to combine possibly small power consumption with the high efficiency, its control is a challenging problem. In practical cases, the temperature and the flow rate of the incoming water should be considered as the disturbances and thus the power supply is the only possible manipulated variable.

One possibility for the control of the electric flow heater is the classical PI controller. This approach is very general and it can provide quite good control performance when the system is operated at one working point. If not, there is a need to apply the gain-scheduling technique, which requires preliminary off-line estimation of the system dynamical properties. The other possibility is to apply one of the advanced adaptive nonlinear modelbased techniques (Joshi *et al.*, 1997; Seborg, 1999). They usually allow for the significant improvement of the control performance but there is always one limitation: at least a part of the complete nonlinear description of the system with the nonlinearity and with the values of the parameters must be known. If not, the model-based approach cannot be applied.

In this paper we suggest the application of the B-BAC methodology to control the electric flow heater. This model-based methodology combines the simplicity and generality of the classical PI controller with very good control performance, adaptability and robustness characteristic for advanced nonlinear adaptive controllers. As in the majority of the practical cases, we assume that the complete mathematical model of the process is unknown. Then, according to the B-BAC methodology, we suggest the very simple and general first-order model and on its basis we derive the final form of the control law and of the estimation procedure. This model is written only on the basis of the general energy conservation law and all unknown nonlinearities and the modeling uncertainties are replaced by the only one time-varying parameter, whose value is estimated on-line to ensure the adaptability of the control law. The simulation results prove very good control performance of the suggested B-BAController in the presence of the changes of the set point and of the disturbances. Additionally, it is shown that the control performance is still very good, even when the measurement bias on the value of the measurable disturbing parameter occurs, which proves the robustness of the suggested control technique.

# 2. THEORETICAL APPROACH TO THE B-BAC METHODOLOGY

As it was said in the previous Section, generally the B-BAC methodology is dedicated to control a wide range of technological processes for which it is possible to define the control goal in the following way: one of the parameters characterizing a process, defined here as Y(t) and called the controlled variable, should be kept equal to its pre-defined setpoint  $Y_{sp}$ . Y(t) can be chosen as one of state variables (a component concentration or the temperature) or as a combination of two or more state variables. In a process a number of isothermal or nonisothermal biochemical reactions and/or heat exchange phenomena with unknown kinetics can take place. A process itself takes place in a reactor tank with time varying volume V(t) [m<sup>3</sup>].

The dynamical behavior of Y(t) can be described by the following well known general ordinary differential equation written on the basis of the mass or of the energy balance considerations:

$$\frac{\mathrm{d}\mathbf{Y}(t)}{\mathrm{d}t} = \frac{1}{\mathbf{V}(t)} \underline{\mathbf{F}}^{\mathrm{T}}(t) \underline{\mathbf{Y}}_{\mathrm{F}}(t) - \mathbf{R}_{\mathrm{Y}}(t) \tag{1}$$

The vector product  $\underline{F}^{T}(t)\underline{Y}_{F}(t)$  represents mass or energy fluxes incoming to or outcoming from the reactor tank. The elements of the vector F(t) are the combination of the volumetric flow rates and, consequently, the vector  $\underline{Y}_{F}(t)$  is the corresponding vector to  $\underline{F}(t)$  and its elements are the combination of the inlet values of Y(t) and of the value of Y(t) itself.  $R_{y}(t)$  is a positive or negative time varying term with an unknown expression form. It represents "one global reaction" including all reversible and/or irreversible reactions or heat exchange and/or production with unknown and nonlinear kinetics that influence the value of Y(t). Let us note that in the case when Y(t) is a state variable, the equation (1) has a generalized and simplified form of a state equation describing Y(t) and taken directly from a mathematical model of a process. However, if Y(t) is a combination of two or more state variables, a number of simplified state equations from a mathematical model must be combined together and rearranged to the form of the equation (1).

Once the equation (1) has been obtained, it can be a basis for the B-BAC under the following assumptions:

- the manipulated variable must be chosen as one of the elements of the vectors <u>F(t)</u> or <u>Y</u><sub>F</sub>(t),
- the other elements of the vectors  $\underline{F}(t)$  and  $\underline{Y}_{F}(t)$  as well as the value of Y(t) must be measurable on-line at least at discrete moments of time or they should be known by choice of the user.

If these requirements are met, at this stage we can apply the same methodology as for the 'model reference linearising control' (Isidori, 1989; Bastin and Dochain, 1990). For our control goal let us assume the following stable first-order closed loop dynamics:

$$\frac{\mathrm{d}\mathbf{Y}(t)}{\mathrm{d}t} = \lambda \big( \mathbf{Y}_{\mathrm{sp}} - \mathbf{Y}(t) \big) \tag{2}$$

where  $\lambda > 0$  is the tuning parameter. After combining the equations (1) and (2) we can obtain the following equation:

$$\underline{F}^{T}(t)\underline{Y}_{F}(t) = \lambda V(t) (Y_{sp} - Y(t)) + V(t)R_{Y}(t) \quad (3)$$

Once the manipulated variable has been chosen, the above equation can be rearranged to obtain the control law describing its value. This control law has the form of the 'model reference linearising controller' and is very well known in the bibliography.

In order to provide the adaptability to the control law resulting from the equation (3) there is a need to estimate the value of the nonmeasurable term  $R_Y(t)$ . This value can be estimated on-line at discrete moments of time by the recursive least-squares method with the forgetting factor  $\alpha$ . The estimation procedure is also based on the discretised form of the simplified model (1) and it needs the same measurement data as the control law (3) (Czeczot, 1997, 1998). When we replace  $R_Y(t)$  by its discrete time estimate  $\hat{R}_Y^i$  in the equation (3), it can be rewritten in the following discrete time form (i – discretisation instant):

$$\underline{F}^{\mathrm{T},i}\underline{Y}_{\mathrm{F}}^{i} = \lambda V^{i} (Y_{\mathrm{sp}} - Y^{i}) + V^{i} \hat{R}_{\mathrm{Y}}^{i}$$

$$\tag{4}$$

The equation (4) is a basis for the B-BAC. There is only a need to rearrange it to obtain the control law in the form that allows us to calculate the value of the manipulated variable.

#### 3. B-BAC APPLICATION TO THE ELECTRIC FLOW HEATER

In this paper, we show how to apply the B-BAC methodology to the control of the electric flow heater. We consider the system, which is schematically presented in the Fig. 1. The water, which is to be warmed, flows through the unit with the volumetric flow rate F  $[m^3/h]$  and the inlet temperature  $T_{in}$  [K]. The power supply  $P_h$  [kW] is converted into the heat flux that warms the water inside the unit and, in the result, the warm water flows out of it with the same volumetric flow rate F and with the outlet temperature Tout [K]. From the practical point of view, the controlled variable is the outlet temperature  $T_{out}$ . The inlet temperature  $T_{in}$  and the flow rate F have to be considered as the disturbances. The power supply  $P_h$  is the only possible manipulated variable.



Fig. 1.Simplified diagram of the flow electric heater

# 3.1 Mathematical model of the considered system

The complete phenomenological mathematical model of the considered electric flow heater, which would be necessary for the simulation experiments, is in fact very difficult to derive. There are a large number of different physical phenomena, which influence the dynamics of the system and for which there is a large uncertainty both on the description and on the values of the parameters. Although it is possible to suggest the simplified model, it is not suitable for the experiments since it does not describe the real system for wide changes of the parameters and input variables. Thus, during the simulation experiments, we apply the empirical model as the real system. This nonlinear model was developed and successfully validated on the real measurement data by Łaszczyk (Łaszczyk, 2000, 2000a). This data was collected from the electric flow heater working as a part of the heat distribution pilot plant (Łaszczyk and Pasek ,1995).

The nonlinear model consists of N+1 dynamic equations, which allows for the considering the imperfect mixing phenomenon.

$$\frac{dT_{j}(t)}{dt} = k_{3}F(t)(T_{j}(t) - T_{j-1}(t))$$
(5a)

$$\frac{dT_{out}(t)}{dt} = k_1 F^{\gamma}(t) (T_N(t) - T_{out}(t)) + k_2 P_h(t)$$
 (5b)

For this model j = 0..N,  $T_0(t) = T_{in}(t)$ . The value of N is the number of the sections. The parameters  $k_1$ ,  $k_2$ ,  $k_3$  and  $\gamma$  are constant and must be tuned experimentally.

#### 3.2 General simplified model of the system

If we assume that the model (5) is unknown and we want to apply the B-BAC methodology to control the system shown in the Fig. 1, the first step is to suggest its simplified model in the form of the general dynamic equation (1). Thus, first we have to assume that our unit is perfectly mixed and has the constant volume V. Then, we can apply the methodology suggested for the heat exchange processes in (Czeczot, 2000, 2001, 2002). We can derive the dynamic equation, describing the total heat Q(t) [J], accumulated inside the unit, on the basis of the general heat conservation law.

$$\frac{dQ(t)}{dt} = Q_{in}^{*}(t) - Q_{out}^{*}(t) + Q_{p}^{*}(t) - Q_{Y}^{*}(t)$$
(6)

In this equation  $Q_{in}^{*}[W]$  denotes the inlet heat flux correlated with the incoming water and  $Q_{out}^{*}[W]$  with the outcoming water.  $Q_{P}^{*}[W]$  denotes the heat flux correlated with the power supply.  $Q_{Y}^{*}$  represents the heat flux that describes the influence of the unknown nonlinearities and of the modeling uncertainties. For simplicity and clarity we assume that the unit is perfectly insulated. However, if the heat lost takes place, it can be also included in  $Q_{Y}^{*}$ .

After very simple rearrangements, we can rewrite the equation (6) in the form, which directly describes the controlled variable  $T_{out}$  and includes the manipulated variable  $P_{h}$ .

$$\frac{dT_{out}(t)}{dt} = \frac{F(t)}{V} (T_{in}(t) - T_{out}(t)) + k P_{h}(t) - R_{Y}(t)$$
(7)

The parameter  $R_Y$  represents the unknown nonlinearities of the process but also the large modeling uncertainty taking place due to the different order of the model (7) and of the system (5) and to the possible heat lost. The equation (7) satisfies all the requirements of the B-BAC methodology and thus it can be used as a basis for the B-BAController.

#### 3.3. Final form of the B-BAController

At this stage we can directly apply the B-BAC methodology, described in the previous Section. As it was said before, we define the controlled variable as  $Y(t) = T_{out}(t)$  and the manipulated variable as  $P_h(t)$ . The control goal is to keep the controlled variable equal to its set point  $Y_{sp}$ . If we define the vectors  $\underline{F}(t) = [F(t); k]^T$  and  $\underline{Y}_F(t) = [T_{in}(t) - Y(t); P_h(t)]^T$ , after discretization we can obtain the general form of the B-BAController (4) and then, since the manipulated variable has been chosen, we can rearrange it into the final form of the control law.

$$P_{h,i} = \frac{\lambda V (Y_{sp} - Y_i) - F_i (T_{in,i} - Y_i) + V \hat{R}_{Y,i}}{kV}$$
(8)

According to the B-BAC methodology the value of  $\hat{R}^{i}_{Y}$  must be estimated on-line by the recursive leastsquares procedure. The volume of the unit V can be estimated on the basis of the geometrical dimensions of the particular electric heater. The parameter k is unknown and it describes the efficiency of the conversion between the power supply P<sub>h</sub> and the heat flux that warms the flowing water.

# 4. SIMULATION RESULTS

In this Section the very good control performance and the robustness of the suggested B-BAController (8) are proved by the computer simulation. The example electric flow heater (5) is considered as the real system with the following values of the parameters: N = 3,  $k_1 = 3*10^{-2}$ ,  $k_2 = 6*10^{-2}$ ,  $k_3 = 1*10^{-3}$ ,  $\gamma = 1.2$ . As it was shown in (Łaszczyk, 2000; 2000a), these values ensure the best modeling accuracy.

For the general and simplified model of the process (7), and consequently for the B-BAController (8) we choose the volume of the unit as V = 28 [L] and the value of the parameter  $k = 1*10^{-4}$ . It must be said that we have no a priori information on the value of k and thus it must be chosen randomly. However, as it is shown, the uncertainty on its value and on the value of V has no influence on the very good control performance of the B-BAController.

In this Section we present the most representative simulation results. Every simulation experiment has been carried out in the similar way. First the system is operated in the open loop to avoid the influence of the inaccurate choice of the initial value  $\hat{R}_{Y}^{0}$  for the

estimation procedure. Let us note that we also have no a priori information for this value and thus it has to be chosen randomly as well. This value influences significantly the first stage of the estimation unless the value  $\hat{R}_{Y}$  has converged to its true value  $R_{Y}$ . Every simulation experiment starts with the system being in the steady state, characterized by the following values of the parameters: F = 0.2,  $P_h = 2.0$ ,  $T_{in} = 300$ ,  $T_{out} = 327.6$ . Then, at t = 5 the control loop is closed and the indicated step changes of the set point  $Y_{sp}$  and of the disturbing parameters are applied to the system.

Figures 2 and 3 allow for the comparison between the control performance of the suggested B-BAController (8) and of the classical PI controller. The classical PI controller is still in use in the vast majority of automatic control loops in the process industries ( $\approx$  90%) (Seborg, 1999) due to its simplicity and generality. Moreover, as it was said before, the simplicity and generality of the B-BAC methodology is rather comparable with the PI approach than with any other advanced model-based control strategies. Thus, we decided to choose the classical PI controller for the comparative studies because, in our opinion, it can be considered as the benchmark for our simulation experiments.

Both controllers have been tuned to ensure possibly the best control performance in the presence of the applied variables changes. A number of simulation experiments were carried out and we have chosen the settings for both controllers to avoid significant overregulations and to decrease the regulation time. The tuning parameter of the B-BAController was chosen as  $\lambda = 0.02$  and the forgetting factor for the estimation procedure as  $\alpha = 0.1$ . The initial value for the estimation procedure is  $\hat{R}_Y^0 = 2.0$ . The sampling time for both controller was chosen as  $T_R = 10$  [sec]. The settings for the classical PI controller are  $k_r = 0.01$  and  $T_R/T_i = 2.86*10^{-3}$ .

Fig. 2 shows the control performance of both controllers in the presence of the step changes of the set point Y<sub>sp</sub>. It can be seen that both controllers are able to keep the process stable and to compensate for the changes of the value of Y<sub>sp</sub>. However, for its step increment the B-BAController provides much shorter regulation time without any overregulations on the value of the controlled variable Y. For t = 1000, when the value of  $Y_{sp}$  is decreased, the regulation time for both controllers is comparable. It results from the practical limitation on the manipulated variable  $P_h \ge 0$ . The changes of the manipulated variable P<sub>h</sub>, presented in the lower diagram, are fully acceptable. For the B-BAController these changes are not smooth but in the practice it is possible to force even the step changes of the power supply. Let us also note that for the PI controller the small

over regulations on the value of Y occur. It shows that this controller was tuned properly and that the additional increment of its gain  $k_{\rm r}$  will result in the unstable behavior.



in the presence of the  $Y_{sp}$  step changes



Fig. 3. B-BAC and PI control performance in the presence of the step changes of the disturbing variable F

During the simulation experiments, it was found that the flow rate F is the most significant disturbance. Thus, Fig. 3 presents the control performance of both controllers in the presence of the step changes of its value. Let us note once again that the B-BAController provides much shorter regulation time and extremely smaller overregulations on the value of the controlled variable Y. In fact, in the case of the B-BAController it can be stated that the significant step change of the disturbing value F has no significant influence on the outlet temperature  $T_{out}$ . In the case of the classical PI controller, this influence is very significant. The evolutions of the manipulated variable P<sub>h</sub>, shown in the lower diagram, are again acceptable from the practical point of view.



Fig. 4. B-BAC control performance in the presence of the measurement bias on the value of F

The B-BAC methodology allows for the feedforward action in the final form of the control law. Thus, in the B-BAController (8) there is a feedforward action from the values of the disturbances F and T<sub>in</sub>. Although this action is introduced in a very natural way and without any additional effort, one can say that it is not fair to compare the B-BAController with the feedforward action with the classical PI controller without it. Therefore, the last experiment deals with the influence of the quality of the measurement data on the control performance of the B-BAController (8). The results of this experiment are presented in Fig. 4. All the parameters and variables, including the set point  $Y_{sp}$  and the disturbances F and  $T_{in}$ , are constant during the experiment. We only consider the uncertainty on the measurement data of the disturbing variable F. In other words, the step

changes of the measurement bias  $\Delta F$  are applied to the control system. Let us note that this measurement uncertainty has no significant influence on the control performance. This control performance is still very good and the robustness have been obtained due to the fact that the estimation procedure is able to compensate for the measurement uncertainty at the price of the bias on the value of the estimate of the parameter  $R_Y$ . In fact, even if there is no measurement data for F and this value has to be chosen randomly, due to the compensating properties of the estimation procedure the control performance of the suggested B-BAController remains very good.

# 5. CONCLUDING REMARKS

It can be stated that the B-BAController has been derived without any knowledge about the nonlinear description of the system. However, it still ensures very good control performance and robustness that are characteristic for the advanced adaptive modelbased techniques. On the other hand, the generality and simplicity of this methodology is comparable with the classical PI approach. The values of the parameter k and of the volume of the unit V can be chosen approximately or just randomly. In both cases the estimation procedure is able to compensate for the uncertainty resulted from their inaccurate values.

Let us also state that the suggested B-BAController is a very promising alternative in the practical control of the electric flow heaters, not only for the classical PI controller but also for more advanced control strategies due to its simplicity and generality as well as to its very good control performance. The form of the suggested control law is based on the very general and simplified model of the process, which ensures its robustness and generality. Moreover, in opposite to other more sophisticated model-based approaches, the simplicity of the control law ensures that it is very easy to implement (jointly with the estimation procedure) on the standard industrial PLC devices or as the PC-based virtual controller (Metzger, 2000; Czeczot, 2002a).

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