

SENSORLESS SPEED AND FLUX REGULATION OF INDUCTION MOTORS: A SLIDING MODE APPROACH

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Abstract: A novel sliding mode observer for current-based sensorless speed control of induction motors is presented in this paper. The control objective is to guarantee asymptotic tracking of prespecified speed and rotor flux references without mechanical sensors. To this end, an adaptive sliding mode speed and flux observer is introduced: it is based on a different and original approach with respect to the widely used equivalent control techniques. As regards the control algorithm, the problem of chattering, typical of sliding mode controllers, is overcome since the derivative of the stator currents are used as discontinuous forcing actions, while the actual control signals are continuous, thus limiting the mechanical stress. *Copyright*© 2005 IFAC

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1. INTRODUCTION

Sensorless control of motor drives has become more and more frequently used in industrial and practical application during recent years. This approach has the advantage of reducing the realization costs of the control system, thanks to the elimination of the sensors relevant to the mechanical variables (Dezza *et al.*, 1999). Yet, the problem of controlling the speed (torque) and the flux in induction motors is quite hard to solve. Generally, no flux sensors are provided, and flux observers convergence risks to be compromised by significant parameters values variations (the most critical, the rotor resistance, may change up to 200% of the nominal value), while the measurements of stator currents turn out to be affected by noise, due to electro-magnetic disturbances or harmonics. As a consequence, high performances and high robustness properties are required to the controller and the observers.

A great number of valid proposals of sensorless control schemes for induction motors have appeared in the literature recently (see, for instance, (Marino *et al.*, 1996a), (Kwon and Kim, 2004), and the references therein cited). In this context, the so-called sliding mode control design methodology, capable of guaranteeing high levels of robustness against matched disturbances and parameters variation, seems to be well applicable (Yan *et al.*, 2000), (Aurora *et al.*, 2001), (Derdiyok *et al.*, 2002), (Bartolini *et al.*, 2003), (Aurora and Ferrara, 2004).

In this paper, a novel adaptive sensorless sliding mode control strategy is presented. A classical current-fed induction motor control scheme, where the stator currents are assumed as control signals, is adopted, maintaining the conventional control loops and simply replacing the control algorithm. To avoid the problem of chattering, the time derivatives of the currents have been regarded as auxiliary control signals, while the

actual control signals are continuous. The key element of the proposed method is the coupling of original sliding mode speed and flux adaptive observers. Convergence of flux, speed, rotor time constant and load torque estimates to the real values is guaranteed. Simulations show that the observer-based control algorithm provides high regulation accuracy and appreciable robustness, even during a zero speed test.

2. MODEL OF THE INDUCTION MOTOR

In a fixed reference frame $a - b$, the fifth order induction motor model is defined by the following equations

$$\begin{cases} \frac{d\psi_a}{dt} = -\alpha\psi_a - \omega\psi_b + M\alpha i_a \\ \frac{d\psi_b}{dt} = -\alpha\psi_b + \omega\psi_a + M\alpha i_b \\ \frac{di_a}{dt} = -\beta\frac{d\psi_a}{dt} + \frac{1}{\sigma L_s}(u_a - R_s i_a) \\ \frac{di_b}{dt} = -\beta\frac{d\psi_b}{dt} + \frac{1}{\sigma L_s}(u_b - R_s i_b) \\ \frac{d\omega}{dt} = \frac{1}{J}\frac{M}{L_r}(\psi_a i_b - \psi_b i_a) - \frac{K_f}{J}\omega - \frac{\Gamma_l}{J} \end{cases} \quad (1)$$

where the state variables are the rotor speed ω , the rotor fluxes (ψ_a, ψ_b) and the stator currents (i_a, i_b) . Stator voltages (u_a, u_b) are the control signals, Γ_l is the load torque; J is the moment of inertia and K_f the friction coefficient; (R_r, R_s) and (L_r, L_s) are the rotor and stator windings resistances and inductances, respectively, and M is the mutual inductance. An induction motor with one pole pair is considered. To simplify notations, the following parameters have been introduced

$$\alpha = \frac{R_r}{L_r}, \quad \sigma = 1 - \frac{M^2}{L_s L_r}, \quad \beta = \frac{1}{\sigma} \frac{M}{L_s L_r}, \quad (2)$$

For current-fed induction motors with high-gain current loops the motor control algorithm can be constructed on the basis of the following reduced order motor model

$$\begin{cases} \frac{d\psi_a}{dt} = -\alpha\psi_a - \omega\psi_b + M\alpha i_a \\ \frac{d\psi_b}{dt} = -\alpha\psi_b + \omega\psi_a + M\alpha i_b \\ \frac{d\omega}{dt} = \frac{1}{J}\frac{M}{L_r}(\psi_a i_b - \psi_b i_a) - \frac{K_f}{J}\omega - \frac{\Gamma_l}{J} \end{cases} \quad (3)$$

by considering the stator currents (i_a, i_b) as control inputs, the rotor fluxes (ψ_a, ψ_b) as the state variables, Γ_l as an external input, and α as an unknown parameter (depending on the rotor resistance value). The quantities $\omega_r(t)$ and $\Psi_r^2(t)$ are the reference signals for the rotor speed and the square modulus of the rotor

flux $\Psi^2 = \psi_a^2 + \psi_b^2$, respectively¹. Then, the tracking errors $\tilde{\omega}$ and $\tilde{\Psi}$ can be defined as

$$\begin{cases} \tilde{\omega} = \omega - \omega_r \\ \tilde{\Psi} = \Psi^2 - \Psi_r^2 = \psi_a^2 + \psi_b^2 - \Psi_r^2 \end{cases} \quad (4)$$

such that their time derivatives are

$$\begin{cases} \dot{\tilde{\omega}} = \frac{1}{J}\frac{M}{L_r}(\psi_a i_b - \psi_b i_a) - \frac{K_f}{J}\omega - \frac{\Gamma_l}{J} - \dot{\omega}_r \\ \dot{\tilde{\Psi}} = -2\alpha(\psi_a^2 + \psi_b^2) \\ \quad + 2M\alpha(\psi_a i_a + \psi_b i_b) - 2\Psi_r \dot{\Psi}_r \end{cases} \quad (5)$$

3. THE PROPOSED STATE FEEDBACK SLIDING MODE CONTROL

Among the various sliding mode control solutions for induction motors proposed in the literature, the one presented in (Utkin, 1992) can be regarded as the reference one. It is briefly recalled in (Aurora and Ferrara, 2004). Its purpose is to directly control the inverter switching by use of three switching reference signals for the stator voltages.

For current-fed induction motors, the stator currents can be regarded as control variables. In order to overcome the problem of chattering, a sliding mode control algorithm can be designed by considering their time derivatives as control inputs.

It is first necessary to derive the sliding functions to impose the desired behavior of speed and flux errors. To this end, let

$$\begin{cases} s_1 = k_\omega \tilde{\omega} + \dot{\tilde{\omega}} \\ s_2 = k_\Psi \tilde{\Psi} + \dot{\tilde{\Psi}} \end{cases} \quad (6)$$

with the dynamics of $s^T = (s_1, s_2)$ described by

$$\frac{ds}{dt} = F + D\dot{i} \quad (7)$$

where $\dot{i}^T = (\dot{i}_a, \dot{i}_b)$ is the two dimensional control. The components of vector $F^T = (f_1, f_2)$ may be regarded as bounded disturbances, which are in turn continuous functions of motor parameters, speed, rotor fluxes, reference signals and of their first and second time derivatives.

$$D = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \begin{bmatrix} \frac{1}{J}\frac{M}{L_r} & 0 \\ 0 & 2\alpha M \end{bmatrix} \begin{bmatrix} -\psi_b & \psi_a \\ \psi_a & \psi_b \end{bmatrix} \quad (8)$$

with k_1 and k_2 positive constants. By transforming the sliding functions through the use of the matrix $\Omega = D^{-1}$, and remembering that matrix Ω exists

¹ To allow for correct operation of the control algorithm, the first and second time derivatives of the speed and flux references are assumed to be bounded.

when $\|\Psi\| \neq 0$, the state motion on the subspace $s^* = 0$ turns out to be characterized by the equation

$$\frac{ds^*}{dt} = \Omega F + \frac{d\Omega}{dt} D s^* + i \quad (9)$$

By choosing the switching control law

$$i = -i_0 \text{sign}(s^*) \quad (10)$$

for sufficiently high values of the design parameter i_0 the objective of reaching the manifold $s^* = 0$ in finite time is attained. In contrast to the nonlinear output feedback control scheme presented in (Marino *et al.*, 1996b), the proposed sliding mode control algorithm does not require the knowledge of the load torque (and of the friction coefficient as well), but only the knowledge of an upper bound of it.

4. THE SLIDING MODE ADAPTIVE SPEED AND FLUX OBSERVER

In contrast to (Yan *et al.*, 2000) and (Derdiyok *et al.*, 2002), the adaptive speed and flux observer here proposed does not rely on the equivalent control method (Utkin, 1992) according to which unknown quantities are obtained by filtering a discontinuous signal. Indeed, nonlinear speed and flux estimation laws are driven by signals computed by resorting to an auxiliary sliding mode current observer.

First, let us suppose that a flux observer is available

$$\frac{d\hat{\psi}_a}{dt} = f_{\psi_a}, \quad \frac{d\hat{\psi}_b}{dt} = f_{\psi_b} \quad (11)$$

where f_{ψ_a} and f_{ψ_b} are functions that will be defined later. Now, let us design a sliding mode current observer

$$\begin{cases} \frac{d\hat{i}_a}{dt} = -\beta f_{\psi_a} + \frac{u_a - R_s \hat{i}_a}{\sigma L_s} - K_i \text{sign}(\tilde{i}_a) \\ \frac{d\hat{i}_b}{dt} = -\beta f_{\psi_b} + \frac{u_b - R_s \hat{i}_b}{\sigma L_s} - K_i \text{sign}(\tilde{i}_b) \end{cases} \quad (12)$$

where, according to (Utkin, 1992), vanishing of the estimate errors $\tilde{i}_a = \hat{i}_a - i_a$ and $\tilde{i}_b = \hat{i}_b - i_b$ is ensured by sufficiently high gain $K_i > 0$ of the discontinuous signal, introduced to enforce a sliding mode behavior. By considering the estimate errors dynamics

$$\begin{cases} \frac{d\tilde{i}_a}{dt} = -\beta \frac{d\tilde{\psi}_a}{dt} - K_i \text{sign}(\tilde{i}_a) \\ \frac{d\tilde{i}_b}{dt} = -\beta \frac{d\tilde{\psi}_b}{dt} - K_i \text{sign}(\tilde{i}_b) \end{cases} \quad (13)$$

analogously to (Marino *et al.*, 1996b), auxiliary quantities are introduced

$$z_a = \tilde{i}_a + \beta \tilde{\psi}_a, \quad z_b = \tilde{i}_b + \beta \tilde{\psi}_b \quad (14)$$

which exhibit the dynamics

$$\frac{dz_a}{dt} = -K_i \text{sign}(\tilde{i}_a), \quad \frac{dz_b}{dt} = -K_i \text{sign}(\tilde{i}_b) \quad (15)$$

and reconstruction of the fluxes estimate errors $\tilde{\psi}_a = \hat{\psi}_a - \psi_a$ and $\tilde{\psi}_b = \hat{\psi}_b - \psi_b$ related to (11) turns out to be feasible, i.e.

$$\tilde{\psi}_a = \frac{1}{\beta} (z_a - \tilde{i}_a), \quad \tilde{\psi}_b = \frac{1}{\beta} (z_b - \tilde{i}_b) \quad (16)$$

Relying on knowledge of variables $\tilde{\psi}_a$ and $\tilde{\psi}_b$, it is possible to define functions $f_{\tilde{\psi}_a}$ and $f_{\tilde{\psi}_b}$ so that the flux observer (11) can be rewritten as

$$\begin{cases} \frac{d\hat{\psi}_a}{dt} = -\hat{\alpha} \hat{\psi}_a - \hat{\omega} \hat{\psi}_b + M \hat{\alpha} i_a - K_\psi \tilde{\psi}_a \\ \frac{d\hat{\psi}_b}{dt} = -\hat{\alpha} \hat{\psi}_b + \hat{\omega} \hat{\psi}_a + M \hat{\alpha} i_b - K_\psi \tilde{\psi}_b \end{cases} \quad (17)$$

where $K_\psi \geq 0$, while $\hat{\omega}$ and $\hat{\alpha}$ are estimated by

$$\begin{cases} \frac{d\hat{\omega}}{dt} = \frac{1}{J} \frac{M}{L_r} (\hat{\psi}_a i_b - \hat{\psi}_b i_a) \\ \quad - \frac{K_f \hat{\omega} + \Gamma_l}{J} + f_\omega \\ \frac{d\hat{\alpha}}{dt} = f_\alpha \end{cases} \quad (18)$$

with f_ω and f_α additional terms, to be defined, introduced to impose the desired behavior to the estimation errors $\tilde{\omega} = \hat{\omega} - \omega$ and $\tilde{\alpha} = \hat{\alpha} - \alpha$, that is

$$\begin{cases} \frac{d\tilde{\omega}}{dt} = \frac{1}{J} \frac{M}{L_r} (\tilde{\psi}_a i_b - \tilde{\psi}_b i_a) - \frac{K_f}{J} \tilde{\omega} + f_\omega \\ \frac{d\tilde{\alpha}}{dt} = f_\alpha \end{cases} \quad (19)$$

Note that, as previously mentioned, two key assumptions have been taken into account: as in (Marino *et al.*, 1996b), α is regarded as an unknown but constant quantity, while the load torque is supposed to be known, as assumed in (Marino *et al.*, 2002). Relying on (17), the flux estimate errors dynamics turns out to be

$$\begin{cases} \frac{d\tilde{\psi}_a}{dt} = -\tilde{\alpha} \hat{\psi}_a - \tilde{\omega} \hat{\psi}_b - \alpha \tilde{\psi}_a - \omega \tilde{\psi}_b \\ \quad + M \tilde{\alpha} i_a - K_\psi \tilde{\psi}_a \\ \frac{d\tilde{\psi}_b}{dt} = -\tilde{\alpha} \hat{\psi}_b + \tilde{\omega} \hat{\psi}_a - \alpha \tilde{\psi}_b + \omega \tilde{\psi}_a \\ \quad + M \tilde{\alpha} i_b - K_\psi \tilde{\psi}_b \end{cases} \quad (20)$$

Now, it is possible to select the following Lyapunov function

$$V_\omega = \frac{1}{2} \left\{ \tilde{\psi}_a^2 + \tilde{\psi}_b^2 + \frac{1}{\gamma_\omega} \tilde{\omega}^2 + \frac{1}{\gamma_\alpha} \tilde{\alpha}^2 \right\} \quad (21)$$

in which $\gamma_\omega > 0$ and $\gamma_\alpha > 0$.

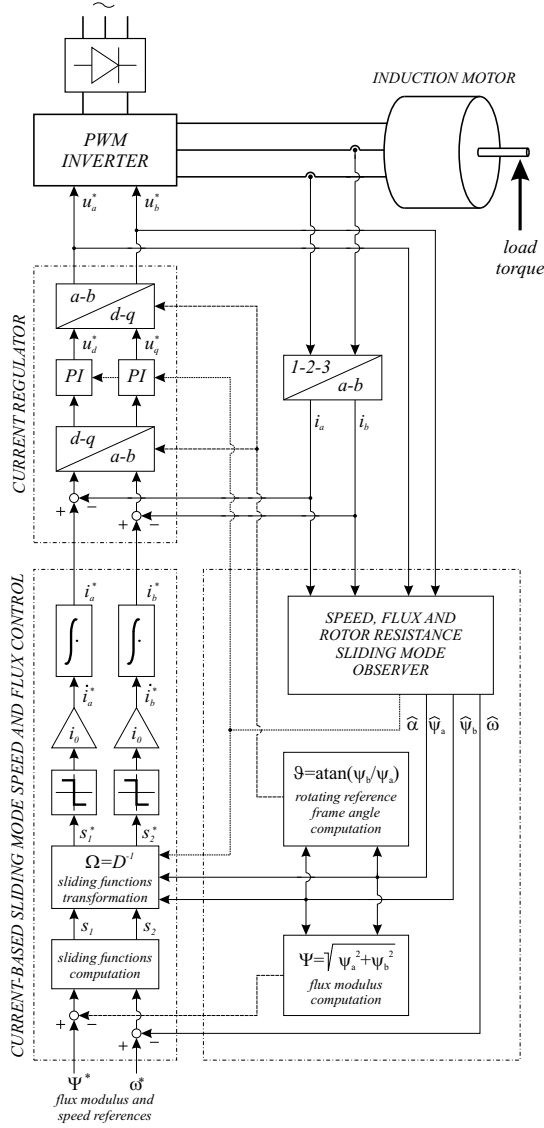


Fig. 1. The proposed induction motor control scheme.

By developing calculations, it is easy to prove that to guarantee $\dot{V}_\omega \leq 0$ one can select $K_\psi = 0$ and

$$f_\omega = -\frac{1}{J} \frac{M}{L_r} (\tilde{\psi}_a i_b - \tilde{\psi}_b i_a) + \gamma_\omega (\tilde{\psi}_a \hat{\psi}_b - \tilde{\psi}_b \hat{\psi}_a) \quad (22)$$

$$f_\alpha = \gamma_\alpha (\tilde{\psi}_a (\hat{\psi}_a - M i_a) + \tilde{\psi}_b (\hat{\psi}_b - M i_b))$$

Equations (18), with additional terms f_ω and f_α defined in (22), provide an estimation law for the mechanical speed ω and an adaptive law for the unknown parameter α . Convergence of the flux observer (17) is so guaranteed. Fig. 4 shows the proposed induction motor control scheme with full details.

4.1 Some comments on convergence

To guarantee the convergent behavior of the update law of α , it is necessary to show that the signal which drives such a law verifies the persistency of excitation

condition (Narendra and Annaswamy, 1989). To this aim, the second equation in (18) can be put in the form

$$\begin{aligned} \frac{d\hat{\alpha}}{dt} &= \gamma_\alpha \left[(\hat{\psi}_a - M i_a) (\hat{\psi}_b - M i_b) \right] \begin{bmatrix} \tilde{\psi}_a \\ \tilde{\psi}_b \end{bmatrix} \\ &= \gamma_\alpha \Gamma(t) \begin{bmatrix} \tilde{\psi}_a \\ \tilde{\psi}_b \end{bmatrix} \end{aligned} \quad (23)$$

Persistency of excitation is guaranteed provided that

$$\int_t^{t+T} \Gamma(\tau) \Gamma^T(\tau) d\tau \quad (24)$$

is positive definite for $T > 0$ and for any $t \geq 0$. This condition is satisfied, since it results $\forall t \geq 0$

$$\int_t^{t+T} \left((\hat{\psi}_a - M i_a)^2 + (\hat{\psi}_b - M i_b)^2 \right) d\tau \geq 0 \quad (25)$$

A similar analysis may be performed about ω .

5. LOAD TORQUE ESTIMATION

Except for a limited field of applications, the value of the load torque, which needs to be known for the speed and flux observer described in the previous section, is usually unavailable. Thus, a load torque observer has been developed and tested in simulation. Although, at the present time, the proposed estimation law is under theoretical investigation, good convergence properties have been proved in simulative tests, combining the load torque estimator with the sliding mode adaptive observer (12) (17) (18).

According to the Field Oriented Control approach (Blaschke, 1972), the induction machine model (1) can be also described in a rotating reference frame of axes $d - q$, where the axis d is oriented like the rotor flux vector. Let ϑ_s be the angle between the rotating d axis and the fixed a axis

$$\vartheta_s = \arctan \frac{\psi_b}{\psi_a} \quad (26)$$

The components i_d and i_q of the stator current are directly proportional to the flux magnitude and to the motor torque, respectively, and can be computed as

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos \vartheta_s & \sin \vartheta_s \\ -\sin \vartheta_s & \cos \vartheta_s \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} \quad (27)$$

The load torque observer here proposed has been designed on the basis of a simulative analysis, in the $d - q$ reference frame, of the transient behavior of the Sliding Mode Observer (12) (17) (18) when the induction motor is loaded but the load torque information is missing. The adopted estimation law is based on a sort of i_q current estimation error

$$\frac{d\hat{\Gamma}_l}{dt} = \gamma_\Gamma \left((\hat{i}_a - i_a) \sin \hat{\vartheta}_s - (\hat{i}_b - i_b) \cos \hat{\vartheta}_s \right) \quad (28)$$

where $\gamma_T > 0$ and

$$\hat{\vartheta}_s = \arctan \frac{\hat{\psi}_b}{\hat{\psi}_a} \quad (29)$$

It guarantees good convergence performances if combined with the sensorless adaptive sliding mode observer previously introduced, as confirmed by simulations.

6. SIMULATION EXAMPLES

6.1 Simulation setup

To validate the proposed control algorithm (10) and the speed and flux adaptive observer, simulations have been carried out by use of Matlab and Simulink, adopting the same parameters of the experimental setup shown in (Marino *et al.*, 1996b), in which a 600 W one pole pair induction motor with a rated speed of 1000 rpm is used.

The main purposes were to inspect both performances and robustness properties in reference tracking and observation accuracy. About the control algorithm, another task is to verify that the limit imposed to the maximum value of the stator current time derivatives does not compromise the dynamical performances during transients.

The speed and flux modulus references and the load torque profile are shown in Fig. 6.1: both the first and the second time derivatives of speed and flux reference signals are bounded. The simulation here shown has been carried out with a value of the rotor resistance equal to 200% of the nominal one.

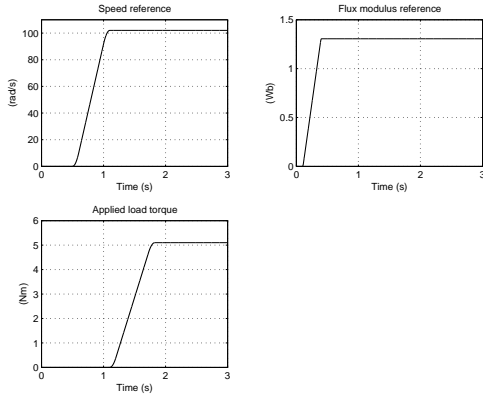


Fig. 2. Speed and rotor flux reference signals and load torque profile in simulations.

6.2 Simulation results

The reference signals are indicated in Fig. 2, while the controlled motor performances are shown in Fig. 3. Fig. 4 reports, in a restricted time interval, both the

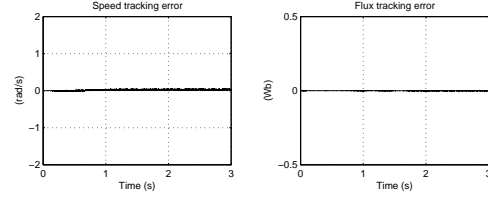


Fig. 3. Speed and rotor flux modulus tracking errors.

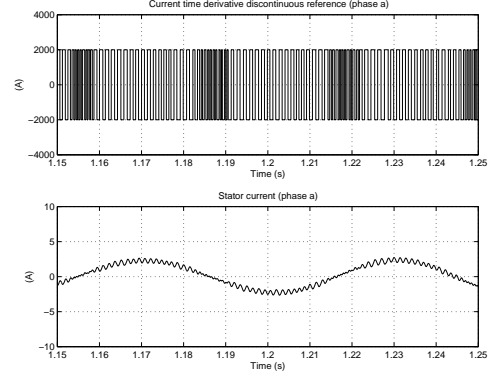


Fig. 4. The switching control input, i.e. the time derivative of the stator current (phase a) and the measured current i_a , showing the high harmonics filtering.

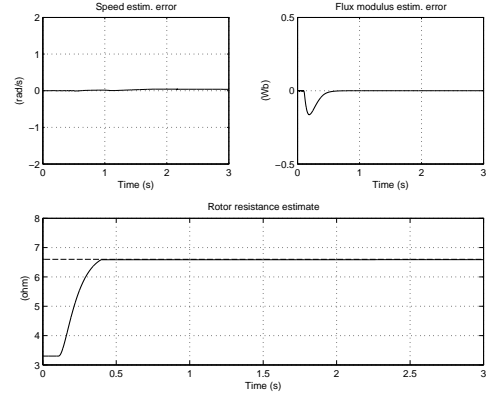


Fig. 5. Speed and flux modulus tracking performances, and rotor resistance estimation, with the new adaptive sensorless sliding mode observer.

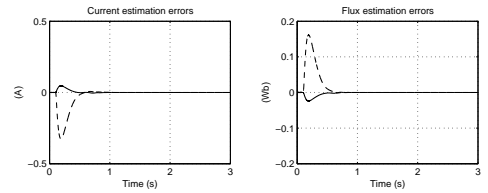


Fig. 6. Stator currents and rotor fluxes components estimate errors (phase a , solid lines; phase b , dashed lines).

discontinuous waveform (phase a) of the control signal, and the stator current i_a of the motor, thus letting us appreciate the filtering action of the integrators, of the current loop and of the same motor. Moreover, the limit imposed on the time derivative of the stator

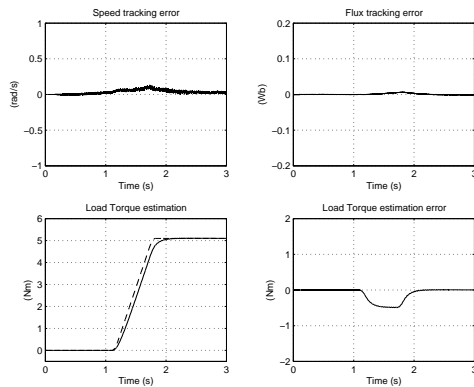


Fig. 7. Speed and flux tracking errors and load torque estimation.

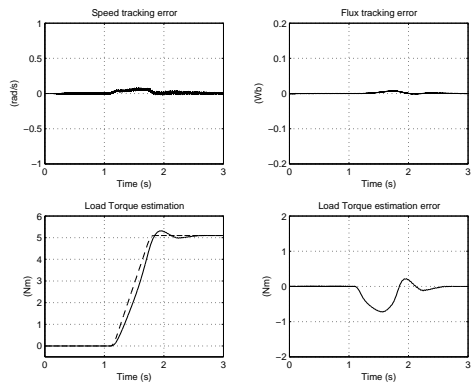


Fig. 8. Speed and flux tracking errors and load torque estimation during a zero speed test.

current (2000 A/s) does not affect negatively the speed regulation during load torque transients.

If the load torque is known, the adaptive sliding mode speed and flux observer proves to be fast and accurate: the previous analysis demonstrates that speed and flux regulation seems not to be affected by the presence of the observer in the control scheme. Fig. 5 shows that the speed and flux modulus estimation performances are satisfactory, even before that the convergence of the rotor resistance estimation is verified, proving good robustness of the speed and flux observer. Both current and flux estimate errors are quickly steered to zero, as shown in Fig. 6.

Fig. 7 shows the simulation results when the load torque is unknown and the observer (28) is introduced in the control scheme.

Excellent performances are highlighted in a zero speed test too, a crucial benchmark for induction motor control algorithms (see Fig. 8).

7. CONCLUSIONS

In this paper a new Adaptive Sensorless Sliding Mode Control algorithm for induction motors is proposed. The control strategy sets a limit to the maximum value of the time derivatives of the stator currents, assumed

as discontinuous control inputs, in order to prevent excessive mechanical stress of the machine. The novel Adaptive Sliding Mode Speed and Flux Observer, based on a robust stator current estimation, provides a fast and precise adaptation of both the mechanical speed and the rotor resistance.

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